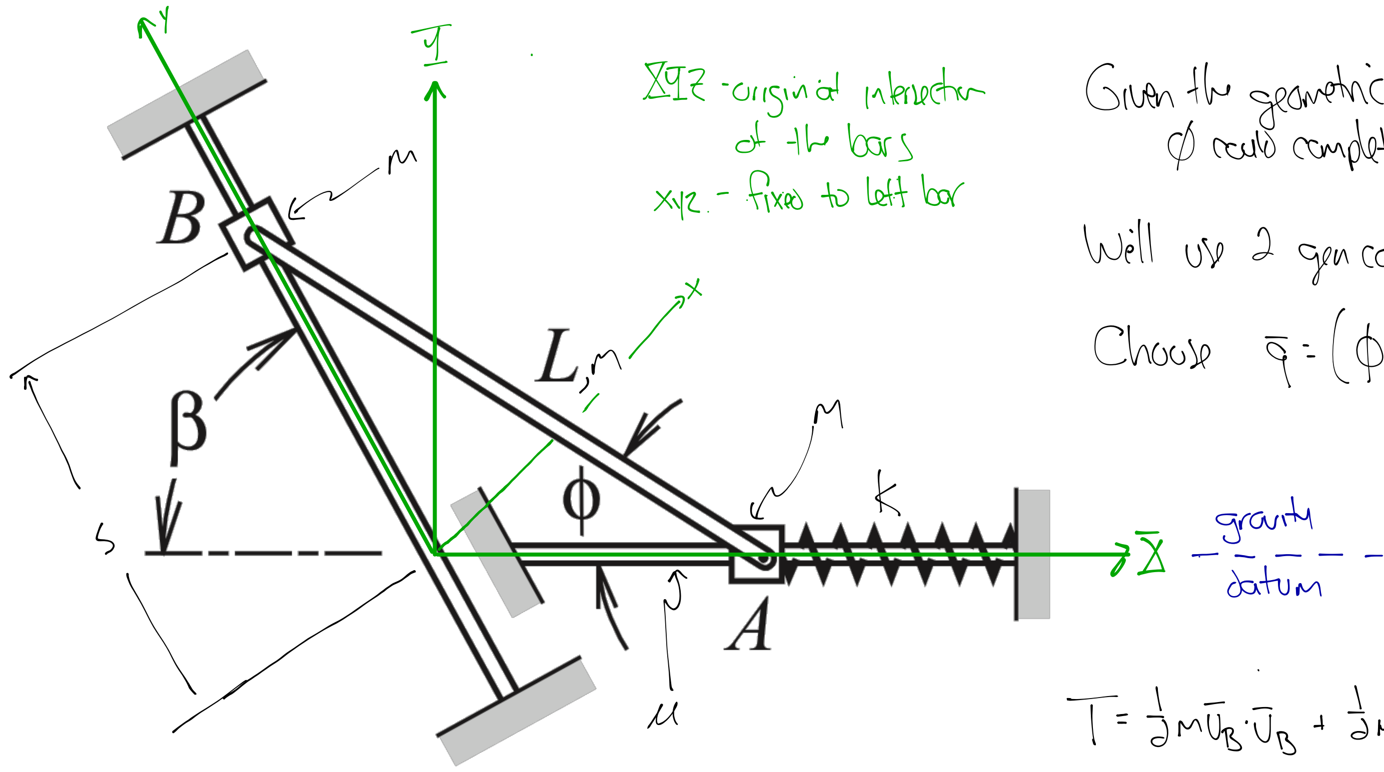
Example 8.7

EXAMPLE 8.7 The coefficient of sliding friction between collar A and its horizontal guide is μ, but friction between collar B and the inclined guide is negligible. The spring, whose stiffness is k, is unstretched when $\phi = 0$. The bar and each collar have identical mass m. Determine the equations of motion of the system. Then consider a specific case in which $kL/mg = 4$, $\beta = 60^{o}$, and the coefficient of sliding friction is $\mu = 0.25$. The bar is released from rest at $\phi = 30^{o}$. Determine the elapsed time until the bar comes to rest and the mechanical energy that is dissipated by friction in that interval.

Well do this part here. See the book for the remainder.

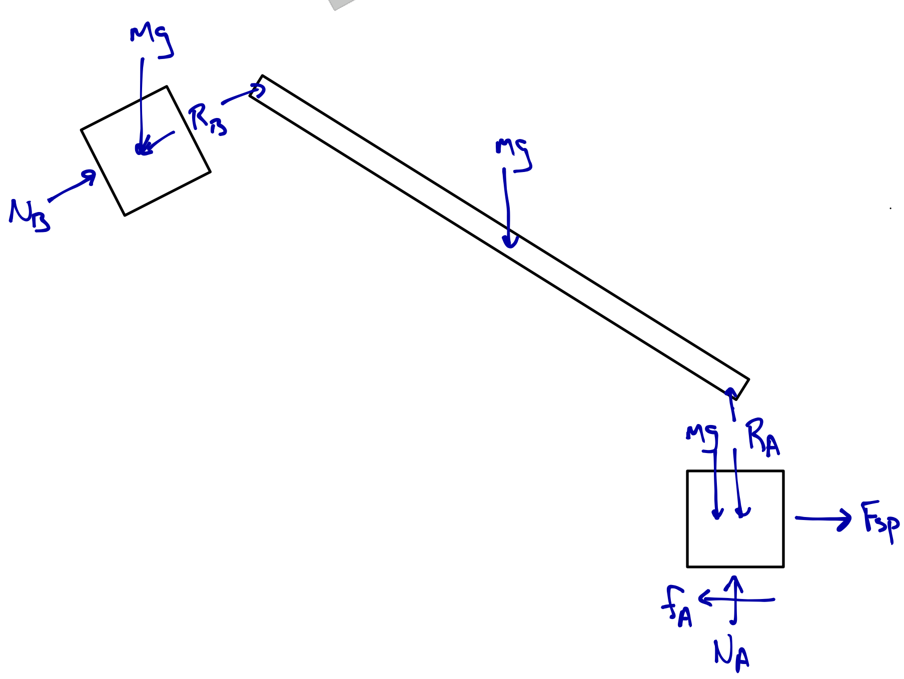


Given the geometric, system has only 1DOF
φ could completely describe the sys. config.

Will use 2 gen coord to account for friction

Choose $\bar{q} = (\phi, s)$

$$T = \frac{1}{2} m \bar{v}_B \cdot \bar{v}_B + \frac{1}{2} m \bar{v}_A \cdot \bar{v}_A + \frac{1}{2} m \bar{v}_G \cdot \bar{v}_G + \frac{1}{2} I_{zz} \dot{\phi}^2$$



$$\bar{v}_B = \dot{s}\bar{j} = \dot{s}(-\cos\beta \bar{I} + \sin\beta \bar{J})$$

$\bar{v}_A = v_A \bar{I}$ ← we need to write this in terms of generalized coords

Q: How?

$$\bar{v}_A = \bar{v}_B + \bar{\omega} \times \bar{r}_{A/B}$$

$$\bar{\omega} = -\dot{\phi}\bar{K} \qquad \bar{r}_{A/B} = L\cos\phi \bar{I} - L\sin\phi \bar{J}$$

$$\bar{v}_A = \dot{s}(-\cos\beta \bar{I} + \sin\beta \bar{J}) + (-L\dot{\phi}\cos\phi \bar{J} - L\dot{\phi}\sin\phi \bar{I})$$

$$\bar{v}_A = (-\dot{s}\cos\beta - L\dot{\phi}\sin\phi)\bar{I} + (\dot{s}\sin\beta - L\dot{\phi}\cos\phi)\bar{J}$$

Similarly,

$\bar{v}_G = \bar{v}_B + \bar{\omega} \times \bar{r}_{G/B}$ ← The only difference from $\bar{v}_A$ is that $\bar{r}_{G/B} = \frac{1}{2}\bar{r}_{A/B}$.

So

$$\bar{v}_A = \left(-\dot{s}\cos\beta - \frac{L}{2}\dot{\phi}\sin\phi\right)\bar{I} + \left(\dot{s}\sin\beta - \frac{L}{2}\dot{\phi}\cos\phi\right)\bar{J}$$

Example 8.7 (cont.)

$V = V_{grav} + U_{sp}$ $\qquad$ $V_{grav} = V_B + V_{bar} + V_A$ $\qquad$ $V_B \equiv$ grav. potential of collar B
$V_{bar} \equiv$ grav. potential of bar
$V_A \equiv$ grav. potential of collar A

$$V_B = mg\left(s\sin\beta\right)$$

$$V_{bar} = mg\left(s\sin\beta - \frac{L}{2}\sin\phi\right)$$

$$V_A = mg\left(s\sin\beta - L\sin\phi\right)$$

← Since we're using constrained coords, we have to include potential $\bar{J}$ direction motion of A

$V_{sp} = \frac{1}{2}k\Delta^2$ where Δ is deflection from spring equil.

spring is a equil when $\phi = 0$

To find $\bar{r}_{A/O}$ in terms of s and ϕ, write

$$\bar{r}_{A/O} = \bar{r}_{B/O} + \bar{r}_{A/B} = \left(-s\cos\beta\,\bar{I} + s\sin\beta\,\bar{J}\right) + \left(L\cos\phi\,\bar{I} - L\sin\phi\,\bar{J}\right)$$

$$= \left(-s\cos\beta + L\cos\phi\right)\bar{I} + \left(s\sin\beta - L\sin\phi\right)\bar{J}$$

Δ is the change in this distance from spring equil in I direction

at $\phi = 0$, $\bar{r}_{A/O}(\phi=0) = \left(-\cancel{s}^{0}\cos\beta + L\right)\bar{I} + \left(\cancel{s}^{0}\sin\beta\right)\bar{J}$ $\qquad$ at $\phi = 0$, $s = 0$

so $\Delta = \left(\bar{r}_{A/O}(\phi=0) - \bar{r}_{A/O}\right)\cdot\bar{I}$

$$= L - L\cos\phi + s\cos\beta$$

$$V_{sp} = \frac{1}{2}k\left(L(1-\cos\phi) + s\cos\beta\right)^2$$

$\mathcal{L} = T - U$

The only nonconservative forces on this system are N_A and f_A

$$\delta W = \left(-f_A\bar{I} + N_A\bar{J}\right)\cdot\delta\bar{r}_A$$

$$\bar{v}_A = \left(-\dot{s}\cos\beta - L\dot{\phi}\sin\phi\right)\bar{I} + \left(\dot{s}\sin\beta - L\dot{\phi}\cos\phi\right)\bar{J}$$

so

$$\delta\bar{r}_A = \left(-\cos\beta\,\delta s - L\sin\phi\,\delta\phi\right)\bar{I} + \left(\sin\beta\,\delta s - L\cos\phi\,\delta\phi\right)\bar{J}$$

$$\delta W = -f_A\left(\cos\beta\,\delta s - L\sin\phi\,\delta\phi\right) + N_A\left(\sin\beta\,\delta s - L\cos\phi\,\delta\phi\right) = \left(+f_A L\sin\phi - N_A L\cos\phi\right)\delta\phi + \left(f_A\cos\beta + N_A\sin\beta\right)\delta s$$

Example 8.7 (cont.)

$$\delta W = \underbrace{\left(f_A L \sin\phi - N_A L \cos\phi\right)}_{Q_1} \delta\phi + \underbrace{\left(f_A \cos\beta \cdot N_A \sin\beta\right)}_{Q_2} \delta s$$

We now need to characterize the constraint

We can get a config. constraint from the geometry (that forces A onto lower bar)

If A is on the horizontal bar

$$s \sin\beta = L \sin\phi \quad \rightarrow \quad s\sin\beta - L\sin\phi = 0$$ ← config constraint

$$\boxed{\dot{s}\sin\beta - L\dot{\phi}\cos\phi = 0}$$ ← differentiate to get vel. constraint

We can also write f_A in terms of N_A

$$f_A = \mu |N_A| \,\mathrm{sgn}\left(v_A \cdot \bar{I}\right)$$

Now write Lagrange's equations for $q = (\phi, s)$

$$\boxed{\begin{aligned} \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}}\right) - \frac{\partial \mathcal{L}}{\partial \phi} &= Q_1 \\ \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{s}}\right) - \frac{\partial \mathcal{L}}{\partial s} &= Q_2 \end{aligned}}$$

The equations in the orange boxes describe the dynamics of this system.