Impulse-Momentum Principles (Sec. 6.4.1)

Linear – $\bar{P}_2 = \bar{P}_1 + \int_{t_1}^{t_2} \Sigma \bar{F} dt$ $\quad P_i \equiv$ linear momentum at time t_i

Rotational – $\underbrace{(H_A)_2 = (H_{A_1}) + \int_{t_1}^{t_2} \Sigma \bar{M}_A dt}_{\text{Both of these are vector equations}}$ $\quad (H_A)_i$ = angular momentum at time t_i

These relationships are really useful when impulses act on a body

Assume the impulsive force/moment is nearly constant over a small time range,

$t_0^- \to t_0^+$

We can write the time range to analyze the response to the impulse as

$t_i < t < t_i + \Delta t$

Find

$$\underbrace{\bar{P}(t = t_i^+)}_{\substack{\text{Momentum}\\\text{right after}\\\text{impulse}}} = \underbrace{P(t = t_i^-)}_{\substack{\text{Momentum}\\\text{right before}\\\text{impulse}}} + \underbrace{(\Sigma \bar{F})_{imp} \Delta t}_{\text{Impulse}}$$

and

$$\underbrace{\bar{H}_A(t = t_i^+)}_{\substack{\text{Momentum}\\\text{right after}\\\text{impulse}}} = \underbrace{\bar{H}_A(t = t_i^-)}_{\substack{\text{Momentum}\\\text{right before}\\\text{impulse}}} + \underbrace{(\Sigma \bar{M}_A)_{imp}}_{\text{Impulse}} \Delta t$$

Because $\Delta t \approx 0$, let $\bar{r}_{P/O}(t = t_i^+) = \bar{r}_{P/O}(t = t_i^-)$

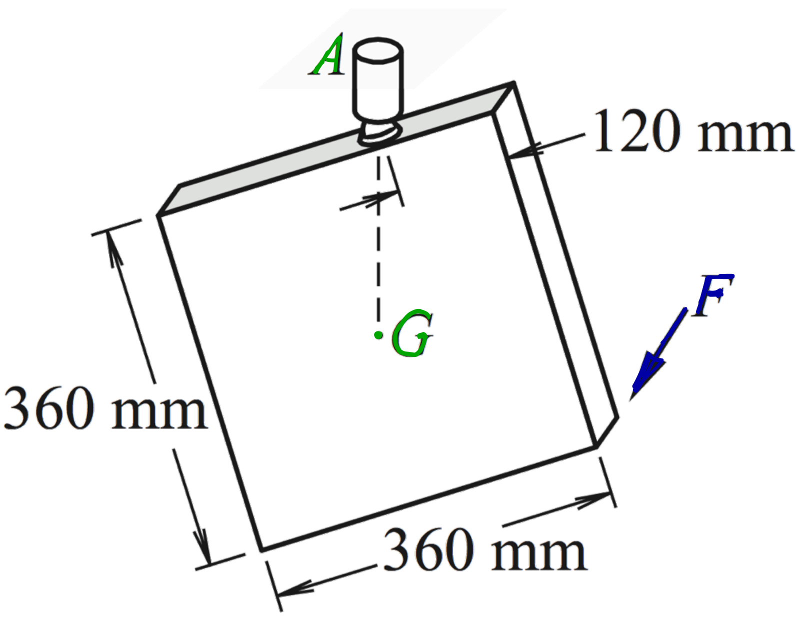
Example 6.11

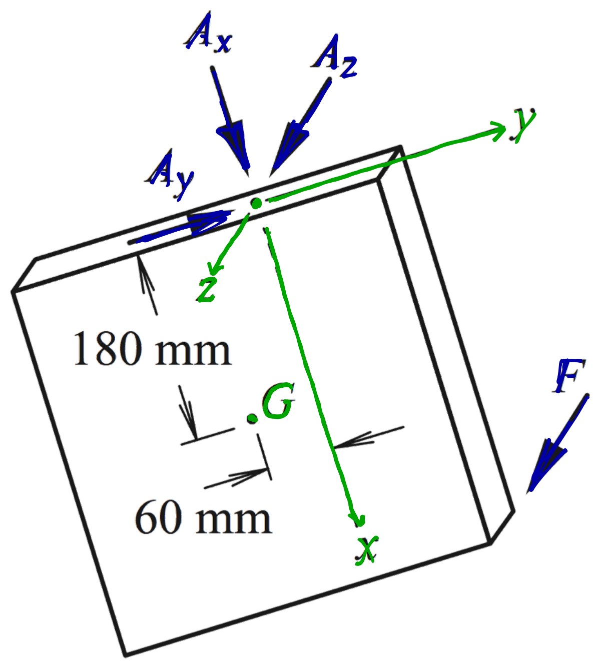
EXAMPLE 6.11 A 10-kg square plate suspended by ball-and-socket joint A is at rest when it is struck by a hammer. The impulsive force $\bar{F}$ generated by the hammer is normal to the surface of the plate, and its average value during the 4-ms interval that it acts is 5000 N. Determine the angular velocity of the plate at the instant following the impact and the average reaction at the support.

$F >> mg$ so ignore gravity

Sum moments about A (ignore moments from reaction)

Define Inertia prop. about A





Initial velocity – $v_1 = 0$

Initial angular velocity – $\omega_1 = 0$

Find angular velocity – $\omega_2 = \omega_x \bar{i} + \omega_y \bar{j} + \omega_z \bar{k}$

Velocity of com is then:

$$\bar{v}_G = \overset{0}{\cancel{\bar{v}_A}} + \overset{0}{\cancel{(\bar{v}_G)_{xyz}}} + \bar{\omega} \times \bar{r}_{G/A} \qquad \bar{r}_{G/A} = 0.18\bar{i} - 0.06\bar{j}$$

$$= (\omega_x \bar{i} + \omega_y \bar{j} + \omega_z \bar{k}) \times (0.18\bar{i} - 0.06\bar{j})$$

$$= -0.06\omega_x \bar{k} - 0.18\omega_y \bar{k} + 0.18\omega_z \bar{j} + 0.06\omega_z \bar{i} = 0.06\omega_z \bar{i} + 0.18\omega_z \bar{j} + (-0.06\omega_x - 0.18\omega_y)\bar{k}$$

Find angular momentum at time 2

Rot. Impulse/Mom.

$(H_A)_2$ ← Get $(H_A)_2$ from angular velocity at time 2 ← Equate components from these

It also $= \Sigma \bar{M}_A \Delta t$

$$\Sigma \bar{M}_A = (0.36\bar{i} + 0.12\bar{j}) \times F k \qquad \Delta t = 0.004 s$$

$$m(\bar{v}_2) = \Sigma \bar{F} \Delta t$$

$$\Sigma \bar{F} = A_x \bar{i} + A_y \bar{j} + (A_z + F)\bar{k}$$

Use these to solve for Reaction forces

Q: What direction with A_z be?

Same as $\bar{F}$!

Chapter 7 - Introduction to Analytical Mechanics

Based in energy, rather than forces

Virtual Work (Sec. 7.1.1)

Requires introduction of virtual movement (virtual displacements and rotations)

virtual movement - infinitessimally small motion (occur in zero time)

denote virtual displacement of P as $\delta \bar{r}_P$

virtual rotation of θ as $\delta \bar{\theta}$

So, work done by forces acting at point P is $\Sigma \bar{F} \cdot \delta \bar{r}_P$

moment causing θ is $\Sigma M_P \cdot \delta \bar{\theta}$

Principle of Virtual Work

$$\delta W = \sum_j \bar{F}_j \cdot \delta \bar{r}_j + \sum_n \bar{M}_n \cdot \delta \bar{\theta}_n = 0$$ In static equil. this is true.

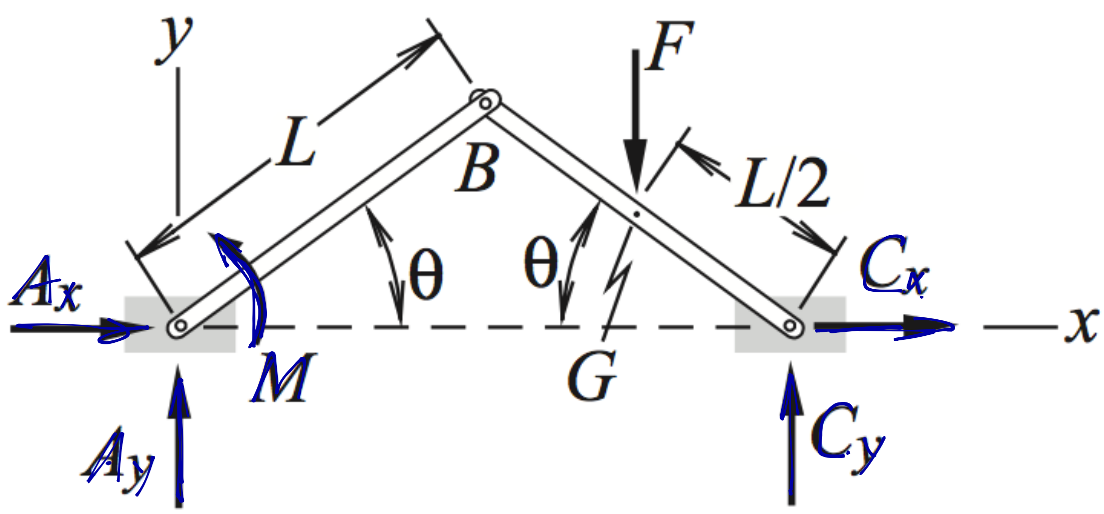
$\delta \bar{r}_j$ = virtual displacement of point of application of F_j

$\delta \theta_n$ = virtual rotation of body on which $\bar{M}_n$ acts

Q: Why/how is this useful? ... Hold on until next week. (mostly)

Virtual Work (cont.)

For virtual work, we can select virtual movements that only involve the forces we're interested in.



In this case, we want to find Cx, but want to avoid Ax, Ay, and Cy

Choose virtual movement such that A is stationary and C moves horizontally.

This movement causes θ to change. Write the virtual rotation as:

$$\delta\theta$$

So, the virtual work by the moment M is $M\delta\theta$

We also need to know the work done by F and Cx

So, we need to find the virtual displacements of their points of application.

We can do this by finding their normal displacements.

$$\bar{r}_{C/A} = 2L\cos\theta\,\hat{\imath}$$

$$\bar{r}_{G/A} = \frac{3}{2}L\cos\theta\,\hat{\imath} + \frac{1}{2}L\sin\theta\,\hat{\jmath}$$

then calculating virtual displacements

$$\delta\bar{r}_{C/A} = \frac{\partial r_{C/A}}{\partial\theta}\delta\theta = \left(-2L\sin\theta\,\hat{\imath}\right)\delta\theta$$

$$\delta\bar{r}_{G/A} = \frac{\partial r_{G/A}}{\partial\theta}\delta\theta = \left(\frac{3}{2}L\cos\theta\,\hat{\imath} + \frac{1}{2}L\sin\theta\,\hat{\jmath}\right)\delta\theta$$

So, $\delta W = \bar{M}\delta\bar{\theta} + \left[C_x\hat{\imath}\cdot\delta\bar{r}_{C/A}\right] + \left[-F\hat{\jmath}\cdot\delta\bar{r}_{G/A}\right] = 0$ ← still only looking at static case!!!

Can solve this to find Cx.

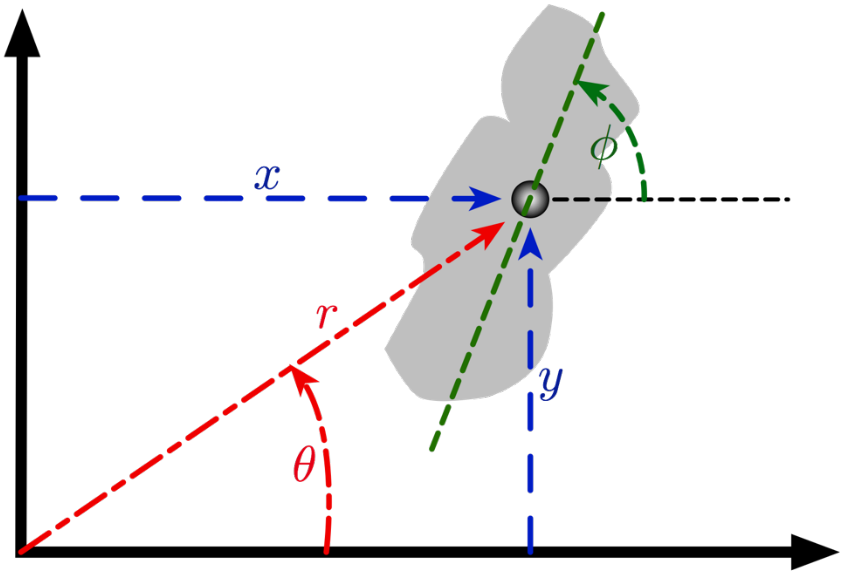
Dynamic Virtual Work (Sec. 7.1.2)

Extension of previous section. Please read.

Generalized Coordinates and Kinematics Constraints (Sec. 7.2)

Generalized coordinates - what is used to describe the orientation of the system?

Degrees-of-freedom - the minimum number of generalized coordinates to completely define the position of the system



A rigid body in a plane has 3 DOF

Q: What are some choices? usually denoted as $\bar{q}$

(x, y, ϕ) (r, θ, ϕ)

Q: Others?

Q: What if it's pinned at one end? How many DOF? $\longrightarrow$ 1 DOF, ϕ

3DOF if unconstrain - 2 constraints (pin limits x and y=0) = 1 DOF

We can choose a set of constrained generalized coords, but then need to include constraints in the eq of motion. This is often undesirable.

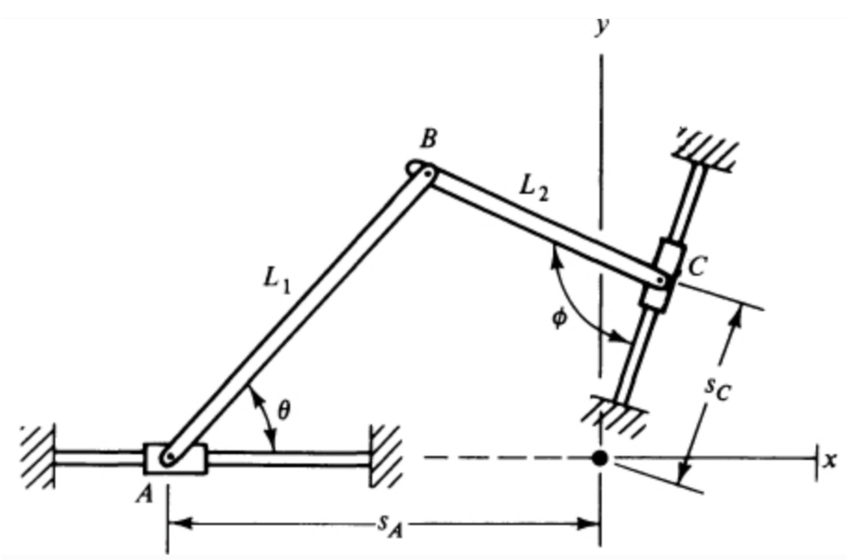


Figure 6.6 Generalized coordinates for a linkage.

Q: Is (s_A, θ) a good choice for generalized coords?

No, need additional information needed to define position

(s_A, s_C) would be a better choice

If # of generalized coords > # DOF the generalized coordinations are constrained

else

unconstrained or independent