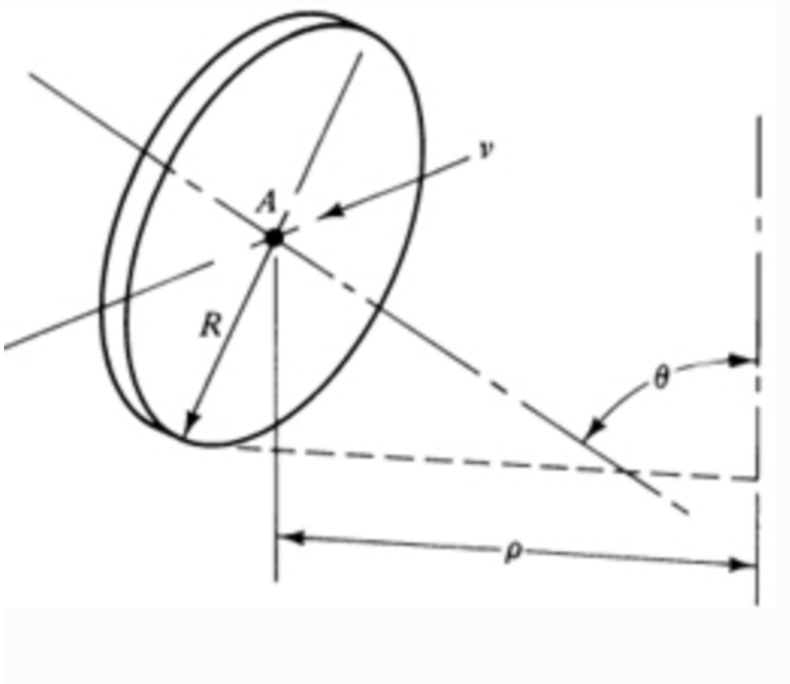
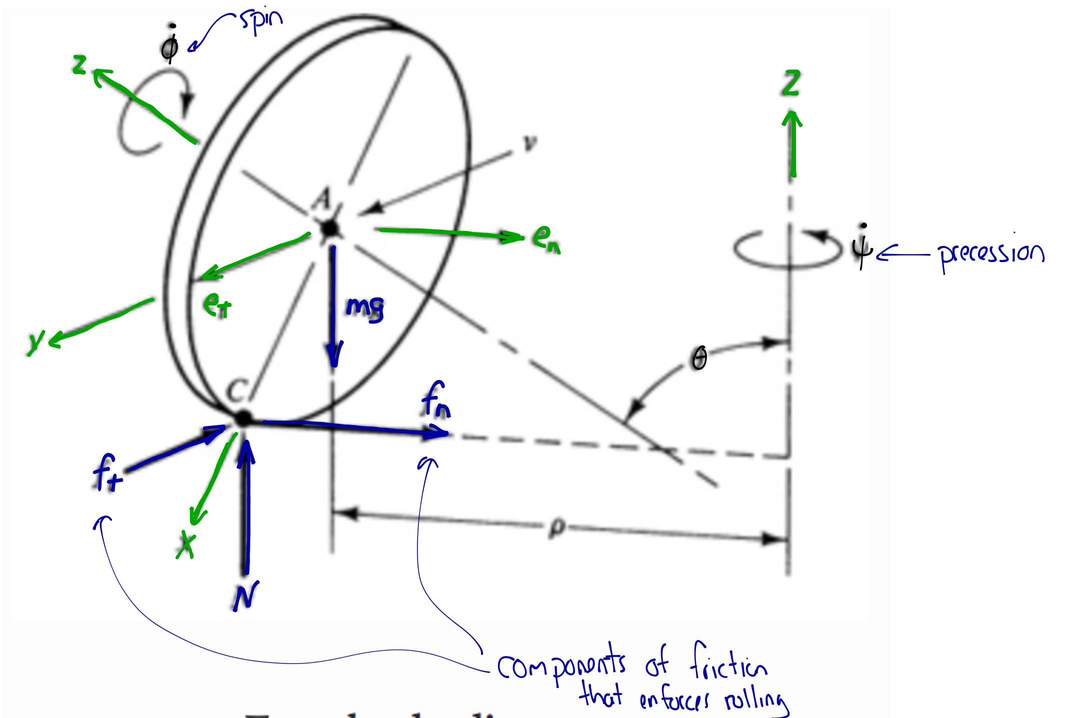
Example 5.8

This problem is from Advanced Engineering Dynamics by Jerry Ginsberg. It is basically the same as Example 6.4 from your book, but with a different frame selection.

A thin homogeneous disk of mass m rolls without slipping on a horizontal plane such that the center A has a constant speed v as it follows a circular path of radius ρ. The angle of inclination of the axis relative to the vertical is a constant value θ. Derive an expression relating v to the other parameters.



Example 5.8



Free-body diagram.

constant speed $v \Rightarrow v = \rho\dot{\psi} \rightarrow \dot{\psi} = v/\rho$

$\bar{\omega}_{disk} = \dot{\psi}\bar{K} + \dot{\phi}\bar{k}$ $\qquad \bar{\alpha}_{disk} = \left[\cancelto{0}{\ddot{\psi}}\bar{K} + \cancelto{0}{\ddot{\phi}}\bar{k}\right] + \left[\dot{\psi}\cancelto{0}{\dot{\bar{K}}} + \dot{\phi}\dot{\bar{k}}\right] = \dot{\phi}(\bar{\omega} \times \bar{k})$

At this instant: $\bar{K} = -\sin\theta\,\bar{i} + \cos\theta\,\bar{k}$

$\bar{\omega}_{disk} = -\dot{\psi}\sin\theta\,\bar{i} + (\dot{\phi} + \dot{\psi}\cos\theta)\bar{k}$ ← only valid at this instant, because is only for this instant

$\bar{\alpha}_{disk} = \dot{\phi}\dot{\psi}\sin\theta\,\bar{j}$

We can write $\bar{v}_A$ based on rotation rates (because of the rolling condition)

$\bar{v}_A = \cancelto{0 \text{ b/c rolling}}{\bar{v}_C} + \bar{\omega} \times \bar{r}_{A/C} = v\bar{j}$ $\qquad \bar{r}_{A/C} = -R\bar{i}$

$= \left[-\dot{\psi}\sin\theta\,\bar{i} + (\dot{\phi} + \dot{\psi}\cos\theta)\bar{k}\right] \times \left[-R\bar{i}\right] = -R(\dot{\phi} + \dot{\psi}\cos\theta)\bar{j} = v\bar{j}$

$= \rho\dot{\psi} \longrightarrow \dot{\phi} = -\left(\frac{\rho}{R} + \cos\theta\right)\dot{\psi}$ ← how fast its spinning depends on how fast its precessing and the radii

Plug ϕ into $\bar{\omega}_{disk}$ and $\bar{\alpha}_{disk}$

$\bar{\omega}_{disk} = \dot{\psi}\left[-\sin\theta\,\bar{i} - \frac{\rho}{R}\bar{k}\right]$

$\bar{\alpha}_{disk} = -\dot{\psi}^2\left[\frac{\rho}{R} + \cos\theta\right]\sin\theta\,\bar{j}$

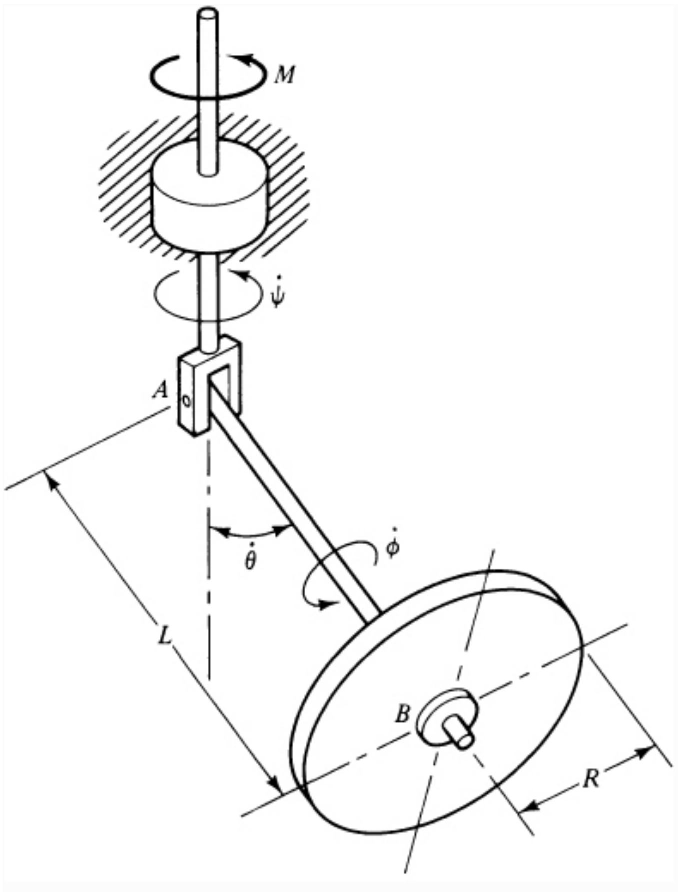
Use these to find $\bar{H}_A$ and $\dot{\bar{H}}_A$ and sub into moment equations

Then, solve $\Sigma\bar{F} = m\bar{a}_G$ for remaining unknowns

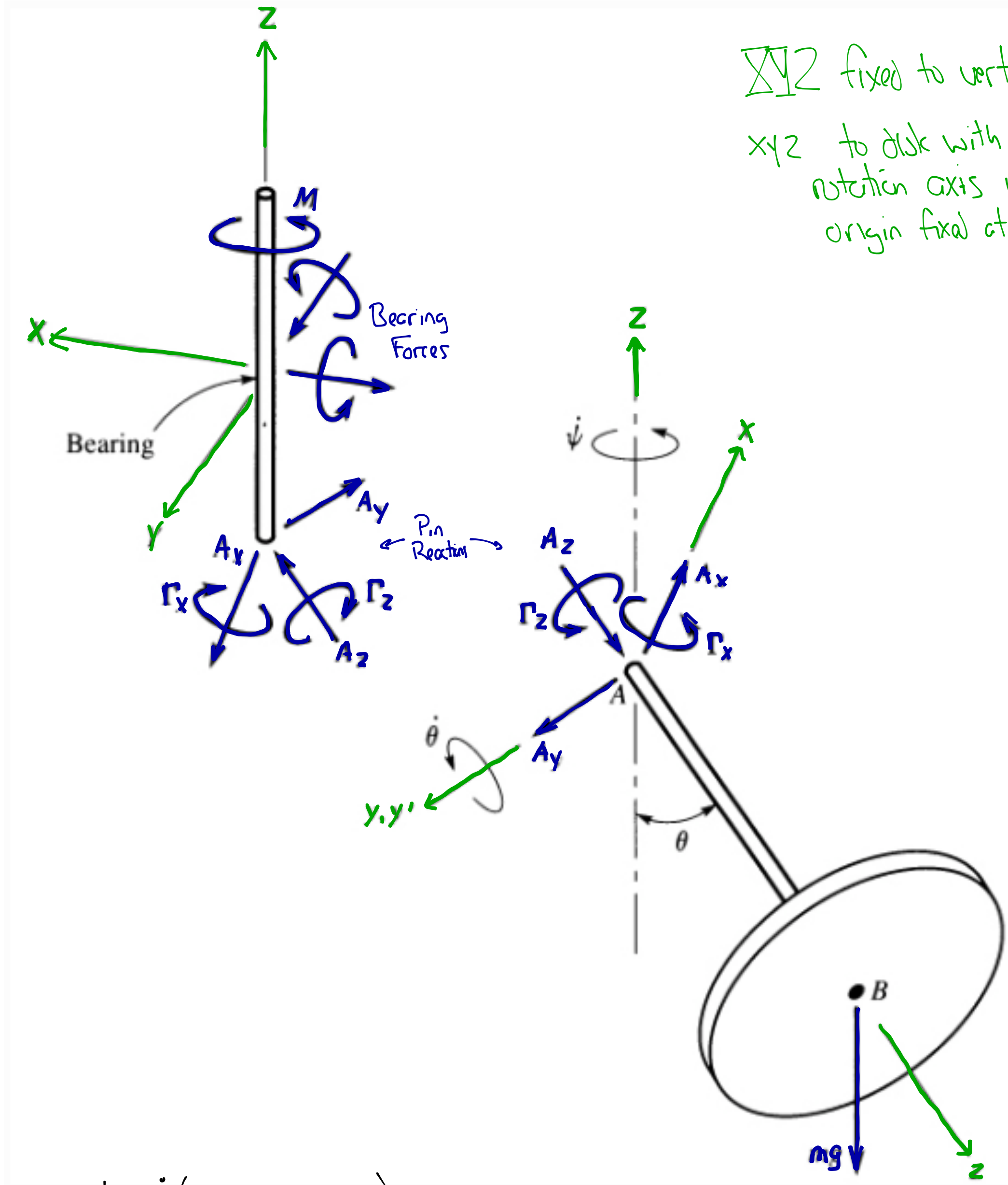
Example 5.9

This problem is from Advanced Engineering Dynamics by Jerry Ginsberg. It is basically the same as Example 6.2 from your book.

A servomotor maintains at a constant value the spin rate $\dot{\phi}$ at which the disk rotates relative to the pivoted shaft AB. The precession rate $\dot{\psi}$ about the vertical axis is also held constant by a torque M(t). Derive the differential equation governing the nutation angle θ. Also derive an expression for M.



Example 5.9



XYZ fixed to vertical shaft

xyz to disk with z along rotation axis with origin fixed at pin joint

$$\bar{\omega} = \dot{\psi}\bar{K} + \dot{\theta}\bar{j} + \dot{\phi}\bar{k} \qquad \underbrace{\bar{\omega}'}_{\text{rotation of frame}} = \dot{\psi}\bar{K}$$

$$\bar{K} = \sin\theta\,\bar{i} - \cos\theta\,\bar{k}$$

$$\bar{\omega} = (\dot{\psi}\sin\theta)\bar{i} + \dot{\theta}\bar{j} + (\dot{\phi} - \dot{\psi}\cos\theta)\bar{k} \qquad \bar{\omega}' = \dot{\psi}(\sin\theta\,\bar{i} - \cos\theta\,\bar{k})$$

$$\bar{\alpha} = \cancelto{0}{\ddot{\psi}}\bar{K} + \cancelto{0}{\dot{\psi}\dot{\bar{K}}} + \ddot{\theta}\bar{j} + \dot{\theta}\dot{\bar{j}} + \cancelto{0}{\ddot{\phi}}\bar{k} + \dot{\phi}\dot{\bar{k}}$$

$$= \ddot{\theta}\bar{j} + \dot{\theta}(\bar{\omega}' \times \bar{j}) + \dot{\phi}(\bar{\omega} \times \bar{k})$$

$$= \ddot{\theta}\bar{j} + \dot{\theta}\left(\dot{\psi}\sin\theta\,\bar{i} - \dot{\psi}\cos\theta\,\bar{k} \times \bar{j}\right) + \dot{\phi}\left[(\dot{\psi}\sin\theta)\bar{i} + \dot{\theta}\bar{j} + (\dot{\phi} - \dot{\psi}\cos\theta)\bar{k} \times \bar{k}\right]$$

$$= \ddot{\theta}\bar{j} + \dot{\theta}\left(\dot{\psi}\sin\theta\,\bar{k} + \dot{\psi}\cos\theta\,\bar{i}\right) + \dot{\phi}\left[-\dot{\psi}\sin\theta\,\bar{j} + \dot{\theta}\bar{i}\right]$$

$$\bar{\alpha} = \left(\dot{\theta}\dot{\psi}\cos\theta + \dot{\phi}\dot{\theta}\right)\bar{i} + \left(\ddot{\theta} - \dot{\phi}\dot{\psi}\sin\theta\right)\bar{j} + \left(\dot{\theta}\dot{\psi}\sin\theta\right)\bar{k}$$

Sum moments about point A: (We can use Eulers Eq since xyz are principal axes)

$$\Sigma M_{Ax} = \Gamma_x = I_{xx}\alpha_x + (I_{yy} - I_{zz})\omega_y\omega_z$$ — Solve with Γ_x

$$\Sigma M_{Ay} = -mgL\sin\theta = I_{yy}\alpha_y + (I_{zz} - I_{xx})\omega_x\omega_z$$ ⟶ Eq of Motion

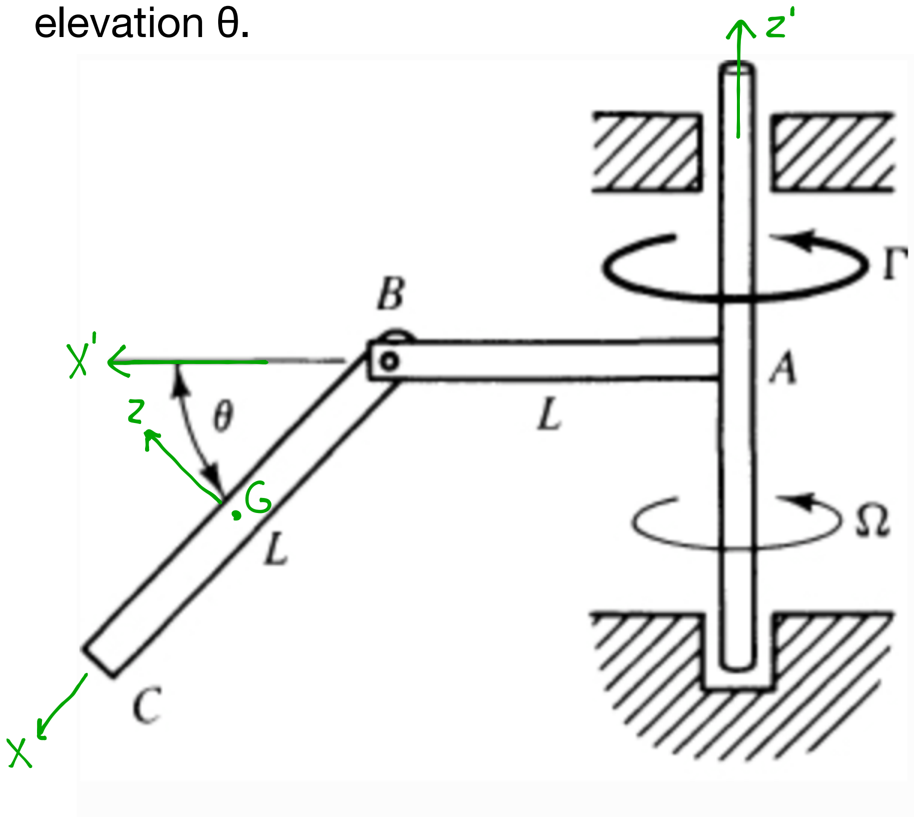
$$\Sigma M_{Az} = \Gamma_z = I_{zz}\alpha_z + (I_{xx} - I_{yy})\omega_x\omega_y$$ — Solve for Γ_z

Sub into moment equation for vertical shaft to calculate M

Problem 5.22

This problem is from Advanced Engineering Dynamics by Jerry Ginsberg. It is basically the same as Exercise 6.14 from your book.

Bar BC is pivoted from the end of the T-bar. The torque Γ is such that the system rotates about the vertical axis at the constant speed Ω. Derive the differential equation of motion for the angle of elevation θ.

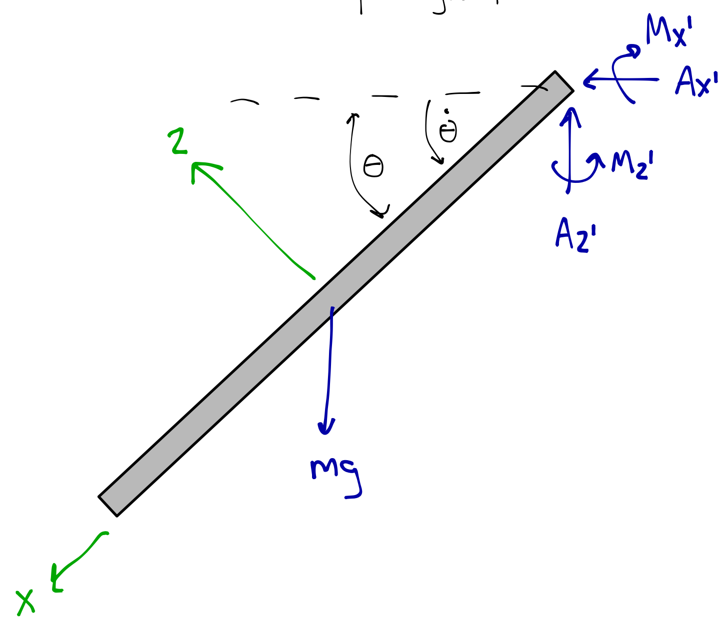


Problem 5.22

$x'y'z'$ – fixed to AB, precesses at Ω

xyz - fixed to BC

Free Body Diagram



$$\bar{\omega} = \Omega k' + \dot{\theta} j$$

$$k' = -\sin\theta\, \bar{i} + \cos\theta\, k' \quad \leftarrow -\theta \text{ rotation about } y, y'$$

$$\bar{\omega} = \Omega(-\sin\theta\, \bar{i} + \cos\theta\, \bar{k}) + \dot{\theta}\bar{j}$$

$$\bar{\omega} = -\Omega\sin\theta\, \bar{i} + \dot{\theta}\bar{j} + \Omega\cos\theta\, \bar{k}$$

$$\bar{\alpha} = \cancelto{0}{\dot{\Omega}}k' + \cancelto{0}{\Omega\dot{\bar{k}}'} + \ddot{\theta}\bar{j} + \dot{\theta}\dot{\bar{j}} = \ddot{\theta}\bar{j} + \dot{\theta}(\bar{\omega}\times\bar{j})$$

$$= \ddot{\theta}\bar{j} + \dot{\theta}\left[(-\Omega\sin\theta\, \bar{i} + \dot{\theta}\bar{j} + \Omega\cos\theta\, \bar{k})\times\bar{j}\right]$$

$$\bar{\alpha} = \ddot{\theta}\bar{j} + \dot{\theta}\left[-\Omega\cos\theta\, \bar{i} - \Omega\sin\theta\, \bar{k}\right]$$

Now write $\bar{a}_G$

$$\bar{a}_G = \bar{a}_A + \cancelto{0}{(a_G)_{xyz}} + \bar{\alpha}\times\bar{r}_{G/A} + \bar{\omega}\times(\bar{\omega}\times\bar{r}_{G/A}) + 2\bar{\omega}\times\cancelto{0}{(v_G)_{xyz}}$$

$$\bar{a}_A = -L\Omega^2\bar{i}' \quad \leftarrow \text{pure rotation}$$

$$\bar{r}_{G/A} = \frac{L}{2}\bar{i}$$

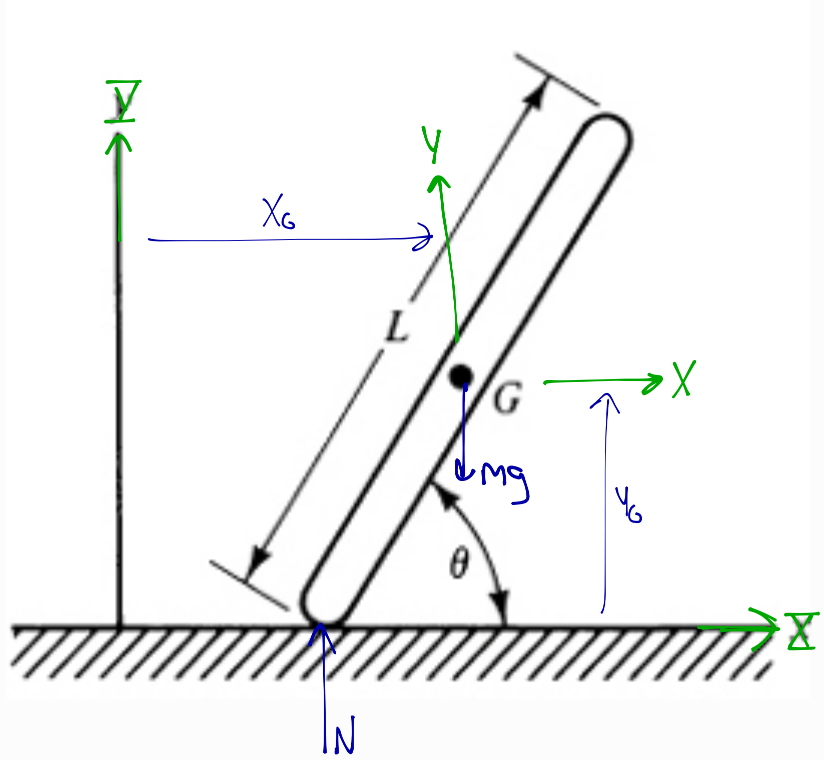
Then, write $\underbrace{\Sigma\bar{F} = m\bar{a}_G}$ and $\Sigma\bar{M}_G = \dot{\bar{H}}_G$

Use to solve for reactions, then substitute into moment equation to get eq. of motion for θ

Problem 5.29

This problem is from Advanced Engineering Dynamics by Jerry Ginsberg. It is very similar to Exercise 6.28 from your book.

The bar of mass m is falling toward the horizontal surface. Friction is negligible. Derive differential equations of motion for the position coordinates (xG, yG) of the center of mass of the bar, and for the angle of inclination θ. Also obtain an expression for the contact force exerted by the ground on the bar in terms of xG, yG, θ, and their derivatives.



Problem 5.29

Write y_G (and deriv) in terms of θ

$$y_G = \frac{L}{2}\sin\theta \qquad \dot{y}_G = \frac{L}{2}\dot{\theta}\cos\theta$$

$$\ddot{y}_G = \frac{L}{2}\ddot{\theta}\cos\theta - \frac{L}{2}\dot{\theta}^2\sin\theta$$

so: $\bar{a}_G = \ddot{X}_G\bar{i} + \left(\frac{L}{2}\ddot{\theta}\cos\theta - \frac{L}{2}\dot{\theta}^2\sin\theta\right)\bar{j}$

$$\Sigma\bar{F} = (N - mg)\bar{j} = m\left[\ddot{X}_G\bar{i} + \left(\frac{L}{2}\ddot{\theta}\cos\theta - \frac{L}{2}\dot{\theta}^2\sin\theta\right)\bar{j}\right]$$

$$\Sigma F_x = 0 = m\ddot{X}_G \longrightarrow \boxed{\ddot{X}_G = 0}$$

$$\Sigma F_y = N - mg = \frac{L}{2}\ddot{\theta}\cos\theta - \frac{L}{2}\dot{\theta}^2\sin\theta$$

$$\boxed{N = \frac{L}{2}\ddot{\theta}\cos\theta - \frac{L}{2}\dot{\theta}^2\sin\theta + mg}$$

Sum moments about G - (planar, so all in $\bar{k}$)

$$\Sigma\bar{M}_{G_k} = \left(-\frac{L}{2}\cos\theta\,\bar{i} - \frac{L}{2}\sin\theta\,\bar{j}\right)\times\left(N\bar{j}\right) =$$

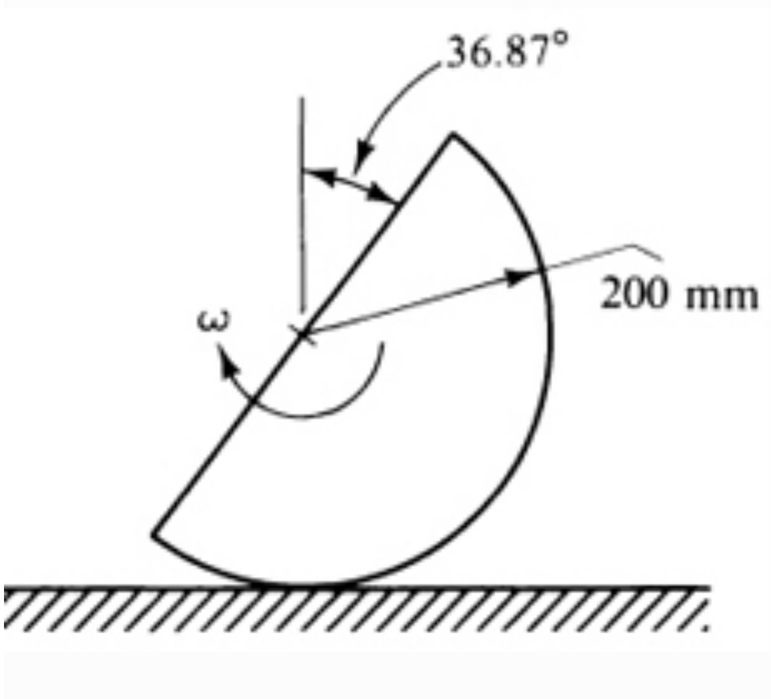
$$= -N\left(\frac{L}{2}\cos\theta\right) = I_{22}\alpha_2$$

$$\boxed{I_{22}\ddot{\theta} = -N\left(\frac{L}{2}\cos\theta\right)} \longleftarrow$$ Sub N from above for "full" solution

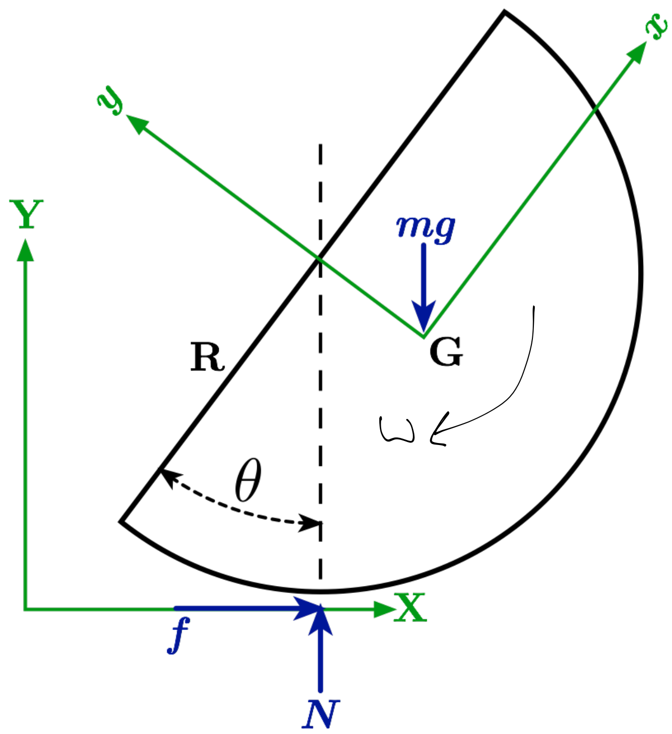
Problem 5.33

This problem is from Advanced Engineering Dynamics by Jerry Ginsberg. It is very similar to Exercise 6.35 from your book.

The 20-kg semicylinder has an angular speed ω = 10 rad/s in the position shown. The coefficient of static friction between the ground and the semicylinder is μ. Determine the minimum value of μ for which slipping between the semicylinder and the ground will not occur in this position. What is the corresponding angular acceleration of the semicylinder?



Problem 5.33



$\bar{\omega} = -\omega \bar{k}$ $\qquad$ $\bar{\alpha} = -\dot{\omega}\bar{k}$

If no slip; Velocity of semicircle center:

$v_O = R\omega \rightarrow \dot{v}_O = R\dot{\omega} = \bar{a}_O \longrightarrow \bar{a}_O = R\dot{\omega}\bar{I}$

$\bar{r}_{G/O} = \frac{-4R}{3\pi}\bar{i}$ ⟵ get from lookup table

$= \frac{-4R}{3\pi}\left(\cos\theta \bar{I} - \sin\theta \bar{J}\right)$

$\bar{a}_G = \bar{a}_O \times \cancel{(\bar{a}_G)_{xyz}}^{0} + \bar{\alpha}\times\bar{r}_{G/O} + \bar{\omega}\times(\bar{\omega}\times\bar{r}_{G/O}) + 2\bar{\omega}\times\cancel{(\bar{v}_G)_{xyz}}^{0}$

$= \bar{a}_O + \bar{\alpha}\times\bar{r}_{G/O} + \bar{\omega}\times(\bar{\omega}\times\bar{r}_{G/O})$

$= \left(R\dot{\omega}\bar{I}\right) + \left[-\dot{\omega}\bar{k} \times \frac{-4R}{3\pi}(\cos\theta\bar{I} - \sin\theta\bar{J})\right] + \left[-\omega\bar{k}\times\left(-\omega\bar{k}\times\left(\frac{-4R}{3\pi}(\cos\theta\bar{I} - \sin\theta\bar{J})\right)\right)\right]$

$= \left(-R\dot{\omega}\bar{I}\right) + \left[\frac{-4R}{3\pi}\dot{\omega}\cos\theta\bar{J} + \frac{+4R}{3\pi}\dot{\omega}\sin\theta\bar{I}\right] + \left[-\omega\bar{k}\times\left(\frac{-4R}{3\pi}\omega\cos\theta\bar{J} + \frac{4R}{3\pi}\sin\theta\bar{I}\right)\right]$

$= \left(-R\dot{\omega}\bar{I}\right) + \left[\frac{-4R}{3\pi}\dot{\omega}\cos\theta\bar{J} + \frac{+4R}{3\pi}\dot{\omega}\sin\theta\bar{I}\right] + \left[-\frac{4R}{3\pi}\omega^2\cos\theta\bar{I} + \frac{4R}{3\pi}\omega^2\sin\theta\bar{J}\right]$

$= \left[-R\dot{\omega} + \frac{4R}{3\pi}\dot{\omega}\sin\theta - \frac{4R}{3\pi}\omega^2\cos\theta\right]\bar{I} + \left[\frac{-4R}{3\pi}\dot{\omega}\cos\theta + \frac{4R}{3\pi}\omega^2\sin\theta\right]\bar{J}$

$\Sigma\bar{F} = f\bar{I} + (N - mg)\bar{J} = m\bar{a}_G$ ⟵ equate I and J components to solve for f and N

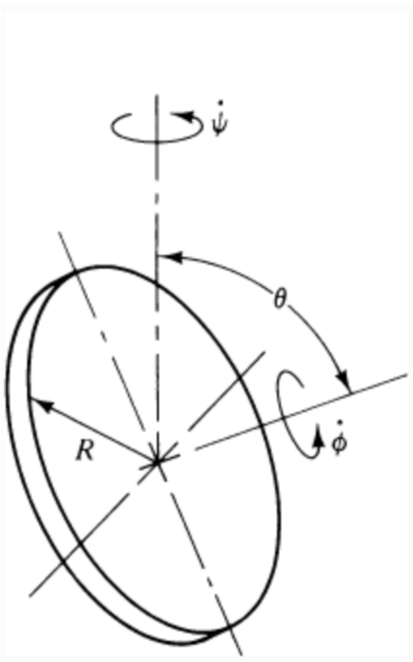
$\Sigma M_{Gz} = -N\left(\frac{4R}{3\pi}\cos\theta\right) + f\left(R - \frac{4R}{3\pi}\sin\theta\right) = I_{zz}\alpha_z$ ⟵ gives ω as a function of f and R

$\min \mu = \frac{f}{N}$

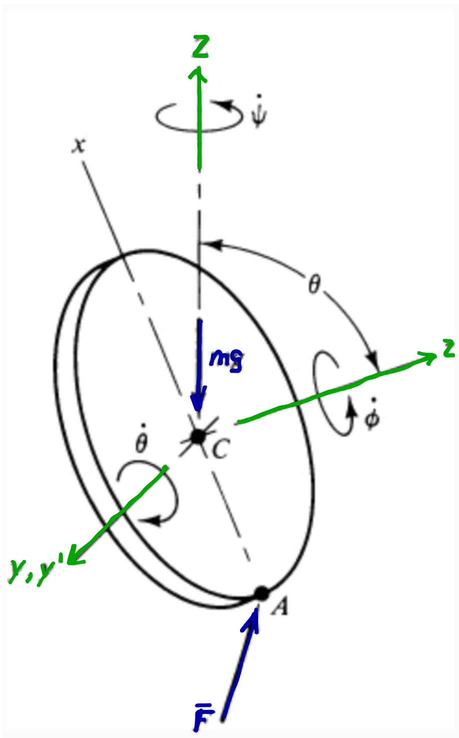
Example 5.15

This problem is from Advanced Engineering Dynamics by Jerry Ginsberg. It is basically the same as Example 6.12 from your book.

The coin is rolling without slipping, but the angle θ at which the plane of the coin is inclined is not constant. Evaluate the kinetic energy of the disk in terms of θ, the precession rate $\dot{\psi}$, and spin rate $\dot{\phi}$. Also, prove that the work done by the friction and normal forces is zero.



Example 5.15



$$\bar{\omega} = \dot{\psi}\bar{K} - \dot{\theta}\bar{j}' + \dot{\phi}\bar{k} = (\dot{\psi}\sin\theta)\bar{i} - \dot{\theta}\bar{j} + (\dot{\psi}\cos\theta + \dot{\phi})\bar{k}$$

$$\bar{v}_C = \omega \times \bar{r}_{C/A} = R(\dot{\psi}\cos\theta + \dot{\phi})\bar{j} + R\dot{\theta}\bar{k}$$

$$\bar{H}_C = I_{xx}\omega_x\bar{i} + I_{yy}\omega_y\bar{j} + I_{zz}\omega_z\bar{k}$$ ← no prod. of inertia

$$\boxed{T = \frac{1}{2}m\bar{v}_C\cdot\bar{v}_C + \frac{1}{2}\bar{\omega}\cdot\bar{H}_C}$$

For the 2nd part of the problem:

Define $\bar{F}$ as vector force combining the Normal and friction forces.

The moment it creates about C is then:

$$\bar{M} = \bar{r}_{A/C} \times \bar{F}$$

And the work done by the reactions is then:

$$dW = \bar{F}\cdot\delta\bar{r}_c + \bar{M}\cdot\delta\bar{\theta} \qquad \delta\bar{r} = \bar{v}_c dt = \delta\bar{\theta}\times\bar{r}_{C/A} \quad \text{and} \quad \delta\bar{\theta} = \bar{\omega}dt$$

So,
$$dW = \bar{F}\cdot(\delta\bar{\theta}\times\bar{r}_{C/A}) + (r_{A/C}\times\bar{F})\cdot\delta\bar{\theta}$$

$$= \bar{F}\cdot(\delta\bar{\theta}\times\bar{r}_{C/A}) - \bar{F}(\delta\bar{\theta}\times r_{C/A}) = 0$$ ← Reactions do no work!