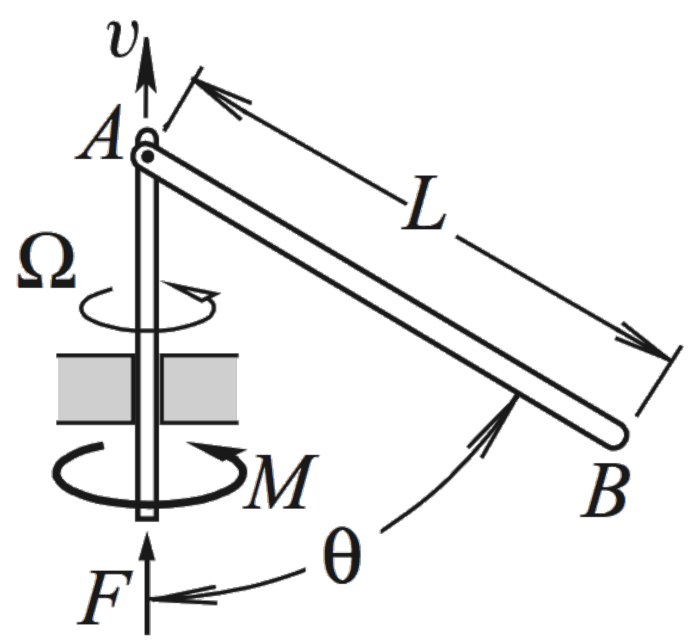
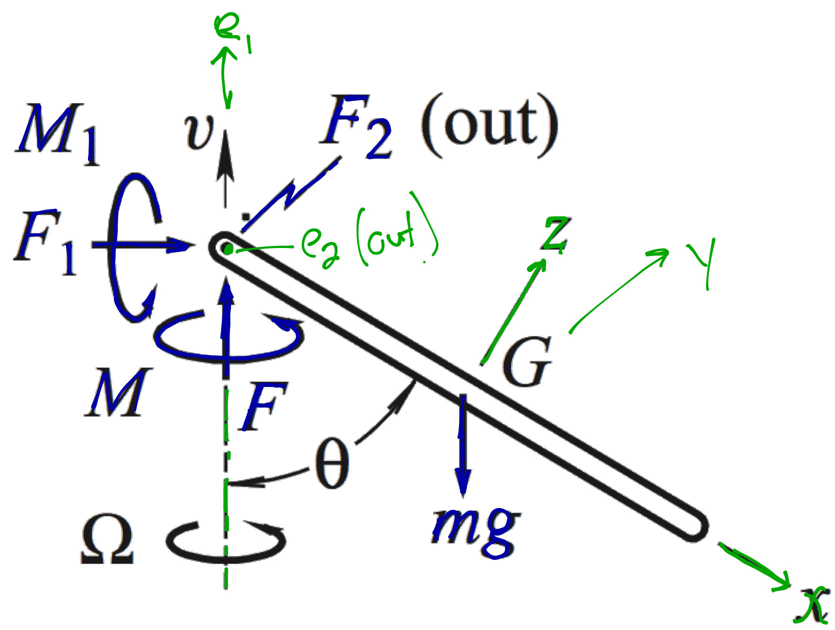
Example 6.3

EXAMPLE 6.3 The vertical force F causes the vertical bar to translate upward at a speed v that is a specified function of time, and the whole system precesses about the vertical axis at the constant speed Ω as a result of the action of the torque M. Derive the equation of motion for the nutation angle θ, as well as an expression for the value of F required to attain this motion. It may be assumed that the mass of the vertical shaft is negligible.





Q: Where should we sum moments about?

A?... we can't use a because it is accelerating upward

So, we have to use G (COM)

$$\bar{\omega} = \Omega \bar{e}_1 + \dot{\theta} \bar{e}_2$$

$$\bar{\alpha} = \ddot{\theta} \bar{e}_2 + \dot{\theta} \dot{\bar{e}}_2 = \ddot{\theta} \bar{e}_2 + \dot{\theta}\left(\Omega \bar{e}_1 \times \bar{e}_2\right)$$

See that $\bar{e}_2 = -\bar{j}$ and $\bar{e}_1 = -\cos\theta \, \bar{i} + \sin\theta \, \bar{k}$

$$\bar{\omega} = \Omega\left(-\cos\theta \, \bar{i} + \sin\theta \, \bar{k}\right) - \dot{\theta} \bar{j} = -\Omega \cos\theta \, \bar{i} - \dot{\theta} \bar{j} + \Omega \sin\theta \, \bar{k}$$

$$\bar{\alpha} = \Omega \dot{\theta} \sin\theta \, \bar{i} - \ddot{\theta} \bar{j} + \Omega \dot{\theta} \cos\theta \, \bar{k}$$

$$\bar{a}_G = \bar{a}_A + \bar{\alpha} \times \bar{r}_{G/A} + \bar{\omega} \times \left(\bar{\omega} \times \bar{r}_{G/A}\right)$$

xyz are principal, so we can use Euler's Eq for ΣM_G.

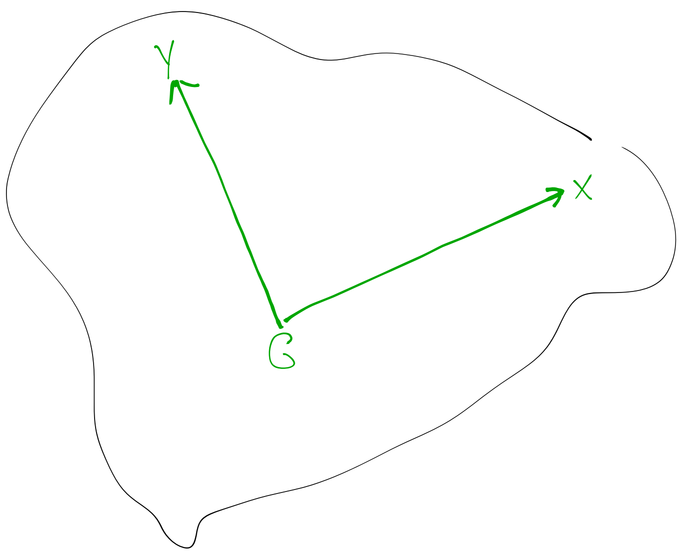
Solve with $\Sigma \bar{F} = m \bar{a}_G$

} see book for full solution

Planar Motion (Sec. 6.2)

Define xyz as body-fixed coord with origin at a point about which we can sum moments.

Also, align z-axis ⊥ to plane of motion



So $\bar{\omega} = \omega \bar{k}$

$\bar{\alpha} = \dot{\omega} \bar{k}$

$\bar{a}_G = a_{Gx} \bar{i} + a_{Gy} \bar{j}$ ← could also define in frame at COM rotated but $z = z'$

$\bar{H}_G = I_{zz} \omega_z \bar{k} - I_{xy} \omega \bar{i} - I_{yz} \omega \bar{j}$

So, the equations of motion are then

$\sum \bar{F} = m \bar{a}_G \longrightarrow$

$\sum \bar{F} \cdot \bar{i} = m a_{Gx} \bar{i}$

$\sum \bar{F} \cdot \bar{j} = m a_{Gy} \bar{j}$

$\sum \bar{F} \cdot \bar{k} = 0$

$\sum M_G = \dot{\bar{H}}_G = \frac{\partial \bar{H}_A}{\partial t} + \bar{\omega} \times \bar{H}_A \longrightarrow$

$\sum \bar{M}_G \cdot \bar{i} = -I_{xz} \dot{\omega} + I_{yz} \omega^2$

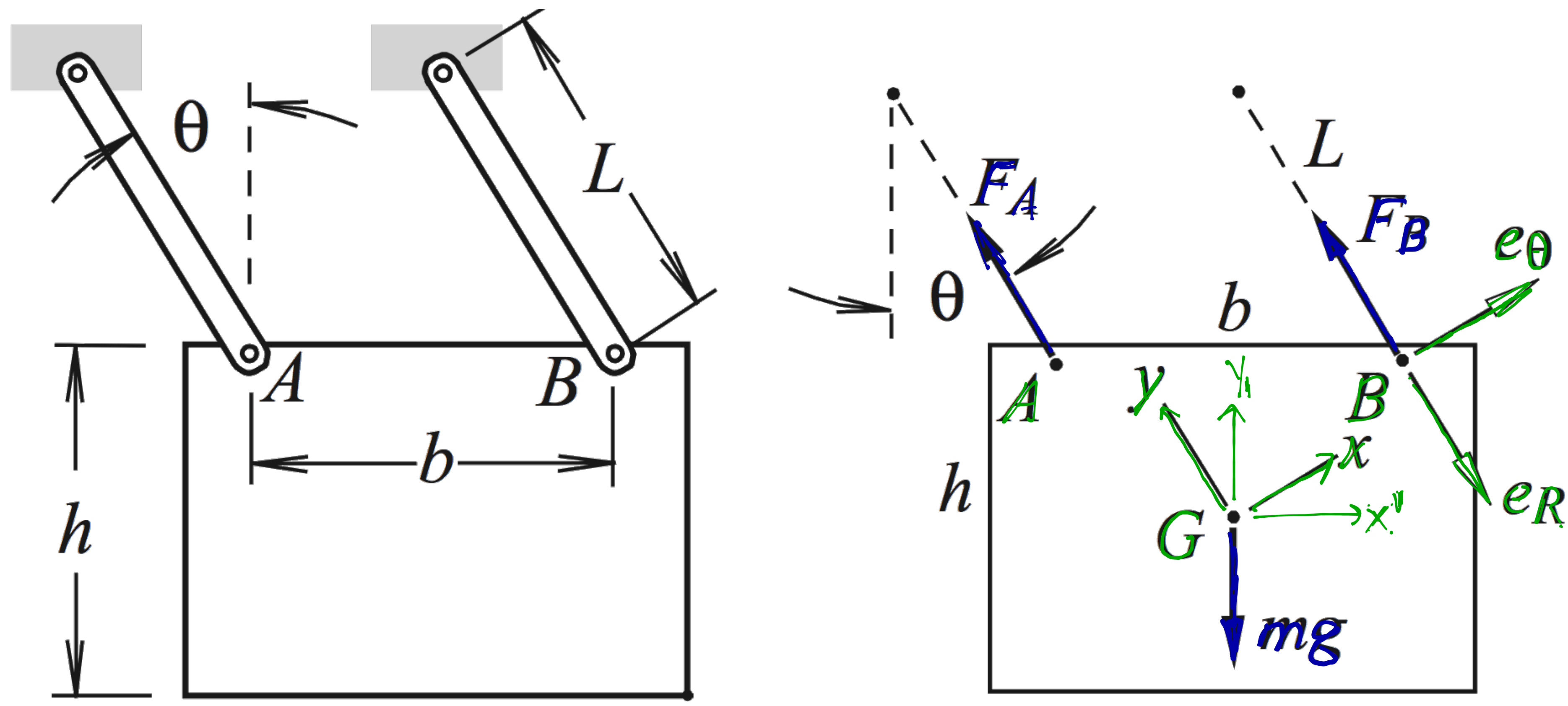
$\sum \bar{M}_G \cdot \bar{j} = -I_{yz} \dot{\omega} + I_{xz} \omega^2$

$\sum \bar{M}_G \cdot \bar{k} = I_{zz} \dot{\omega}$

If z-axis is principal, then $I_{xz} = I_{yz} = 0$, so only moment eq is $\sum \bar{M}_G = I_{zz} \dot{\omega}$

Example 6.6

EXAMPLE 6.6 The rectangular plate, whose mass is m, serves as a fire door. In case of an emergency, the cable holding the plate is severed and the door swings down under the restraint of the rigid links that suspend the plate from the ceiling. Derive a differential equation of motion governing the angle of inclination θ of the links. Also derive expressions for the forces exerted by the links on the plate. The mass of each link is negligible.



Because of the geometry of this system (4-bar with equal-length opposite links), the body only translates.

So, $\bar{a}_G = \bar{a}_A = \bar{a}_B$

This allows us to write $\bar{a}_G$ by knowing that B is in pure rotation

$$\bar{a}_B = -L\dot{\theta}^2 \bar{e}_r + L\ddot{\theta}\bar{e}_\theta$$

By placing xyz as we did, we can easily write

$$a_G = -L\dot{\theta}^2 \bar{e}_r + L\ddot{\theta}\bar{e}_\theta = L\dot{\theta}^2 \bar{j} + L\ddot{\theta}\bar{i}$$

Because the body is in pure rotation

$$H_G = 0$$

Example 6.6 (cont.)

$$\Sigma \bar{M}_G = \bar{r}_{A/G} \times F_A \bar{J} + \bar{r}_{B/G} \times F_B \bar{J}$$

$$\bar{r}_{A/G} = -\frac{b}{2}\bar{i}' + \frac{h}{2}\bar{J}'$$

$$\bar{r}_{B/G} = \frac{b}{2}\bar{i}' + \frac{h}{2}\bar{J}'$$

$$\begin{bmatrix} \bar{i}' \\ \bar{J}' \\ \bar{k}' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ +\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{i} \\ \bar{J} \\ \bar{k} \end{bmatrix}$$

← $-\theta$ rotation about k, k'

$$\bar{r}_{A/G} = \frac{-b}{2}\left(\cos\theta\,\bar{i} - \sin\theta\,\bar{J}\right) + \frac{h}{2}\left(\sin\theta\,\bar{i} + \cos\theta\,\bar{J}\right)$$

$$\bar{r}_{B/G} = \frac{b}{2}\left(\cos\theta\,\bar{i} - \sin\theta\,\bar{J}\right) + \frac{h}{2}\left(\sin\theta\,\bar{i} + \cos\theta\,\bar{J}\right)$$

$$\Sigma \bar{M}_G = \left(-\frac{b}{2}F_A\cos\theta\,\bar{k} + \frac{h}{2}F_A\sin\theta\,\bar{k}\right) + \left(\frac{b}{2}F_B\cos\theta\,\bar{k} + \frac{h}{2}F_B\sin\theta\,\bar{k}\right) = 0$$

Note: all rotation in $\bar{k}$ direction. It better be!

$$\Sigma \bar{F} = m\bar{a}_G = -mg\bar{J}' + F_A\bar{J} + F_B\bar{J} =$$

$$= -mg\sin\theta\,\bar{i} + mg\cos\theta\,\bar{J} + F_A\bar{J} + F_B\bar{J} = mL\ddot{\theta}\,\bar{i} + mL\dot{\theta}^2\bar{J}$$

$$\Sigma \bar{F}\cdot\bar{i} = -mg\sin\theta = mL\ddot{\theta}$$

$$\Sigma \bar{F}\cdot\bar{J} = -mg\cos\theta + F_A + F_B = mL\dot{\theta}^2$$

Solve this system of equations to find:

$$\ddot{\theta} + \frac{g}{l}\sin\theta = 0$$

← same motion as a simple pendulum

$$F_A = \frac{1}{2}\left(1 + \frac{h}{b}\tan\theta\right)\left(mg\cos\theta + mL\dot{\theta}^2\right)$$

$$F_B = \frac{1}{2}\left(1 - \frac{h}{b}\tan\theta\right)\left(mg\cos\theta + mL\dot{\theta}^2\right)$$

Newton-Euler Equations for a System (Sec. 6.3)

Can sometimes simplify analysis by considering bodies as a system rather than individually. Not much in this section other than that. Please read it

Momentum and Energy Principles (Sec. 6.4)

Integrals of force and moment equations. Generally used to supplement, not replace.

↑ Because we often need to know the motion to implement them.

Can be used to simplify some special cases.

Impulse-Momentum Principles (Sec. 6.4.1)

Linear – $\bar{P}_2 = \bar{P}_1 + \int_{t_1}^{t_2} \Sigma \bar{F} dt$ $\quad$ P_i = linear momentum at time t_i

Rotational – $(H_A)_2 = (H_{A_1}) + \int_{t_1}^{t_2} \Sigma \bar{M}_A dt$ $\quad$ $(H_A)_i$ = angular momentum at time t_i

Both of these are vector equations

These relationships are really useful when impulses act on a body

Q: What is an impulse?

strictly – infinitely large signal of zero duration

practically – large enough force over small enough interval that other, non-impulsive forces can be ignored

Q: Are reaction forces to an impulse impulsive?

Yes. By definition, they are large enough to enforce the constraints.

Impulse-Momentum Principles (cont.)

Assume the impulsive force/moment is nearly constant over a small time range,

$t_0^- \rightarrow t_0^+$

We can write the time range to analyze the response to the impulse as

$t_i < t < t_i + \Delta t$

Find

$$\underbrace{\bar{P}(t = t_i^+)}_{\text{Momentum right after impulse}} = \underbrace{P(t = t_i^-)}_{\text{Momentum right before impulse}} + \underbrace{\left(\sum \bar{F}\right)_{imp} \Delta t}_{\text{Impulse}}$$

and

$$\underbrace{\bar{H}_A(t = t_i^+)}_{\text{Momentum right after impulse}} = \underbrace{\bar{H}_A(t = t_i^-)}_{\text{Momentum right before impulse}} + \underbrace{\left(\sum \bar{M}_A\right)_{imp} \Delta t}_{\text{Impulse}}$$

Because $\Delta t \approx 0$, let $\bar{r}_{P/O}(t = t_i^+) = \bar{r}_{P/O}(t = t_i^-)$

(More detail in the book)