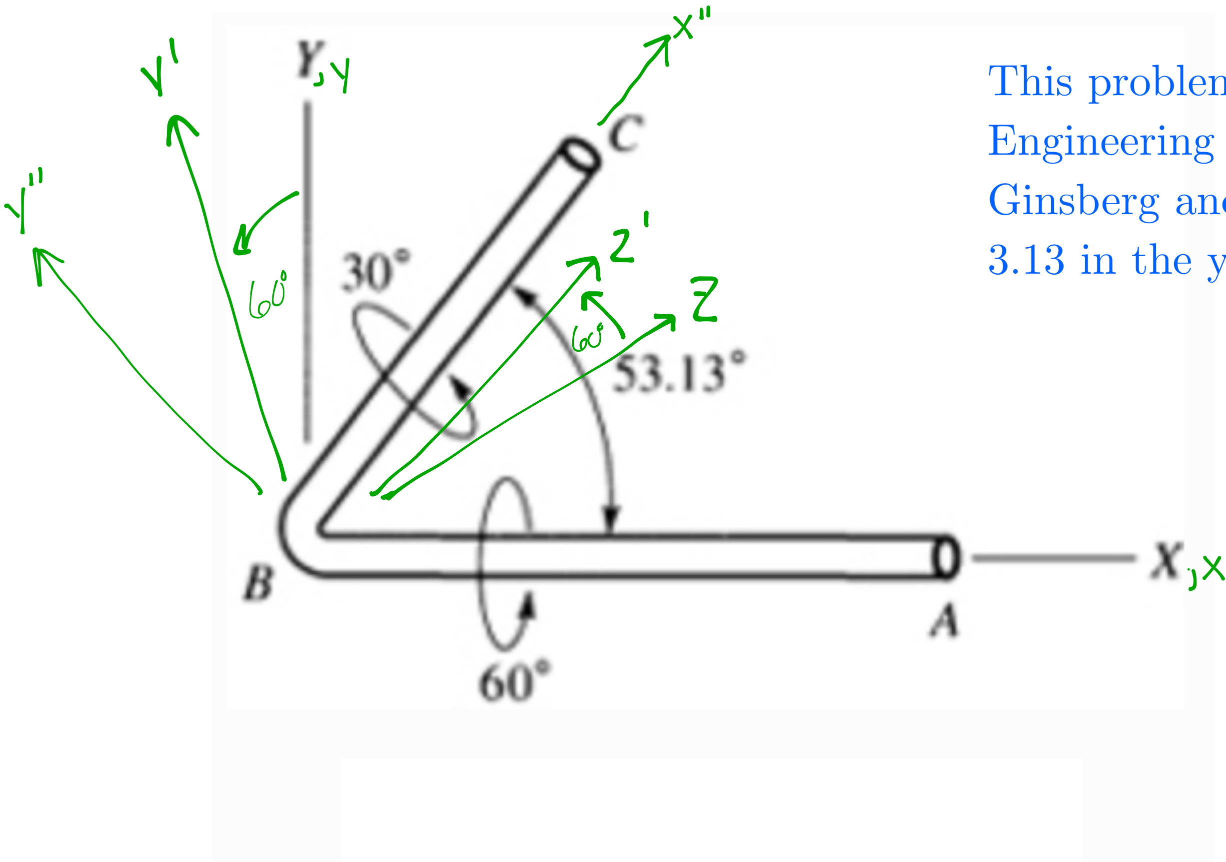
Problem 3.8

The bent rod is given a pair of rotations, first by 60° about line AB, and then 30° about line BC, with the sense of each rotation as shown in the sketch. Let xyz be a coordinate system fixed to the rod that initially aligned with the fixed XYZ system shown. Determine the transformation by which vector components with respect to XYZ may be converted to components with respect to xyz.



This problem is from Advanced Engineering Dynamics by Jerry Ginsberg and is similar to Exercise 3.13 in the your book.

General procedure:
- body-fixed rotation about AB
- rotate to align axis with BC
- rotate about BC,
- undo rotation to BC

Body fixed rotation about $\bar{z}$

$$R_{AB} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 60^\circ & \sin 60^\circ \\ 0 & -\sin 60^\circ & \cos 60^\circ \end{bmatrix}$$

Rotate to align x-axis with BC:

$$R_B = \begin{bmatrix} \cos 53.13 & \sin 53.13 & 0 \\ -\sin 53.13 & \cos 53.13 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

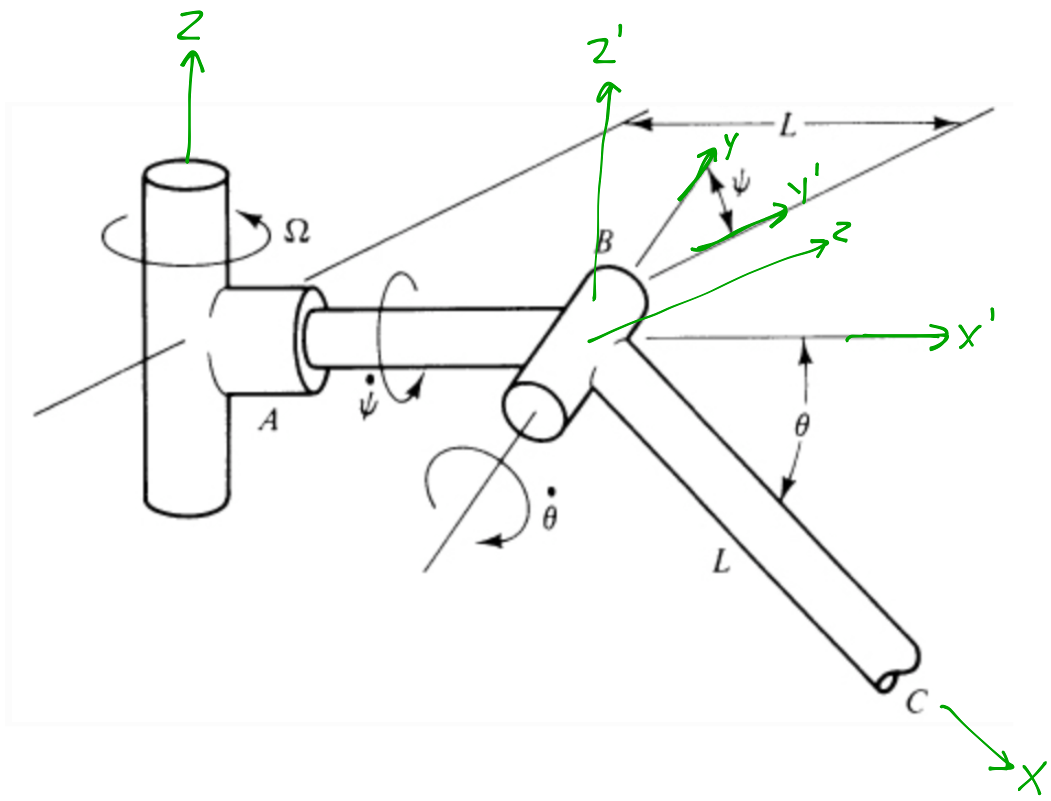
Rotate about BC

$$R_{BC} = \begin{bmatrix} \cos 30^\circ & \sin 30^\circ & 0 \\ -\sin 30^\circ & \cos 30^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Undo rotation to BC - R_B^T $\longrightarrow$ So, the total rotation matrix is $\boxed{R = R_B^T R_{BC} R_B R_{AB}}$

Problem 3.36 This problem is from Advanced Engineering Dynamics by Jerry Ginsberg

Consider the system in Problem 3.35 in a situation where $\dot{\psi}$ is constant at 2 rad/s, while is constant at 4 rad/s. The entire assembly is rotating about the vertical axis at the constant rate Ω = 8 rad/s. Determine the velocity and acceleration of end C at the instant when ψ = 53.13° and θ = 20°.



x'y'z' is fixed to link AB

xyz is fixed to link BC

$\bar{\omega}' = \Omega \bar{K} = \Omega \bar{k}'$ $\qquad \bar{\omega} = \Omega \bar{k}' + \dot{\psi}\bar{i}' + \dot{\theta}\bar{j}$

$\bar{\alpha}' = 0$ $\qquad \bar{\alpha} = \dot{\psi}(\bar{\omega}\times\bar{i}') + \dot{\theta}(\bar{\omega}\times\bar{j})$

$\bar{j} = \cos\psi\,\bar{j}' + \sin\psi\,\bar{k}'$

$\bar{v}_B = L\Omega\bar{j}'$ $\qquad \bar{a}_B = L\Omega(\Omega\bar{k}'\times\bar{j}') = L\Omega(-\Omega\bar{i}') = -L\Omega^2\bar{i}'$

$\bar{r}_{C/B} = L(\cos\theta\,\bar{i}' + \sin\theta\sin\psi\,\bar{j}' - \sin\theta\cos\psi\,\bar{k}')$

$(\bar{v}_C)_{xyz} = (\bar{a}_C)_{xyz} = 0$

$$\bar{v}_C = \bar{v}_B + \cancelto{0}{(v_C)_{xyz}} + \bar{\omega}'\times\bar{r}_{C/B} = L\Omega\bar{j}' + \left[\left(\dot{\psi}\bar{i}' + \dot{\theta}\cos\psi\,\bar{j}' + (\Omega+\dot{\theta}\sin\psi)\bar{k}'\right)\times L\left(\cos\theta\,\bar{i}' + \sin\theta\sin\psi\,\bar{j}' - \sin\theta\cos\psi\,\bar{k}'\right)\right]$$

$$= L\Omega\bar{j}' + \left[L\dot{\psi}\sin\theta\sin\psi\,\bar{k}' + L\dot{\psi}\sin\theta\cos\psi\,\bar{j}' - L\dot{\theta}\cos\psi\cos\theta\,\bar{k}' - L\dot{\theta}\cos^2\psi\sin\theta\,\bar{i}' - L(\Omega+\dot{\theta}\sin\psi)\cos\theta\,\bar{j}' - L(\Omega+\dot{\theta}\sin\psi)\sin\theta\sin\psi\,\bar{i}'\right]$$

$$\boxed{\bar{v}_C = \left[-L\dot{\theta}\cos^2\psi\sin\theta - L(\Omega+\dot{\theta}\sin\psi)\sin\theta\sin\psi\right]\bar{i}' + \left[L\Omega + L\dot{\psi}\sin\theta\cos\psi - L(\Omega+\dot{\theta}\sin\psi)\cos\theta\right]\bar{j}' + \left[L\dot{\psi}\sin\theta\sin\psi - L\dot{\theta}\cos\psi\cos\theta\right]\bar{k}'}$$

$$\bar{a}_C = \bar{a}_B + \cancelto{0}{(a_C)_{xyz}} + \bar{\alpha}\times\bar{r}_{C/B} + \bar{\omega}\times(\bar{\omega}\times\bar{r}_{C/B}) + 2\bar{\omega}\times\cancelto{0}{(v_C)_{xyz}}$$