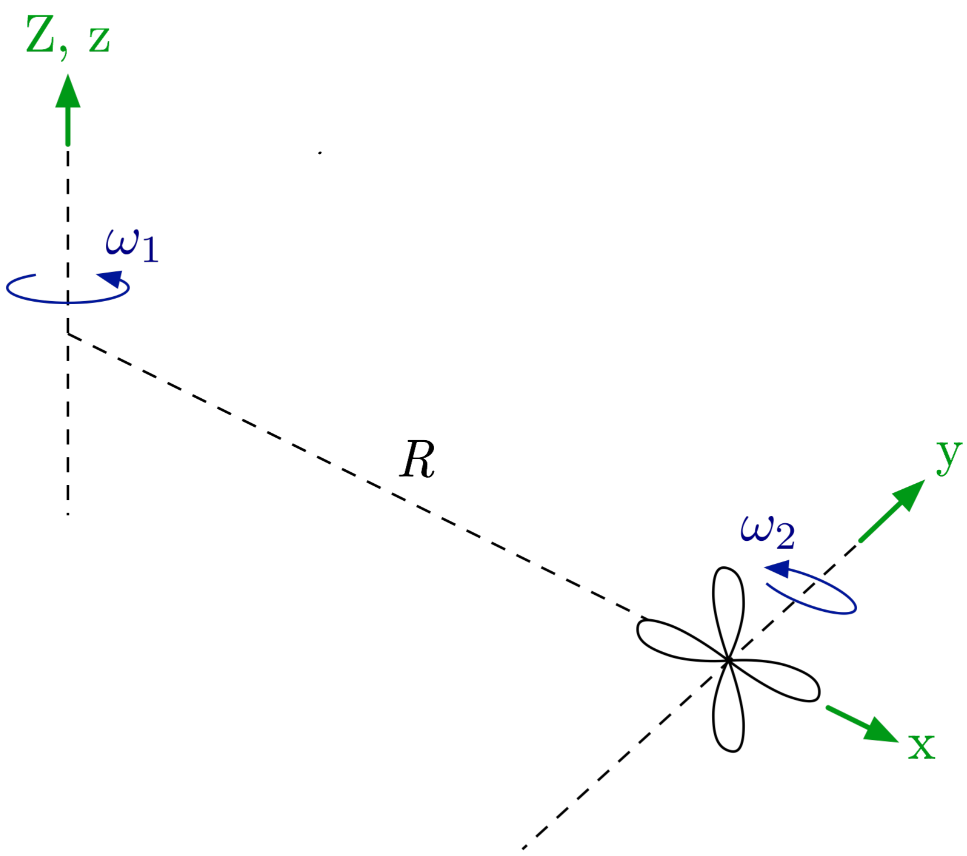
Example

The radiator fan of an automobile engine whose crankshaft is aligned with the longitudinal axis is rotating at 1,000 rev/min, clockwise as viewed from the front of the vehicle. The automobile is following a 60-m radius left turn at a constant speed of 75 km/hr. Determine the angular velocity and angular acceleration of the fan.



fan rotation rate → $\omega_2 = 1000\left(\frac{2\pi}{60}\right)\frac{rad}{s}$

rotation rate due to the circular path can be determined by its speed and the radius of curvature.

$$v = \left(75\ \frac{km}{h}\right)\left(\frac{1\ h}{3600\ s}\right)\left(\frac{1000\ m}{1\ km}\right) = 75\left(\frac{1}{36}\right)\frac{m}{s}$$

axes xyz are fixed with y along the fan axis of rotation

$$\bar{\omega} = (-\omega_2)\bar{J} + \omega_1\bar{K} \longrightarrow \omega_1 = \frac{v}{R}$$

$\bar{k} = \bar{K}$

$$\boxed{\omega = -\omega_2\bar{J} + \frac{v}{R}\bar{K}}$$

To find the accel:

both ω_1 and ω_2 are constant, so $\dot{\omega}_1 = \dot{\omega}_2 = 0$

$$\bar{\alpha} = \frac{d\bar{\omega}}{dt} = \frac{d}{dt}\left(-\omega_2\bar{J} + \omega_1\bar{K}\right) = \frac{d}{dt}\left(-\omega_2\bar{J}\right) + \cancel{\frac{d}{dt}\left(\omega_2\bar{K}\right)} \rightarrow 0$$

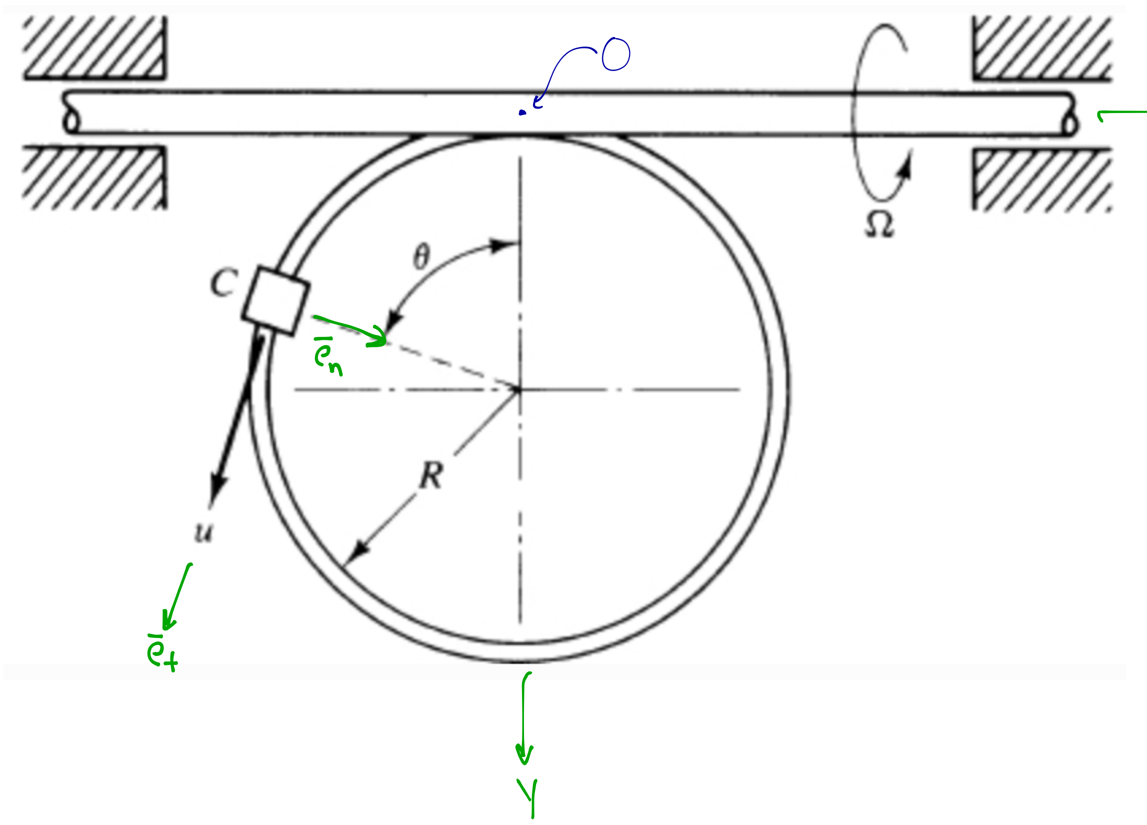
$\dot{\omega}_1 = 0$ and $\dot{\bar{K}} = 0$

$$= -\underbrace{\cancel{\dot{\omega}_2}}_{\dot{\omega}_2=0}\bar{J} - \bar{\omega}_2\left(\underbrace{\omega_1\bar{k}}_{\omega\text{ of the frame}} \times \bar{J}\right)$$

$$\boxed{\bar{\alpha} = -\omega_1\left(-\omega_2\bar{i}\right) = \omega_1\omega_2\bar{i}}$$

Example

Collar C slides relative to the curved rod at a constant speed u, while the rod rotates about the horizontal axis at the constant rate Ω. Determine the acceleration of the collar in terms of θ.



xyz is fixed to the rod

$\bar{\omega} = \Omega \bar{i}$ ← angular velocity of the frame

$\bar{\alpha} = 0 \qquad \bar{v}_O = \bar{a}_O = 0$

$\bar{r}_{C/O} = -R\sin\theta\,\bar{i} + (R - R\cos\theta)\,\bar{j}$

$(\bar{v}_C)_{xyz} = u\,\bar{e}_t = u(-\cos\theta\,\bar{i} + \sin\theta\,\bar{j})$

$(\bar{a}_C)_{xyz} = \frac{u^2}{R}\bar{e}_n = \frac{u^2}{R}(\sin\theta\,\bar{i} + \cos\theta\,\bar{j})$

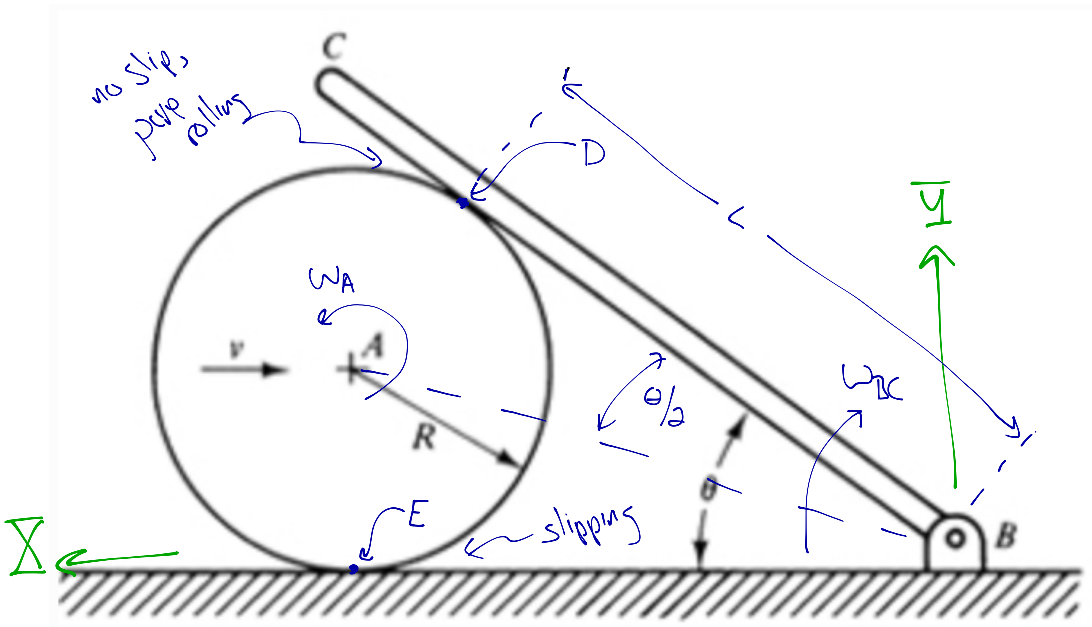
$$\bar{a}_C = \cancel{\bar{a}_O}^{\,0} + (\bar{a}_C)_{xyz} + \cancel{\bar{\alpha}\times\bar{r}_{C/O}}^{\,0} + \bar{\omega}\times(\bar{\omega}\times\bar{r}_{C/O}) + 2\bar{\omega}\times(\bar{v}_C)_{xyz}$$

$$= \frac{u^2}{R}(\sin\theta\,\bar{i} + \cos\theta\,\bar{j}) + \Omega\bar{i}\times\left(\Omega(R - R\cos\theta)\bar{k}\right) + 2\Omega u\sin\theta\,\bar{k}$$

$$\boxed{\bar{a}_C = \frac{u^2}{R}\sin\theta\,\bar{i} + \left(\frac{u^2}{R}\cos\theta - R\Omega^2(1-\cos\theta)\right)\bar{j} + 2\Omega u\sin\theta\,\bar{k}}$$

Example

In the position shown, cylinder A is moving to the right such that its center has a speed v. There is no slipping between the cylinder and bar BC, but there is slipping between the cylinder and the ground. Determine the angular velocity and angular acceleration of bar BC, and the velocity and acceleration of the cylinder at the point where it contacts the ground.



$$\bar{V}_A = -v\bar{I}$$

$$\bar{V}_D = V_A + \bar{\omega}_A \times \bar{r}_{D/A} = \bar{\omega}_{BC} \times \bar{r}_{D/C}$$

$$\bar{r}_{D/C} = L\cos\theta \bar{I} + L\sin\theta \bar{J}$$

$$\bar{r}_{D/A} = -R\sin\theta \bar{I} + R\cos\theta \bar{J}$$

$$L = R\cot\frac{\theta}{2}$$

$$\bar{V}_D = -v\bar{I} - \omega_A \bar{K} \times \left[(-R\sin\theta \bar{I} + R\cos\theta \bar{J})\right] = -v\bar{I} + R\omega_A \sin\theta \bar{J} - R\omega_A\cos\theta \bar{I} = (-v_A + R\omega_A\cos\theta)\bar{I} + R\omega_A\sin\theta\bar{J}$$

$$\bar{V}_D = \omega_{BC}\bar{K} \times (L\cos\theta\bar{I} + L\sin\theta\bar{J}) = L\omega_{BC}\cos\theta\bar{J} - L\omega_{BC}\sin\theta\bar{I}$$

Equate components in each direction:

I direction

$$-v + R\omega_A\cos\theta = -L\omega_{BC}\sin\theta$$

J-direction

$$R\omega_A\sin\theta = L\omega_{BC}\cos\theta$$

Solved using SymPy

$$\omega_A = \frac{v}{R\cos\theta} \quad \text{and} \quad \omega_{BC} = \frac{v}{L\sin\theta}$$

Now, use $\bar{\omega}_A$ to find $\bar{V}_e$:

$$\bar{V}_E = \bar{V}_A + \bar{\omega}_A \times \bar{r}_{E/A} = -v\bar{I} + \left(\frac{-v}{R\cos\theta}\right)\bar{K} \times -R\bar{J} = -v\bar{I} + -v\cos\theta\bar{I} \rightarrow \boxed{\bar{V}_E = -v(1+\cos\theta)\bar{I}}$$