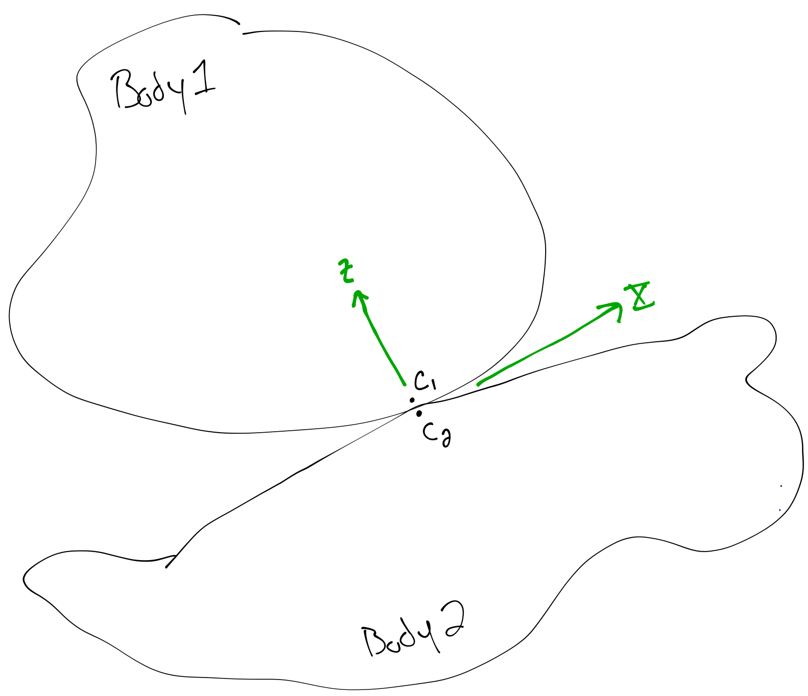
Rolling (Sec. 4.4)

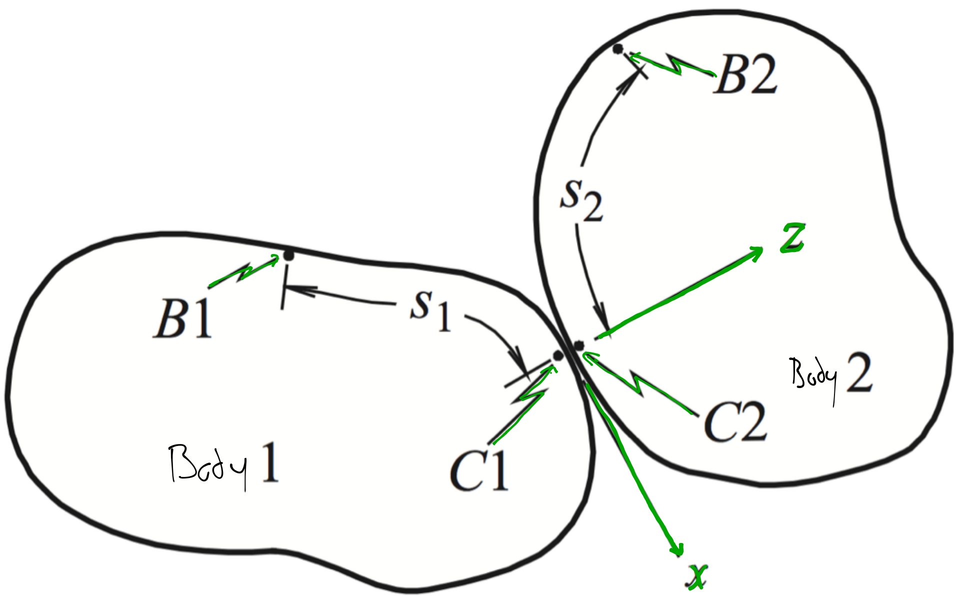


C_1 - contact point on body 1
C_2 - Contact point on body 2

The two bodies cannot penetrate one another, so velocities of the points of contact must be equal along the contact axis

$$\bar{v}_{C_1} \cdot \bar{K} = \bar{v}_{C_2} \cdot \bar{K}$$

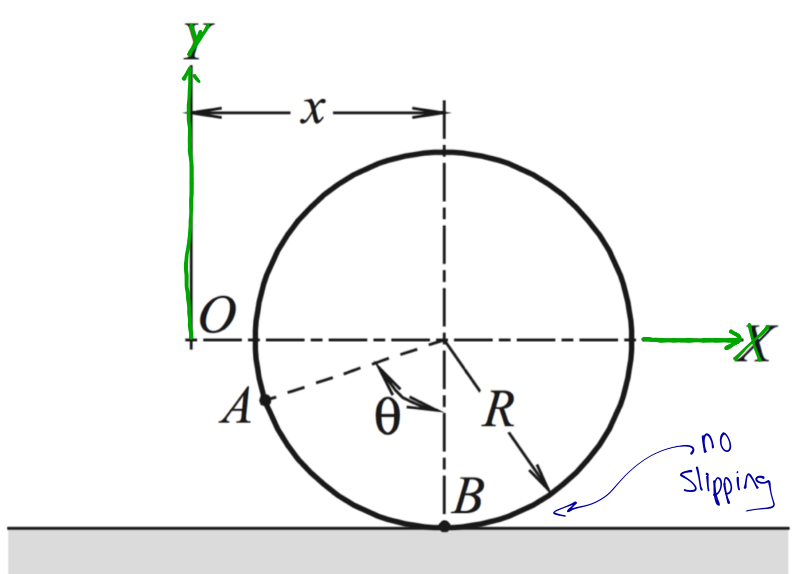
Rolling without Slipping



If there is no slipping, then the arc length along the two bodies during rolling must also be equal.

$s_1 = s_2 \rightarrow$ arclength $B_1 \rightarrow C_1$ = arclength $B_2 \rightarrow C_2$

Rolling without Slipping along the Ground



Q: If no slipping, what is the distance x?

arclength of rolling $\theta \rightarrow r\theta$

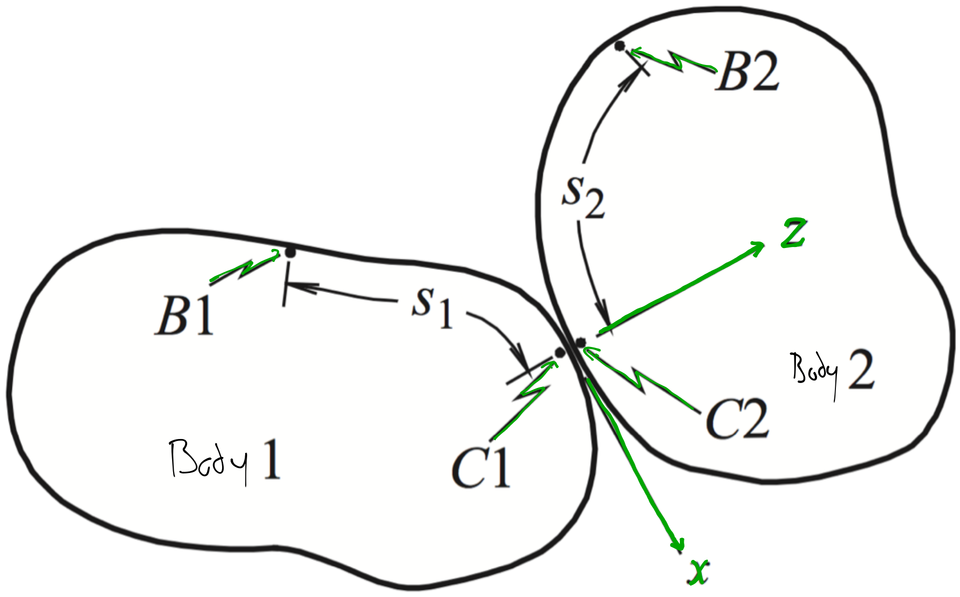
So,

$$\bar{r}_{A/O} = (x - R\sin\theta)\bar{I} + (-R\cos\theta)\bar{J}$$

$$= (R\theta - R\sin\theta)\bar{I} + (-R\cos\theta)\bar{J}$$

We can take derivatives of this to find $\bar{v}_A$ and $\bar{a}_A$, but this "arclength" method gets more difficult for "general" rolling

Rolling (cont.)



In the general case…

Consider C_i and B_i to be very close.

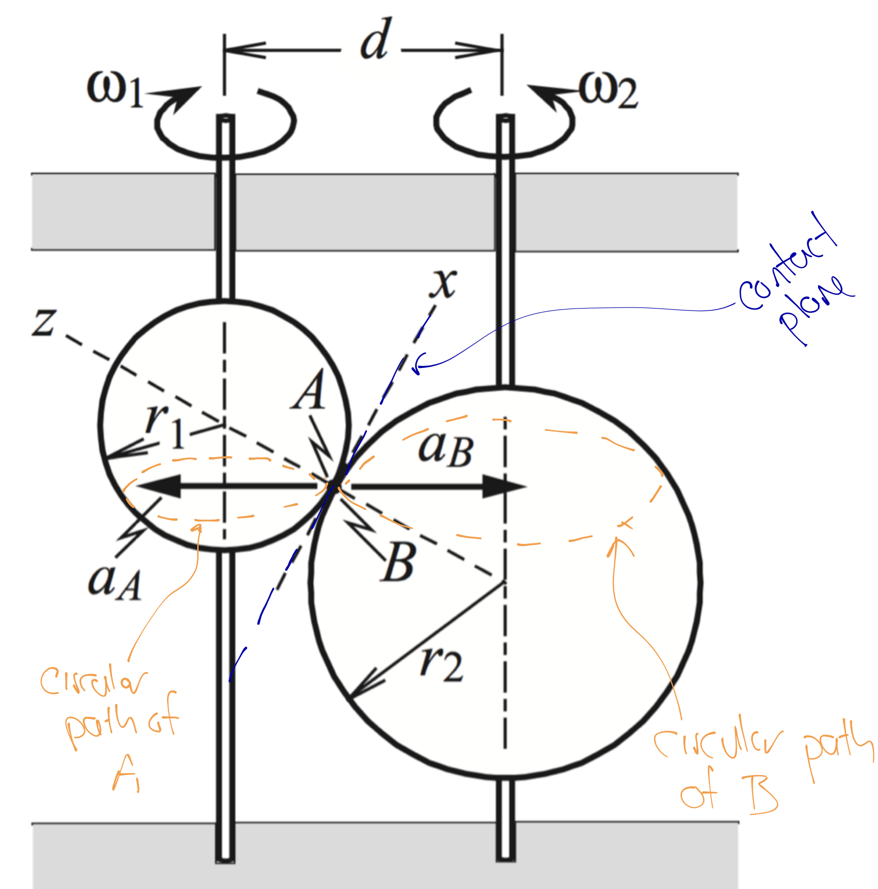
In the limit, the points on the two bodies have equal velocity in all directions

$\bar{v}_{C_1} = \bar{v}_{C_2}$ = in all directions if no slipping

Q: What about the accel. of the contact point?

Not always $\perp$ to contact plane (as planar-only analysis suggests)

Consider:

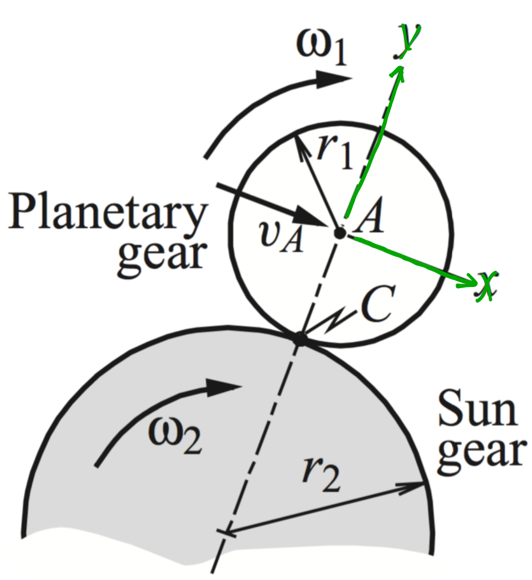


Accel of A and B are not $\perp$ to contact plane.

A and B follow circular paths, so accel is toward the center of that path

Q: So, how do we find the accel of the contact points?

Use known constraints on the paths of the points.



We know that A will follow a circular path of radius r_1+r_2

So, $\bar{v}_A = v_A \hat{i}$ and $\bar{a}_A = \dot{v}_A \hat{i} - \frac{v_A^2}{r_1+r_2}\hat{j}$

Because there is no slipping, we can relate $\bar{v}_A$ to $\bar{v}_C$ by:

$$\bar{v}_A = \bar{v}_C + (-\omega_1 \hat{k}) \times \bar{r}_{A/C} = \bar{v}_C + r_1\omega_1 \hat{i} = (r_1\omega_1 + r_2\omega_2)\hat{i}$$

$$\bar{v}_C = \bar{v}_O + (-\omega_2 \hat{k}) \times \bar{r}_{C/O} = r_2\omega_2 \hat{i}$$