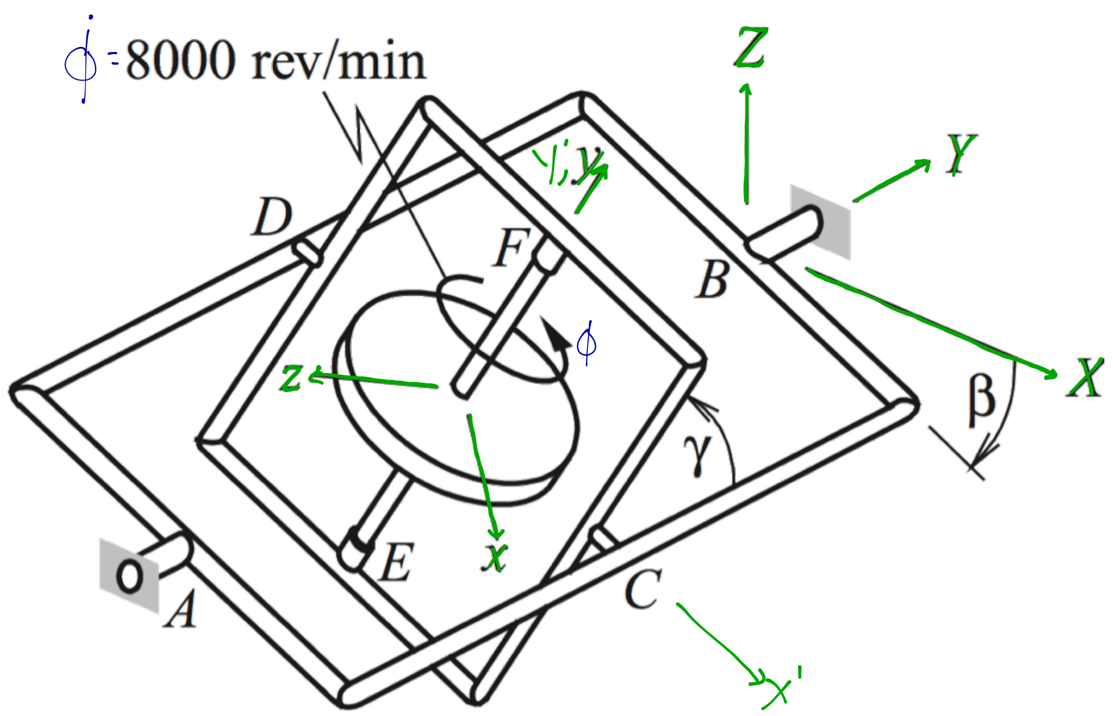
Example 4.2

EXAMPLE 4.2 A free gyroscope consists of a flywheel that rotates relative to the inner gimbal at the constant angular speed of 8000 rev/min, and the rotation of the inner gimbal relative to the outer gimbal is $\gamma = 0.2\sin(100\pi t)$ rad. The rotation of the outer gimbal is $\beta = 0.5\sin(50\pi t)$ rad. Use the Eulerian angle formulas to determine the angular velocity and angular acceleration of the flywheel at $t = 4$ ms. Express the results in terms of components relative to the body-fixed xyz and space-fixed XYZ reference frames, where the x axis was directed from bearing D to bearing C at $t = 0$.



- Rotation about a fixed axis = precession

  Here, that's β about the Y axis

- Line of nodes is $\perp$ to precession axis — nutation

  Here, that axis aligns to CD

  $\bar{e}_{C/D} = x'$ axis

- Spin axes $\perp$ to line of nodes

  Here, that's the y axis

Precession – β about Y

Nutation – γ about x' ($\bar{e}_{C/D}$)

Spin – ϕ about y

So,

$$\bar{\omega} = R_\phi R_\gamma R_\beta \begin{bmatrix} 0 \\ \dot{\beta} \\ 0 \end{bmatrix}_{XYZ} + R_\phi R_\gamma \begin{bmatrix} \dot{\gamma} \\ 0 \\ 0 \end{bmatrix}_{x'y'z'} + \begin{bmatrix} 0 \\ \dot{\phi} \\ 0 \end{bmatrix}_{xyz}$$

$$R_\beta = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \quad R_\gamma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & \sin\gamma \\ 0 & -\sin\gamma & \cos\gamma \end{bmatrix} \quad R_\phi = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix}$$

To complete, write $\bar{\omega}$ and $\bar{\alpha} = \frac{d\bar{\omega}}{dt}$ in xyz and substitute numerical values at $t = 4$ ms

Then, rotate from xyz to XYZ using rotation matrices

See back and Jupyter Notebook for numerical sol.