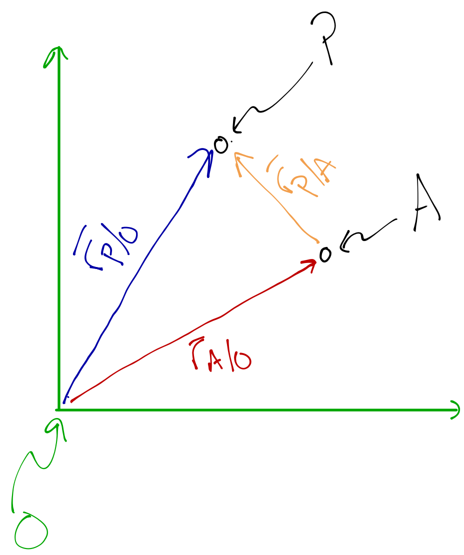
Chapter 1 - Basic Considerations

Dr. Ginsberg begins with a quick review of vector math. We will not cover most of it, but you need to make sure you understand it.

Notation



$\bar{r}_{P/O}$ – position of point P relative to point O

$$\bar{r}_{P/O} = \bar{r}_{A/O} + \bar{r}_{P/A}$$

← Fixed, Inertial Frame

$$\bar{v}_P = \frac{d}{dt}\left(\bar{r}_{P/O}\right) = \dot{\bar{r}}_{P/O}$$ ← This is the absolute velocity. We'll talk about relative velocities later.

Note: no "with respect to" subscript for velocity ← It's the same for all observers in the fixed frame

$$\bar{a}_P = \frac{d}{dt}\left(\bar{v}_P\right) = \ddot{\bar{r}}_{P/O}$$ ← Acceleration

This note applies for accel too.

Newton's Laws

1) If $\sum \bar{F} = 0$, then $\bar{v} \equiv$ constant ← "Objects in motion..."

2) $\sum \bar{F} = m\bar{a}$

3) $\bar{F}_{12} = \bar{F}_{21}$ ← "Equal but opposite..."

I'm being a bit sloppy with notation here

• Strictly only address particles

• All are vector eq.

Energy and Momentum (Sec. 1.2.3)

Just integrals of Newton's 2^{nd} Law

Useful when how the force varies as a function of particle position is known

We know $\sum \bar{F}(\bar{r})$

Write the displacement of a particle in time Δt as

$$\Delta \bar{r} = \bar{r}(t+\Delta t) - \bar{r}(t)$$

Shrink $\Delta t \to dt \implies \Delta \bar{r} \to d\bar{r}$

Now, take the dot product of this with 2^{nd} Law

$$\left(\sum \bar{F} = m\bar{a}\right) \cdot d\bar{r} \longrightarrow \sum \bar{F} \cdot d\bar{r} = m\bar{a} \cdot d\bar{r}$$

$$\sum \bar{F} \cdot d\bar{r} = m\left(\frac{d\bar{v}}{dt}\right) \cdot \left(\bar{v}\,dt\right) \qquad d\bar{r} = \bar{v}\,dt$$

$$\sum \bar{F} \cdot d\bar{r} = \frac{1}{2} m \left[\frac{d}{dt}\left(\bar{v} \cdot \bar{v}\right)\right] \qquad \frac{d\bar{v}}{dt} \cdot \bar{v} = \bar{v} \cdot \frac{d\bar{v}}{dt}$$

Taking the path integral;

$$\oint_1^2 \sum \bar{F} \cdot d\bar{r} = \frac{1}{2} m \left(\bar{v}_2 \cdot \bar{v}_2\right) - \frac{1}{2} m \left(\bar{v}_1 \cdot \bar{v}_0\right) \longleftarrow$$

1 = starting point
2 = ending point

Kinetic Energy (of a particle)

$$T = \frac{1}{2} m \left(\bar{v} \cdot \bar{v}\right) = \frac{1}{2} m |\bar{v}|^2$$

Then the work done to move a particle from $1 \to 2$ is

$$W_{1\to 2} = \oint_1^2 \sum \bar{F}(\bar{r}) \cdot d\bar{r} \quad \text{or} \quad T_2 = T_1 + W_{1\to 2} \longleftarrow$$

Work-energy Priciple

Energy and Momentum (cont.)

Now, let's consider the case where we know $\underbrace{\Sigma \bar{F}(t)}_{\text{How force changes with time}}$ not $\Sigma \bar{F}(\bar{r})$

Simply integrate Newton's 2^{nd} Law with respect to time (after mult. by dt)

$$\Sigma \bar{F} = m\bar{a} \rightarrow \Sigma \bar{F} dt = m\bar{a}\, dt \rightarrow \Sigma \bar{F} dt = m \frac{d\bar{v}}{\cancel{dt}} \cancel{dt} \rightarrow \Sigma \bar{F} dt = m\, d\bar{v}$$

Now integrate over $t_1 \rightarrow t_2$

$$\int_{t_1}^{t_2} \Sigma \bar{F} dt = \int_{t_1}^{t_2} m\, d\bar{v} = m(\bar{v}_2 - \bar{v}_1)$$

velocity at t_2 velocity at t_1

Linear Momentum (of a particle)

$$P = m\bar{v}$$

So, we can write

$$\bar{P}_2 = \bar{P}_1 + \int_{t_1}^{t_2} \Sigma \bar{F} dt \leftarrow \text{Linear Impulse-momentum Principle}$$

Note:

- This is a vector equation (really 3 components)
- Rare to know all components of $\Sigma \bar{F}$ for all time
- Instead, often use this for impulsive forces (high amp, short duration) $\leftarrow$ Ex.) collisions

Energy and Momentum (cont.)

Let's look at the 2nd Law's relationship to moments

$$\Sigma \bar{M}_O = \bar{r}_{P/O} \times \Sigma \bar{F}$$

This will always work for calculating moments

So

$$\Sigma \bar{M}_O = \bar{r}_{P/O} \times \Sigma \bar{F} = \bar{r}_{P/O} \times (m\bar{a})$$

$$= \bar{r}_{P/O} \times \left(m \frac{d\bar{v}}{dt}\right)$$

$$\Sigma \bar{M}_O = \bar{r}_{P/O} \times \left(m \frac{d\bar{v}}{dt}\right)$$

bring $\frac{d}{dt}$ out front. Need to add 2nd term to maintain same equation

$$= \frac{d}{dt}\left[\bar{r}_{P/O} \times m\bar{v}\right] - \underbrace{\frac{d\bar{r}_{P/O}}{dt} \times m\bar{v}}$$

This is $\bar{v} \times m\bar{v} \equiv 0$ ← always 0

So

$$\Sigma \bar{M}_O = \frac{d}{dt}\left[\bar{r}_{P/O} \times m\bar{v}\right]$$

Angular Momentum

(also moment of momentum)

$$\bar{H}_O = \bar{r}_{P/O} \times m\bar{v}$$

So

$$\Sigma \bar{M}_O = \frac{d}{dt}\bar{H}_O = \dot{\bar{H}}_O$$

← we'll use this way more than this

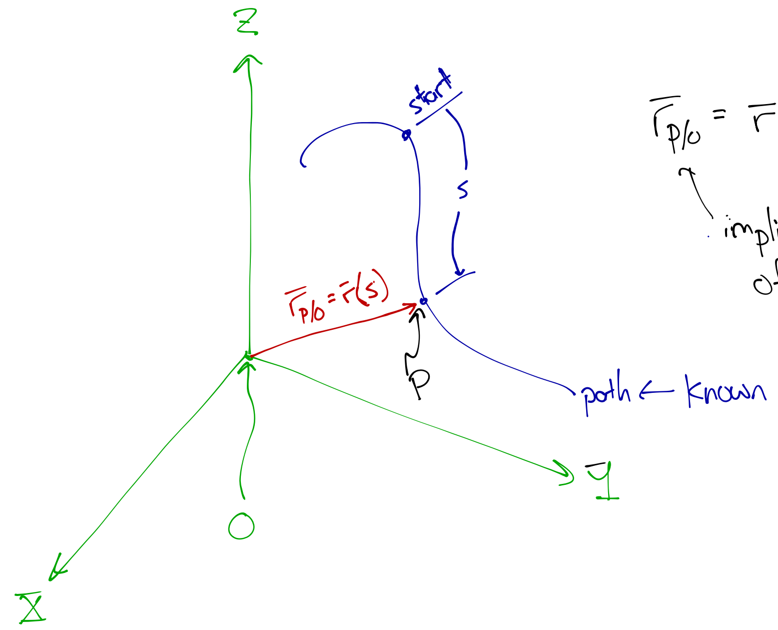
Integrate over $t_1 \to t_2$

$$(\bar{H}_O)_2 = (\bar{H}_O)_1 + \int_1^2 \Sigma \bar{M}_O \, dt$$

← Angular Impulse-momentum Principle

Chapter 2 - Particle Kinematics

Path Variables (Sec 2.1) ← Also called <u>intrinsic coordinates</u>



$\bar{r}_{P/O} = \bar{r}(s)$ and $s(t)$

implicit function of time, because s is a function of time

+s = "forward" along path
−s = "backward" along path

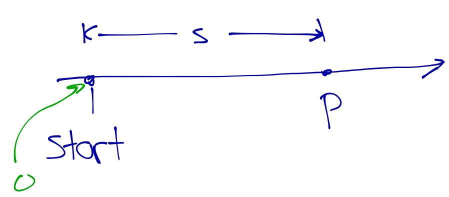
Q: What's the velocity of point P?

$$\bar{v} = \frac{d\bar{r}}{dt} = \frac{d\bar{r}}{ds}\frac{ds}{dt} = \dot{s}\underbrace{\frac{d\bar{r}}{ds}}_{\text{determined by the position dependence on path}}$$

Q: What about acceleration of P?

$$\bar{a} = \frac{d\bar{v}}{dt} = \frac{d}{dt}\left(\dot{s}\frac{d\bar{r}}{ds}\right) = \ddot{s}\frac{d\bar{r}}{ds} + \dot{s}\frac{d}{dt}\underbrace{\left(\frac{d\bar{r}}{ds}\right)}_{\text{determined solely by } s} = \ddot{s}\frac{d\bar{r}}{ds} + \dot{s}^2\underbrace{\left(\frac{d^2\bar{r}}{ds^2}\right)}_{\text{Another property of the path}}$$

Look at a straight line in direction $\bar{e}$, so



$\bar{r}_{P/O} = s\bar{e}$,

$\frac{d\bar{r}}{ds} = \bar{e}$

$\bar{e}$'s direction is constant, so $\frac{d^2\bar{r}}{ds^2} = 0$

$$\underbrace{\bar{v} = \dot{s}\bar{e} \quad \text{and} \quad \bar{a} = \ddot{s}\bar{e}}_{\text{only aligned here b/c } \bar{e} = \text{const in this problem}}$$

<u>Not generally true!</u>

Tangent and Normal Components (Sec. 2.1.1)

Define $\bar{e}_t$ as the direction tangent to the path at any point along it

$$\bar{e}_t = \frac{d\bar{r}}{ds}$$ ← defines unit vector tangent to path

Now, look at the $\frac{d^2\bar{r}}{ds^2}$ from the earlier equation:

$$\frac{d^2\bar{r}}{ds^2} = \frac{d\bar{e}_t}{ds}$$

Q: What's the relationship of directions $\bar{e}_t$ and $\frac{d\bar{e}_t}{ds}$?

$\perp \rightarrow \bar{e}_t \cdot \frac{d\bar{e}_t}{ds} = 0$

So, use this to define the normal direction

$$\underbrace{\bar{e}_n = \rho \frac{d\bar{e}_t}{ds}}_{\text{points toward "center" of curve}} \qquad \rho \equiv \frac{1}{\left|\frac{d\bar{e}_t}{ds}\right|}$$

← needed to scale $\bar{e}_n$ to a unit vector

Q: What is this physically?

radius of curvature

Let's use these unit vectors to rewrite the particle velocity and accel:

$$\bar{v} = v\bar{e}_t, \quad \text{where } \underbrace{v}_{\text{scalar}} = \dot{s} \leftarrow \text{speed}$$

$$\bar{a} = \dot{v}\bar{e}_t + \frac{v^2}{\rho}\bar{e}_n$$

These two vectors always lie in the osculating plane formed by $\bar{e}_t$ and $\bar{e}_n$

Note: The osculating plane is only constant if the path is planar

Define a 3rd unit vector in the binormal direction:

$$\bar{e}_b = \bar{e}_t \times \bar{e}_n$$

Tangent and Normal Components (cont.)

We can use Newton's 2^{nd} Law with these coords

Key point: We can pick the coordinate systems used to describe systems. A smart choice can save a lot of work.

$\sum \bar{F} = m\bar{a}$ ← all components must be equal so

$$\sum \bar{F} = m\left[\dot{v}\bar{e}_t + \frac{v^2}{\rho}\bar{e}_n\right]$$

$\sum \bar{F}_t = \sum \bar{F}\cdot\bar{e}_t = m\dot{v}$ ← tangential

$\sum F_n = \sum \bar{F}\cdot\bar{e}_n = m\frac{v^2}{\rho}$ ← normal

$\sum F_b = \sum \bar{F}\cdot\bar{e}_b = 0$ ← binormal

usually forces in these directions will be constraint forces, keeping the particle on the path

Review Sections 2.1.2 and 2.1.3 on your own for now.
We may return to them later.