MCHE 513: Intermediate Dynamics
Fall 2018 – Homework 4

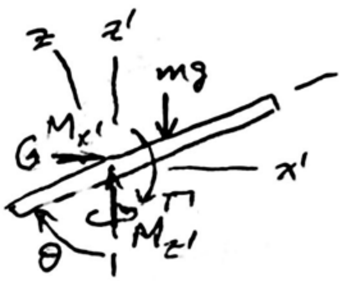
Assigned:	Thursday, October 18th
Due:	Friday, October 26th, 5pm

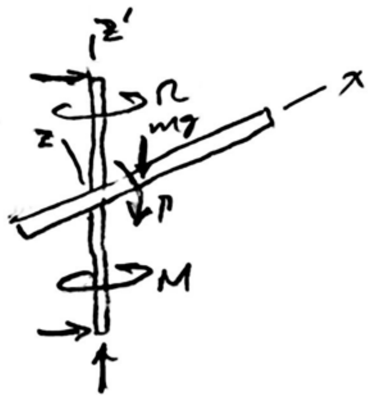
Assignment: From "Engineering Dynamics" by Jerry Ginsberg, problems:
6.12, 6.13, 6.32, 6.41

Submission: Emailed *single* pdf document:

- to `joshua.vaughan@louisiana.edu`
- with subject line and filename `ULID-MCHE513-HW4`, where `ULID` is your ULID
- *Note:* Submissions with incorrect filenames or submitted as multiple images/pdfs will be rejected.

Exercise 6.12





Given $\theta=\frac{\pi}{2}\sin(\omega t)$, constant Ω.

Find Γ and M at $\theta=\frac{\pi}{2}$ and $\pi/3$.

Solution: Consider the isolated bar to find Γ, then the bar on the shaft to find M.

$I_{xx}=0,\ I_{yy}=I_{zz}=\frac{1}{12}mL^2+m\left(\frac{L}{6}\right)^2$

$\bar{\omega}=\Omega\bar{k}'+\dot{\theta}\bar{j}$, $\bar{\alpha}=\ddot{\theta}\bar{j}+\Omega\dot{\theta}(\bar{k}'\times\bar{j})$

Set $\bar{k}'=\cos\theta\,\bar{i}+\sin\theta\,\bar{k}$

$\bar{\omega}=\Omega\cos\theta\,\bar{i}+\dot{\theta}\bar{j}+\Omega\sin\theta\,\bar{k}$

$\bar{\alpha}=\Omega\dot{\theta}(-\sin\theta\,\bar{i}+\cos\theta\,\bar{k})+\ddot{\theta}\bar{j}$

Isolated bar: $\sum\bar{M}_c\cdot\bar{j}=mg\left(\frac{L}{6}\sin\theta\right)+\Gamma=\dot{\bar{H}}_c\cdot\bar{j}$

Assembly: $\sum\bar{M}_c\cdot\bar{k}'=M$ but $\bar{k}'=\cos\theta\,\bar{i}+\sin\theta\,\bar{k}$

so $M=\dot{\bar{H}}_c\cdot(\cos\theta\,\bar{i}+\sin\theta\,\bar{k})$

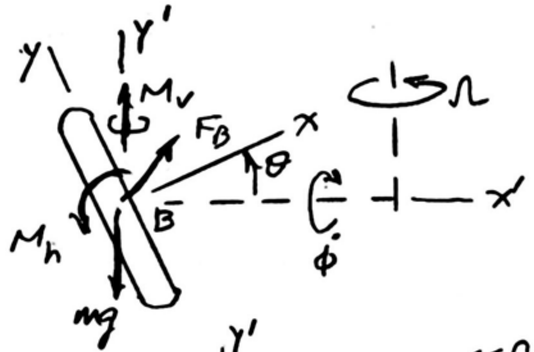
$\bar{H}_c=\frac{1}{9}mL^2(\dot{\theta}\bar{j}+\Omega\sin\theta\,\bar{k})$, $\frac{\partial\bar{H}_c}{\partial t}=\frac{1}{9}mL^2(\ddot{\theta}\bar{j}+\Omega\dot{\theta}\cos\theta\,\bar{k})$

$\dot{\bar{H}}_c=\frac{\partial\bar{H}_c}{\partial t}+\bar{\omega}\times\bar{H}_c=\frac{1}{9}mL^2[\ddot{\theta}-\Omega^2\sin\theta\cos\theta]\bar{j}+\frac{2}{9}mL^2\Omega\dot{\theta}\cos\theta\,\bar{k}$

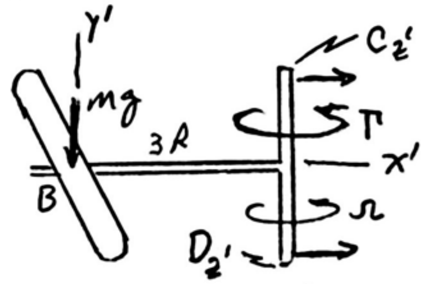
so $\Gamma=\frac{1}{9}mL^2(\ddot{\theta}-\Omega^2\sin\theta\cos\theta)$ ◁

$M=\frac{2}{9}mL^2\Omega\dot{\theta}\sin\theta\cos\theta$ ◁

Exercise 6.13



Given constant Ω, $\phi = 0$ in illustrated position.
Find equation of motion for ϕ and an expression for Γ.



Solution: Because the mass of the T-bar is zero, it can be included in the free body diagram of the disk

In the FBD of the isolated disk, M_v & M_h are couple reactions exerted by the shaft. These are internal for the FBD of the assembly.

Attach xyz to the disk and $x'y'z'$ to the shaft.

Transformation: Starting from $x'y'z'$, rotate by $+\theta$ about z', then $-\phi$ about $x' \Rightarrow$ body-fixed, so

$$[x\,y\,z]^T = [R][x'\,y'\,z']^T, \quad [R] = [R_z(\theta)][R_x(-\phi)]$$

$$[R] = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

Then $\bar\omega = \Omega \bar j' - \dot\phi \bar i'$, $\bar\alpha = -\ddot\phi \bar i' - \dot\phi(\Omega \bar j' \times \bar i') = -\ddot\phi \bar i' + \Omega\dot\phi \bar k'$

Convert to xyz components:

$$\begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix} = [R] \begin{Bmatrix} -\dot\phi \\ \Omega \\ 0 \end{Bmatrix} = \begin{Bmatrix} \Omega\cos\phi\sin\theta - \dot\phi\cos\theta \\ \Omega\cos\phi\cos\theta + \dot\phi\sin\theta \\ \Omega\sin\phi \end{Bmatrix}$$

$$\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{Bmatrix} = [R] \begin{Bmatrix} -\ddot\phi \\ 0 \\ \Omega\dot\phi \end{Bmatrix} = \begin{Bmatrix} -\ddot\phi\cos\theta - \Omega\dot\phi\sin\phi\sin\theta \\ \ddot\phi\sin\theta - \Omega\dot\phi\sin\phi\cos\theta \\ \Omega\dot\phi\cos\phi \end{Bmatrix}$$

$I_{xx} = \frac{1}{2}mR^2$, $I_{yy} = I_{zz} = \frac{1}{4}mR^2$

Exercise 6.13 (cont.)

$$\{\dot{H}_B\} = [I]\{\alpha\} + \{\omega\} \otimes [I]\{\omega\}$$

$$= \frac{1}{4} mR^2 \begin{Bmatrix} 2\dot{\phi}\cos\theta + \Omega\dot{\phi}\sin\phi\sin\theta \\ \ddot{\phi}\sin\theta + \Omega^2\sin\phi\cos\phi\sin\theta - 2\Omega\dot{\phi}\sin\phi\cos\theta \\ \cos\theta\left[\dot{\phi}^2\sin\theta - \Omega^2(\cos\phi)^2\sin\theta + 2\Omega\dot{\phi}\cos\phi\cos\theta\right] \end{Bmatrix}$$

From the FBD of the isolated disk

$$\Sigma\bar{M}_B \cdot \bar{i}' = 0 = \dot{\bar{H}}_B \cdot \bar{i}'$$

From the FBD of the assembly

$$\Sigma\bar{M}_O \cdot \bar{j}' = \Gamma - (C_{z'} + D_{z'})\,3R = \dot{\bar{H}}_B \cdot \bar{j}'$$

but $\Sigma\bar{F} = m\bar{a}_B$ with $\bar{a}_B = (3R)\Omega^2\bar{i}'$

so $\Sigma\bar{F} \cdot \bar{k}' = C_{z'} + D_{z'} = 0 \Rightarrow \Gamma = \dot{\bar{H}}_B \cdot \bar{j}'$

Use $[R]$ to obtain components.

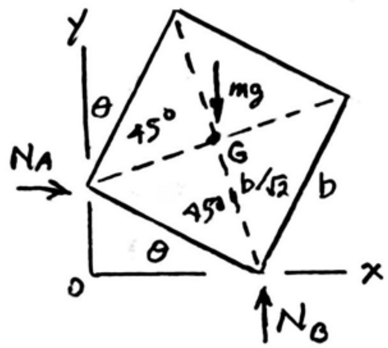
$$\begin{Bmatrix} \dot{\bar{H}}_B \cdot \bar{i}' \\ \dot{\bar{H}}_B \cdot \bar{j}' \\ \dot{\bar{H}}_B \cdot \bar{k}' \end{Bmatrix} = [R]^T\{\dot{H}_B\} = \begin{Bmatrix} 0 \\ \Gamma \\ ? \end{Bmatrix}$$

Carry out:

$$\ddot{\phi}\left[1 + (\cos\theta)^2\right] + \Omega^2\sin\theta\cos\theta(\sin\phi)^2 = 0 \qquad \triangleleft$$

$$\Gamma = -\frac{1}{4}mR^2\left\{(\ddot{\phi}\cos\phi - \dot{\phi}^2\sin\phi)\sin\theta\cos\theta - 2\Omega\dot{\phi}\sin\phi\cos\phi(\sin\theta)^2\right] \qquad \triangleleft$$

Exercise 6.32



Find diff eq for θ, and N_A & N_B.

Solution: First describe $\bar{a}_G$:

$$\bar{r}_{G/O} = \frac{b}{\sqrt{2}} \sin(\theta + \tfrac{\pi}{4})(\bar{i} + \bar{j})$$

$$\bar{v}_G = \dot{\bar{r}}_{G/O} = \frac{b}{\sqrt{2}} \dot{\theta} \cos(\theta + \tfrac{\pi}{4})(\bar{i} + \bar{j})$$

$$\bar{a}_G = \dot{\bar{v}}_G = \frac{b}{\sqrt{2}}\left[\ddot{\theta}\cos(\theta + \tfrac{\pi}{4}) - \dot{\theta}^2 \sin(\theta + \tfrac{\pi}{4})\right](\bar{i} + \bar{j})$$

Then $\Sigma \bar{F} \cdot \bar{i} = N_A = m\bar{a}_G \cdot \bar{i}$, $\Sigma \bar{F} \cdot \bar{j} = N_B - mg = m\bar{a}_G \cdot \bar{j}$

$$\Sigma \bar{M}_G \cdot \bar{k} = N_A \frac{b}{\sqrt{2}} \cos(\theta + \tfrac{\pi}{4}) + N_B \frac{b}{\sqrt{2}} \cos(\theta + \tfrac{\pi}{4}) = \frac{1}{6} m b^2 (-\ddot{\theta})$$

Thus $N_A = \frac{1}{\sqrt{2}} mb\left[\ddot{\theta}\cos(\theta + \tfrac{\pi}{4}) - \dot{\theta}^2 \sin(\theta + \tfrac{\pi}{4})\right]$ ◁

$N_B = \frac{1}{\sqrt{2}} mb\left[\ddot{\theta}\cos(\theta + \tfrac{\pi}{4}) - \dot{\theta}^2 \sin(\theta + \tfrac{\pi}{4})\right] + mg$ ◁

Substitute into moment eq:

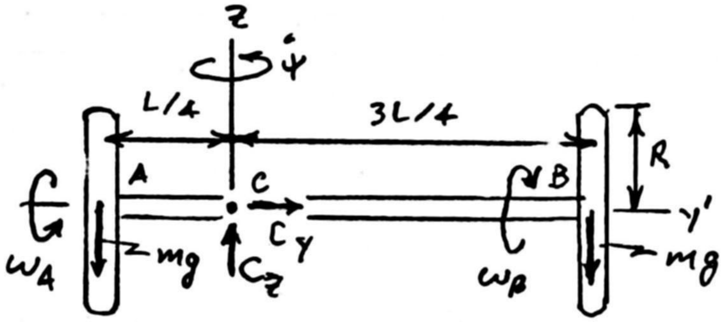
$$\frac{2}{\sqrt{2}} mb\left[\ddot{\theta}\cos(\theta + \tfrac{\pi}{4}) - \dot{\theta}^2 \sin(\theta + \tfrac{\pi}{4})\right]\frac{b}{\sqrt{2}}\cos(\theta + \tfrac{\pi}{4}) + mg\frac{b}{\sqrt{2}}\cos(\theta + \tfrac{\pi}{4}) = -\frac{1}{6} m b^2 \ddot{\theta}$$

or

$$\left[\frac{1}{6} + (\cos(\theta + \tfrac{\pi}{4}))^2\right]\ddot{\theta} - \sin(\theta + \tfrac{\pi}{4})\cos(\theta + \tfrac{\pi}{4})\dot{\theta}^2 = -\frac{g}{\sqrt{2}\, b}\cos(\theta + \tfrac{\pi}{4})$$

$$\left(\frac{2}{3} - \frac{1}{2}\sin 2\theta\right)\ddot{\theta} - \frac{1}{2}(\cos 2\theta)\dot{\theta}^2 = -\frac{1}{2}\frac{g}{b}(\cos\theta - \sin\theta)$$ ◁

Exercise 6.41



Given: constant $\omega_A, \omega_B, \dot\psi$.

Find: ω_B vs ω_A

Solution: Attach $x'y'z'$ to shaft & instantaneously align xyz for the disks with $x'y'z'$.

$$\Sigma \bar M_C = mg\left(-\frac{3L}{4} + \frac{L}{4}\right)\bar i$$

The centers of mass are at points A & B, so

$$\Sigma \bar M_C = (\dot{\bar H}_A)_A + \bar r_{A/C} \times m\bar a_A + (\dot{\bar H}_B)_B + \bar r_{B/C} \times m\bar a_B$$

$$\bar a_A = \frac{L}{4}\dot\psi^2 \bar i', \quad \bar a_B = -\frac{3L}{4}\dot\psi^2 \bar i \Rightarrow \bar r_{A/C} \times m\bar a_A = \bar r_{B/C} \times m\bar a_B = \bar 0$$

Thus $(\dot{\bar H}_A)_A + (\dot{\bar H}_B)_B = -\frac{1}{2}mgL\bar i$

$$\bar\omega_A = \dot\psi \bar k' + \omega_A \bar j \Rightarrow \bar\alpha_A = -\omega_A(\dot\psi \bar k' \times \bar j) = -\omega_A \dot\psi \bar i$$

$$\bar\omega_B = \dot\psi \bar k' - \omega_B \bar j \Rightarrow \bar\alpha_B = -\omega_B(\dot\psi \bar k' \times \bar j) = \omega_B \dot\psi \bar i$$

For each disk $I_{xx} = I_{zz} = \frac{1}{4}mR^2$, $I_{yy} = \frac{1}{2}mR^2$ $\Big\{$ centroidal axes

$$\bar H_A = mR^2\left[+\frac{1}{2}\omega_A \bar j + \frac{1}{4}\dot\psi \bar k\right], \quad \bar H_B = mR^2\left[-\frac{1}{2}\omega_B \bar j + \frac{1}{4}\dot\psi \bar k\right]$$

$$\dot{\bar H}_A = \frac{\partial \bar H_A}{\partial t} + \bar\omega_A \times \bar H_A = -\frac{1}{2}mR^2\omega_A\dot\psi \bar i, \quad \dot{\bar H}_B = \frac{1}{2}mR^2\omega_B\dot\psi \bar i$$

Thus $\frac{1}{2}mR^2(\omega_B - \omega_A)\dot\psi \bar i = -\frac{1}{2}mgL\bar i$

$$\omega_A - \omega_B = \frac{gL}{R^2\dot\psi}$$