MCHE 513: Intermediate Dynamics
Fall 2018 – Homework 3

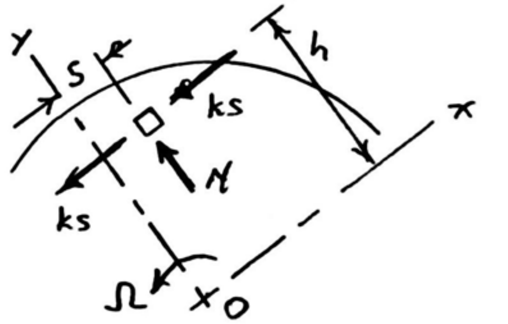
Assigned:	Thursday, September 20th
Due:	Friday, September 28th, 5pm

Assignment: From "Engineering Dynamics" by Jerry Ginsberg, problems:
3.35, 3.49, 3.56, 4.3, 4.9, 4.16, 4.32

Submission: Emailed *single* pdf document:

- to `joshua.vaughan@louisiana.edu`
- with subject line and filename `ULID-MCHE513-HW3`, where `ULID` is your ULID
- *Note:* Submissions with incorrect filenames or submitted as multiple images/pdfs will be rejected.

Exercise 3.35



Given arbitrary $\Omega(t)$.

Find differential eq for s.

Solution: Attach xyz to the turntable $\Rightarrow \bar{\omega} = \Omega\bar{k}$, $\bar{\alpha} = \dot{\Omega}\bar{k}$

$\bar{r}_{P/O} = s\bar{i} + h\bar{j}$, $(\bar{v}_P)_{xyz} = \dot{s}\bar{i}$, $(a_P)_{xyz} = \ddot{s}\bar{i}$

$$\bar{a}_P = \ddot{s}\bar{i} + \dot{\Omega}\bar{k}\times(s\bar{i} + h\bar{j}) - \Omega^2(s\bar{i} + h\bar{j}) + 2\Omega\bar{k}\times\dot{s}\bar{i}$$

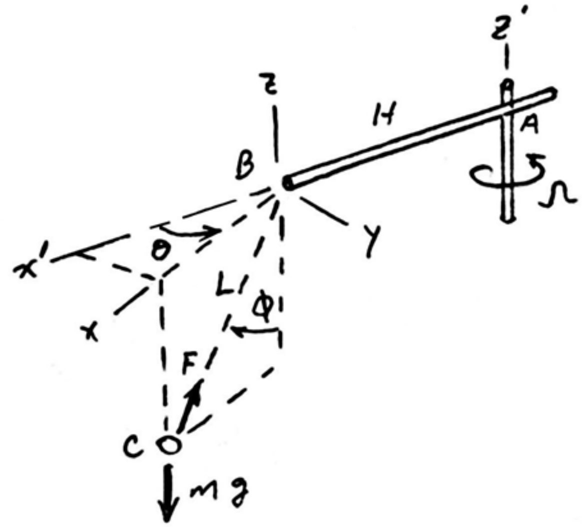
$$\sum \bar{F}\cdot\bar{i} = -2ks = m\bar{a}_P\cdot\bar{i} = m(\ddot{s} - \dot{\Omega}h - \Omega^2 s)$$

so $\ddot{s} - \dot{\Omega}h - \Omega^2 s + \frac{2k}{m}s = 0$ ◁

If Ω is constant $\Rightarrow \ddot{s} + \left(\frac{2k}{m} - \Omega^2\right)s = 0 \Rightarrow$ nat freq $= \left(\frac{2k}{m} - \Omega^2\right)^{1/2}$

If $\Omega > \left(\frac{2k}{m}\right)^{1/2}$, then $s(t)$ has two exponential solutions, one of which grows.

Exercise 3.49



Given constant Ω.

Find eqs of motion for ϕ & θ.

Solution; Form $\Sigma\bar F = m\bar a$, then eliminate unknown F.

Attach $x'y'z'$ to the crane, and let xyz rotate by θ relative to $x'y'z'$.

$\bar\omega' = \Omega\bar k$, $\bar\omega = \bar\omega' + \dot\theta\bar k = (\Omega+\dot\theta)\bar k$, $\bar\alpha = \ddot\theta\bar k$

$\bar a_B = -H\Omega^2\bar i' = -H\Omega^2(\cos\theta\,\bar i - \sin\theta\,\bar j)$

Circular motion in xz plane relative to xyz:

$\bar r_{C/B} = L(\sin\phi\,\bar i - \cos\phi\,\bar k)$, $(\bar v_c)_{xyz} = \frac{\partial\bar r_{C/B}}{\partial t} = L\dot\phi(\cos\phi\,\bar i + \sin\phi\,\bar k)$

$(\bar a_B)_{xyz} = \frac{\partial}{\partial t}(\bar v_c)_{xyz} = L\ddot\phi(\cos\phi\,\bar i + \sin\phi\,\bar k) + L\dot\phi^2(-\sin\phi\,\bar i + \cos\phi\,\bar k)$

$\bar a_c = -H\Omega^2(\cos\theta\,\bar i - \sin\theta\,\bar j) + [(L\ddot\phi\cos\theta - L\dot\phi^2\sin\phi)\bar i$

$+ (L\ddot\phi\sin\phi + L\dot\phi^2\cos\phi)\bar k] + \ddot\theta\bar k\times(L\sin\phi\,\bar i - L\cos\phi\,\bar k)$

$+ (\Omega+\dot\theta)\bar k\times[(\Omega+\dot\theta)\bar k\times(L\sin\phi\,\bar i - L\cos\phi\,\bar k)] + 2(\Omega+\dot\theta)\bar k$

$\times L\dot\phi(\cos\phi\,\bar i + \sin\phi\,\bar k$

$= [L\ddot\phi\cos\phi - H\Omega^2\cos\theta - L(\Omega^2 + \dot\phi^2 + \dot\theta^2)\sin\phi - 2L\Omega\dot\theta\sin\phi]\bar i$

$+ [L\ddot\theta\sin\phi + H\Omega^2\sin\theta + 2L(\dot\phi+\Omega)\cos\phi]\bar j + L(\ddot\phi\sin\phi + \dot\phi^2\cos\phi)\bar k$

Then $\Sigma\bar F = F\bar e_{B/C} - mg\bar k = m\bar a_C$ where $\bar e_{B/C} = -\sin\phi\,\bar i + \cos\phi\,\bar k$

Eliminate $\bar F$ by taking components perpendicular to $\bar e_{B/C}$ in $\bar j$ and $\bar e_\phi$ directions, where $\bar e_\phi = \cos\phi\,\bar i + \sin\phi\,\bar k$

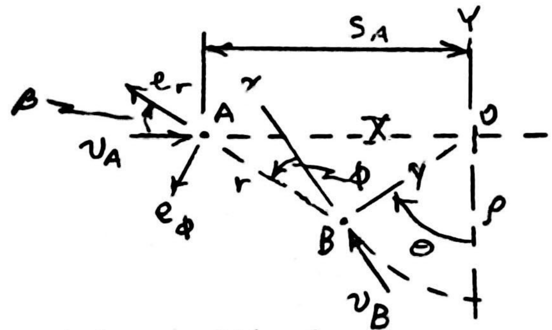
$\Sigma\bar F\cdot\bar j = 0 = m\bar a_C\cdot\bar j$ & $\Sigma\bar F\cdot\bar e_\phi = -mg\sin\phi = m\bar a_c\cdot\bar e_\phi$

Exercise 3.49 (cont.)

Carry out dot products; $L\ddot{\theta}\sin\phi + H\Omega^2\sin\theta + 2L(\dot{\phi}+\Omega)\cos\phi = 0$ ◁

and $L\ddot{\phi} - H\Omega^2\cos\theta\cos\phi - L(\dot{\theta}+\Omega)^2\sin\phi\cos\phi + g\sin\phi = 0$ ◁

Exercise 3.56



Given constant $v_A = \frac{560}{3.6}$ m/s & $v_B = \frac{1440}{3.6}$ m/s, $\rho = 3200$ m, $\theta = 0$ & $s_A = 5200$ m @ $t = 0$.

Find $r, \dot{r}, \phi, \dot{\phi}$ as functions of t.

Solution: Attach xyz to airplane B. Use XYZ as the global coordinates. r & ϕ form polar coordinates relative to xyz, so find $(\bar{v}_A)_{xyz}$ then convert to polar coordinates.

$\bar{\omega} = \dot{\theta}\bar{K} = \frac{v_B}{\rho}\bar{K} \Rightarrow \theta = \frac{v_B t}{\rho}$ Also $\dot{s}_A = -v_A \Rightarrow s_A = (s_A)_0 - v_A t$

$\bar{v}_B = v_B(\cos\theta\,\bar{I} + \sin\theta\,\bar{J})$, $\bar{v}_A = -v_A\bar{I}$

$\bar{r}_{A/B} = (s_A - \rho\sin\theta)\bar{I} + \rho\cos\theta\,\bar{J}$

$$(\bar{v}_A)_{xyz} = \bar{v}_A - \bar{v}_B - \bar{\omega}\times\bar{r}_{A/B}$$

$$= \left[-v_A - v_B\cos\theta - \left(\frac{v_B}{\rho}\right)(-\rho\cos\theta)\right]\bar{I} + \left[-v_B\sin\theta - \left(\frac{v_B}{\rho}\right)(s_A - \rho\sin\theta)\right]\bar{J}$$

$$= -v_A\bar{I} - v_B\frac{s_A}{\rho}\bar{J}$$

Convert to polar coordinates: $\bar{e}_r = \cos\beta\,\bar{I} + \sin\beta\,\bar{J}$, $\bar{e}_\phi = \sin\beta\,\bar{I} - \cos\beta\,\bar{J}$

Evaluate $r = |\bar{r}_{A/B}| = [(s_A - \rho\sin\theta)^2 + (\rho\cos\theta)^2]^{1/2}$

$$= [s_A^2 + \rho^2 - 2\rho s_A\sin\theta]^{1/2} \quad \triangleleft$$

Law of sines for OAB: $\sin\beta = \frac{\rho}{r}\sin\left(\frac{\pi}{2} - \theta\right) = \frac{\rho}{r}\cos\theta$

Then $\phi = \theta - \beta = \theta - \sin^{-1}\left(\frac{\rho}{r}\cos\theta\right)$

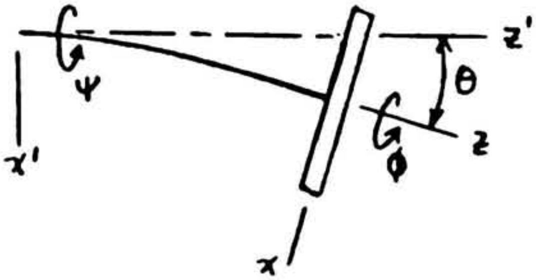
Next set $(\bar{v}_A)_{xyz} = \dot{r}\bar{e}_r + r\dot{\phi}\bar{e}_\phi$, so

$$\dot{r} = (\bar{v}_A)_{xyz}\cdot\bar{e}_r = -v_A\cos\beta - v_B\frac{s_A}{\rho}\sin\beta$$

$$\dot{\phi} = \frac{1}{r}(\bar{v}_A)_{xyz}\cdot\bar{e}_\phi = \frac{1}{r}\left(-v_A\sin\beta + v_B\frac{s_A}{\rho}\cos\beta\right)$$

Substitute v_A, v_B, $s_A = 5200 - v_A t$, $\theta = \frac{v_B t}{\rho}$, then compute other variables.

Exercise 4.3



Given $x'y'z'$ precesses only, $\bar{\omega} = 17\bar{i}' - 20\bar{j}' + 48\bar{k}'$ rad/s when $\theta = 10°$ & $\phi = -5°$. Find $\dot{\psi}$, $\dot{\theta}$, & $\dot{\phi}$, then xyz components of $\bar{\omega}$.

Solution: $\bar{\omega} = \dot{\psi}\bar{k}' + \dot{\theta}\bar{j}' + \dot{\phi}\bar{k}$.

Need transformation from $x'y'z'$ to xyz for $\theta = 10°$, $\phi = -5°$

Starting from $x'y'z'$, rotate by θ about y', then by ϕ about new z:

$[x\,y\,z]^T = [R][x'\,y'\,z']^T$ where $[R] = [R_z(\phi)][R_y(\theta)]$

$$[R] = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\phi\cos\theta & \sin\phi & -\cos\phi\sin\theta \\ -\sin\phi\cos\theta & \cos\phi & \sin\phi\sin\theta \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} 0.9810 & -0.0872 & -0.1730 \\ 0.0858 & 0.9962 & -0.0151 \\ 0.1736 & 0 & 0.9848 \end{bmatrix}$$

Thus $\bar{k} = 0.1736\bar{i}' + 0.9848\bar{k}'$

Set $\bar{\omega} = 0.1736\dot{\phi}\bar{i}' + \dot{\theta}\bar{j}' + (\dot{\psi} + 0.9848\dot{\phi})\bar{k}' = 17\bar{i}' - 20\bar{j}' + 48\bar{k}'$

Match components: $0.1736\dot{\phi} = 17$, $\dot{\theta} = -20$, $\dot{\psi} + 0.9848\dot{\phi} = 48$

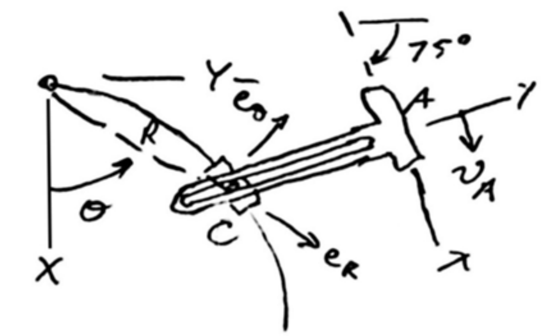
$\dot{\phi} = 97.90$, $\dot{\theta} = -20$, $\dot{\psi} = -48.42$ rad/s ◁

Transform $\bar{\omega}$ components:

$[\omega_x\ \omega_y\ \omega_z]^T = [R][17\ \ -20\ \ 48]^T = [10.12\ \ -19.19\ \ 50.22]^T$

so $\bar{\omega} = 10.12\bar{i} - 19.19\bar{j} + 50.22\bar{k}$ rad/s ◁

Exercise 4.9



Given $R = 0.1\sin(2\theta)$ meter,

$v_A = 20$ m/s, $\dot{v}_A = 0$

Find $\bar{v}$ and $\bar{a}$ at $\theta = 1$ rad.

Solution: Attach xyz to the slider

$$\bar{v}_C = \dot{R}\,\bar{e}_R + R\dot{\theta}\,\bar{e}_\theta$$

$$= \bar{v}_A + (\bar{v}_C)_{xyz} \quad (\bar{\omega} = \bar{0})$$

Convert to XYZ components: $\bar{e}_R = \cos\theta\,\bar{I} + \sin\theta\,\bar{J}$

$\bar{e}_\theta = -\sin\theta\,\bar{I} + \cos\theta\,\bar{J}$, $\bar{v}_A = v_A(\sin 75°\,\bar{I} + \cos 75°\,\bar{J})$

$(\bar{v}_C)_{xyz} = u\bar{j} = u(-\cos 75°\,\bar{I} + \sin 75°\,\bar{J})$

Also $\dot{R} = \frac{d}{dt}(0.1\sin(2\theta)) = 0.2\,\dot{\theta}\cos(2\theta)$

so $\bar{v}_C = 0.2\dot{\theta}\cos(2\theta)(\cos\theta\,\bar{I} + \sin\theta\,\bar{J}) + (0.1\sin 2\theta)\dot{\theta}(-\sin\theta\,\bar{I} + \cos\theta\,\bar{J})$

$$= 20(\sin 75°\,\bar{I} + \cos 75°\,\bar{J}) + u(-\cos 75°\,\bar{I} + \sin 75°\,\bar{J})$$

Match components:

$$\left.\begin{array}{l} \bar{v}_C\cdot\bar{I} = -0.12148\,\dot{\theta} = -0.2588\,u + 19.318 \\ \bar{v}_C\cdot\bar{J} = -0.02091\,\dot{\theta} = 0.9659\,u + 5.176 \end{array}\right\} \begin{array}{l} \dot{\theta} = -162.93 \text{ rad/s} \\ u = -1.833 \text{ m/s} \end{array}$$

Need $\dot{R}$ to form $\bar{a}_C$: $\dot{R} = 0.2\dot{\theta}\cos(2\theta) = 13.560$ m/s

Then $\ddot{R} = \frac{d}{dt}(\dot{R}) = 0.2\ddot{\theta}\cos(2\theta) - 0.4\dot{\theta}^2\sin(2\theta) = -0.08323\,\ddot{\theta} - 9655$

so $\bar{a}_C = (\ddot{R} - R\dot{\theta}^2)\bar{e}_R + (R\ddot{\theta} + 2\dot{R}\dot{\theta})\bar{e}_\theta = \bar{a}_A + (\bar{a}_C)_{xyz} = \bar{0} + \dot{u}\bar{j}$

$\bar{a}_C\cdot\bar{I} = (\ddot{R} - R\dot{\theta}^2)\cos\theta + (R\ddot{\theta} + 2\dot{R}\dot{\theta})(-\sin\theta) = \dot{u}(-\cos 75°)$

$$-0.12148\,\ddot{\theta} - 2803 = -0.2588\,\dot{u}$$

$\bar{a}_C\cdot\bar{J} = (\ddot{R} - R\dot{\theta}^2)\sin\theta + (R\ddot{\theta} + 2\dot{R}\dot{\theta})\cos\theta = \dot{u}\sin 75°$

$$-0.02091\,\ddot{\theta} - 12543 = 0.9659\,\dot{u}$$

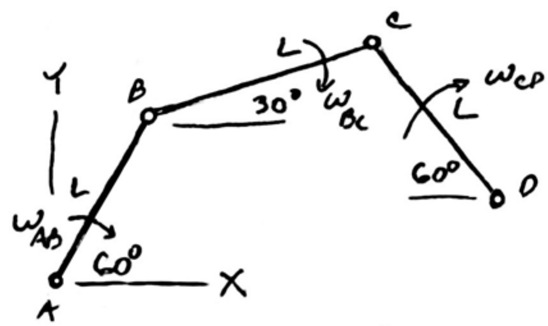
Exercise 4.9 (cont.)

Solve: $\ddot{\theta} = -48.50(10^3)$ rad/s^2, $\dot{u} = -11.94(10^3)$ m/s^2

Thus $\bar{v}_C = (\bar{v}_C \cdot \bar{I})\bar{I} + (\bar{v}_C \cdot \bar{J})\bar{J} = 19.793\bar{I} + 3.406\bar{J}$ m/s ◁

$\bar{a}_C = (\bar{a}_C \cdot \bar{I})\bar{I} + (\bar{a}_C \cdot \bar{J})\bar{J} = (3.089\bar{I} - 11.529\bar{J})$ m/s^2 ◁

Exercise 4.16



Given constant ω_{AB}

Find $\bar{\omega}$ and $\bar{\alpha}$ for all bars in the given position.

Solution;

$\bar{v}_B = -\omega_{AB}\bar{k} \times (0.5L\bar{I} + 0.866\bar{J})$

$$\bar{v}_C = \bar{v}_B + (-\omega_{BC}\bar{K}) \times (0.866L\bar{I} + 0.5\bar{J})$$
$$= (-\omega_{CD}\bar{K}) \times (-0.5L\bar{I} + 0.866L\bar{J})$$

$$\left.\begin{aligned}\bar{v}_C \cdot \bar{I} &= 0.866\omega_{AB}L + 0.5\omega_{BC}L = 0.866\omega_{CD}L \\ \bar{v}_C \cdot \bar{J} &= -0.5\omega_{AB}L - 0.866\omega_{BC}L = 0.5\omega_{CD}L\end{aligned}\right\}$$

$$\omega_{BC} = -0.866\omega_{AB}L \qquad \omega_{CD} = 0.50\omega_{AB}L \qquad \triangleleft$$

Then $\bar{a}_B = -\omega_{AB}^2\bar{r}_{B/A} = -\omega_{AB}^2(0.5L\bar{I} + 0.866L\bar{J})$

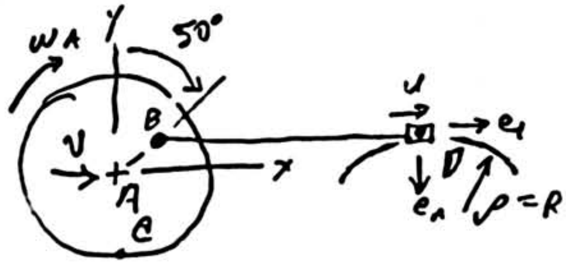
$$\bar{a}_C = \bar{a}_B + (-\dot{\omega}_{BC}\bar{K}) \times (0.866L\bar{I} + 0.5L\bar{J}) - \omega_{BC}^2(0.866L\bar{I} + 0.5\bar{J})$$
$$= (-\dot{\omega}_{CD}\bar{K}) \times (-0.5L\bar{I} + 0.866L\bar{J}) - \omega_{CD}^2(-0.5L\bar{I} + 0.866L\bar{J})$$

$$\bar{a}_C \cdot \bar{I} = -0.5\omega_{AB}^2L + 0.5\dot{\omega}_{BC}L - 0.866\omega_{BC}^2L = 0.866\dot{\omega}_{CD}L + 0.5\omega_{CD}^2$$

$$\bar{a}_C \cdot \bar{J} = -0.866\omega_{AB}^2L - 0.866\dot{\omega}_{BC}L - 0.5\omega_{BC}^2 = 0.5\dot{\omega}_{CD}L - 0.866\omega_{CD}^2L$$

$$\dot{\omega}_{BC} = -0.250\omega_{AB}^2L, \quad \dot{\omega}_{CD} = -1.616\omega_{AB}^2L \qquad \triangleleft$$

Exercise 4.32



Given constant $v_D = u$, no slippage at contact C

Find $\bar{v}_A$ and $\bar{a}_A$ at this position

Solution: $\bar{v}_D = u\bar{i}$ & $\bar{a}_D = \frac{u^2}{R}(-\bar{j})$

No slippage $\Rightarrow$ $\omega_A = \frac{v}{R}$ & $\dot{\omega}_A = \frac{\dot{v}_A}{R}$

$$\bar{v}_B = \bar{v}_A + \omega_A \times \bar{r}_{B/A} = v\bar{i} + \left(-\frac{v}{R}\bar{k}\right) \times 0.5R(\sin 50^\circ\,\bar{i} + \cos 50^\circ\,\bar{j})$$

$$= \bar{v}_D + \bar{\omega}_{BD} \times \bar{r}_{B/D} = u\bar{i} + \omega_{BD}\bar{k} \times 3R(-\bar{i})$$

$$\left.\begin{array}{l} \bar{v}_B \cdot \bar{i} = 1.3214\, v = u \\ \bar{v}_B \cdot \bar{j} = -0.3830\, v = -3R\omega_{BD} \end{array}\right\} \begin{array}{l} v = 0.7568u \quad \triangleleft \\ \omega_{BD} = 0.09662\frac{u}{R} \end{array}$$

Then $$\bar{a}_B = \bar{a}_A + \left(-\frac{\dot{v}}{R}\bar{k}\right) \times 0.5R(\sin 50^\circ\,\bar{i} + \cos 50^\circ\,\bar{j})$$

$$- \frac{v^2}{R^2}(0.5R)(\sin 50^\circ\,\bar{i} + \cos 50^\circ\,\bar{j})$$

$$= \bar{a}_D + \dot{\omega}_{BD}\bar{k} \times (-3R\bar{i}) - \omega_{BD}^2(-3R\bar{i})$$

$$\left.\begin{array}{l} \bar{a}_B \cdot \bar{i} = 1.3214\,\dot{v} - 0.2194\frac{u^2}{R} = 0.0280\frac{u^2}{R} \\ \bar{a}_B \cdot \bar{j} = -0.3830\,\dot{v} - 0.18407\frac{u^2}{R} = -3R\dot{\omega}_{BD} - \frac{u^2}{R} \end{array}\right\} \dot{v} = 0.1872\frac{u^2}{R} \quad \triangleleft$$