MCHE 513: Intermediate Dynamics
Fall 2018 – Homework 2

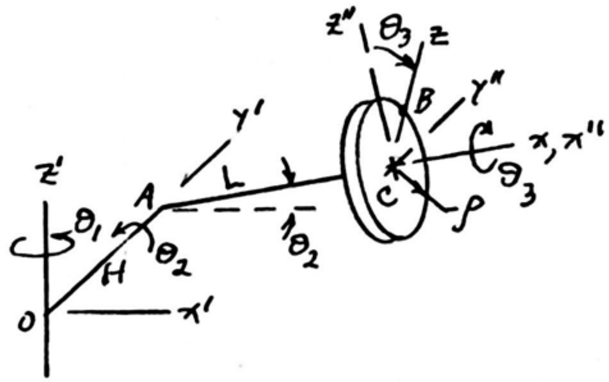
Assigned: Thursday, August 30th
Due: Friday, September 14th, 5pm

Assignment: From "Engineering Dynamics" by Jerry Ginsberg, problems:
3.8, 3.10, 3.16, 3.20, 3.37, 3.38, 3.52

Submission: Emailed *single* pdf document:

- to `joshua.vaughan@louisiana.edu`
- with subject line and filename `ULID-MCHE513-HW2`, where `ULID` is your ULID
- *Note:* Submissions with incorrect filenames or submitted as multiple images/pdfs will be rejected.

Exercise 3.8



Given θ_2 about OA, then θ_3 about AC.

Find (a) transformation $[x' y' z']^T = [R]\{x\ y\ z\}^T$, (b) prove that $[R]$ is independent of sequence, (c) $(\Delta \bar{r}_B)_{x'y'z'}$

Solution: Rotate by θ_2 about $-y'$ axis $\Rightarrow x'' y'' z''$, then rotate by θ_3 about $-x'' \Rightarrow xyz$. Thus $[x\ y\ z] = [R_x(-\theta_3)][R_y(-\theta_2)][x' y' z']^T$

so $[x' y' z']^T = [R](x\ y\ z)^T$ where $[R] = [R_y(-\theta_2)]^T [R_x(-\theta_3)]^T$

where $[R] = [[R_x(-\theta_3)][R_y(-\theta_2)]]^T = [R_y(\theta_2)][R_x(\theta_3)]$ ◁

Alternative sequence: θ_3 about $-x'$, then θ_2 about $-y' \Rightarrow$ space-fixed

$[x\ y\ z]^T = [R_x(-\theta_3)][R_y(-\theta_2)][x' y' z']^T \Rightarrow$ same ◁

Displacement: At any position: $\bar{r}_{B/O} = H\bar{j}' + L\bar{i} + \rho\bar{k}$

Initially $\bar{i} = \bar{i}'$ & $\bar{k} = \bar{k}' \Rightarrow (\bar{r}_{B/O})_0 = L\bar{i}' + H\bar{j}' + \rho\bar{k}'$

Final $\bar{i} = \cos\theta_2\, \bar{i}' + \sin\theta_2\, \bar{k}'$ & $\bar{k} = \sin\theta_3\, \bar{j}' + \cos\theta_3(-\sin\theta_2\, \bar{i}' + \cos\theta_2\, \bar{k}')$

$$(\bar{r}_{B/O})_f = H\bar{j}' + L(\cos\theta_2\, \bar{i}' + \sin\theta_2\, \bar{k}') + \rho[\sin\theta_3\, \bar{j}' + \cos\theta_3(-\sin\theta_2\, \bar{i}' + \cos\theta_2\, \bar{k}')]$$

$$(\Delta \bar{r}_{B/O})_{x'y'z'} = (\bar{r}_{B/O})_f - (\bar{r}_{B/O})_0$$

$$= [-L(1-\cos\theta_2) - \rho\sin\theta_2\cos\theta_3]\,\bar{i}' + \rho\sin\theta_3\, \bar{j}'$$

$$+ [L\sin\theta_2 - \rho(1 - \cos\theta_2\cos\theta_3)]\,\bar{k}' \quad ◁$$

Alternative derivation: Treat $x'y'z'$ like the fixed system. Then

$(\Delta \bar{r}_A)_{x'y'z'} = \bar{0}$ & $\bar{r}_{B/A} = L\bar{i} + \rho\bar{k} \Rightarrow (\Delta \bar{r}_{B/A})_{x'y'z'} = \bar{0}$ so

$$\begin{Bmatrix} (\Delta \bar{r}_B)_{x'y'z'} \cdot \bar{i}' \\ (\Delta \bar{r}_B)_{x'y'z'} \cdot \bar{j}' \\ (\Delta \bar{r}_B)_{x'y'z'} \cdot \bar{k}' \end{Bmatrix} = [[R]_f - [U]] \begin{Bmatrix} L \\ 0 \\ \rho \end{Bmatrix} \quad ◁$$

Exercise 3.10

Given ψ is rotation of the outer gimbal about fixed axis Y, θ is rotation of inner gimbal relative to outer about axis CD, and ϕ is rotation of the flywheel relative to the inner gimbal about y axis, with $[x\ y\ z] = [X\ Y\ Z]$ when $\psi = \theta = \phi = 0$

Find transformation from $[X\ Y\ Z]$ to $[x\ y\ z]$ for 6 sequences

Solution: Attach $x_0\ y_0\ z_0$ to the outer gimbal with $\bar{j}_0 \equiv \bar{J}$ & $\bar{i}_0 = \bar{e}_{D/C}$.

Attach $x_i\ y_i\ z_i$ to the inner gimbal with $\bar{i}_i \equiv \bar{i}_0$ & $\bar{j}_i \equiv \bar{j}$

Regardless of the sequence in which the rotations occur, it is always true that the simple rotation transformations are

$[x_0\ y_0\ z_0]^T = [R_y(\psi)][X\ Y\ Z]^T$, $[x_i\ y_i\ z_i]^T = [R_x(\theta)][x_0\ y_0\ z_0]^T$, and $[x\ y\ z] = [R_y(\phi)][x_i\ y_i\ z_i]$

Eliminate $[x_0\ y_0\ z_0]$ and $[x_i\ y_i\ z_i]$ to find

$$[x\ y\ z]^T = [R_y(\phi)][R_x(\theta)][R_y(\psi)][X\ Y\ Z]^T \text{; any sequence} \quad \triangleleft$$

The longer solution considers each sequence individually. For example, if θ is first, then ψ, then ϕ, the transformations are:

After θ: $[x\ y\ z]^T = [x_i\ y_i\ z_i]^T = [R_x(\theta)][x_0\ y_0\ z_0]^T$ & $[x_0\ y_0\ z_0] = [X\ Y\ Z]$

After ψ: $[x\ y\ z]^T = [x_i\ y_i\ z_i]^T = [R_x(\theta)][x_0\ y_0\ z_0]^T$

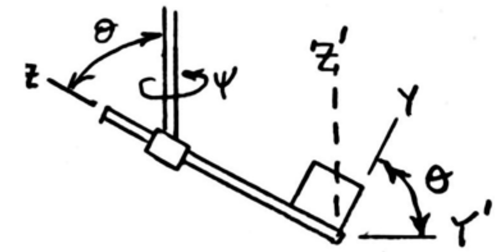
& $[x_0\ y_0\ z_0]^T = [R_y(\psi)][X\ Y\ Z]^T$

After ϕ: $[x\ y\ z]^T = [R_y(\phi)][x_i\ y_i\ z_i]^T$ & $[x_i\ y_i\ z_i]^T = [R_x(\theta)]^T[x_0\ y_0\ z_0]^T$

& $[x_0\ y_0\ z_0]^T = [R_y(\psi)][X\ Y\ Z]^T$

Eliminating $[x_i\ y_i\ z_i]$ & $[x_0\ y_0\ z_0]$ leads to the same $[R]$.

Exercise 3.16



Given $\psi = 75°$ with $\theta = 30°$

Find $[R]$ such that $[x\ y\ z]_f^T = [R][x\ y\ z]_0^T$, and angle between original and final y axes.

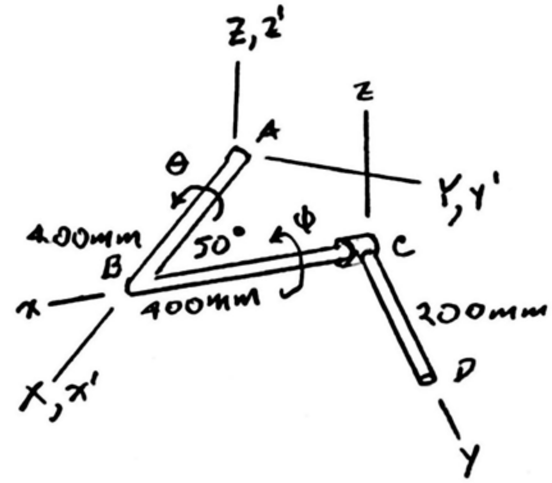
Solution: Define XYZ to coincide with the original xyz. The axis for ψ does not coincide with one of these coordinate axes, so $[x\ y\ z]^T = [R][X\ Y\ Z]^T$ where $[R] = [R']^T[R_z(\psi)][R']$ where Z' is the vertical rotation axis and $[X'\ Y'\ Z']^T = [R'][X\ Y\ Z]^T$. From the sketch, rotate by θ about $-X$ axis $\Rightarrow$ $[R'] = [R_x(-\theta)]$, so

$$[R] = [R_x(-30°)]^T[R_z(75°)][R_x(-30°)]$$

$$= \begin{bmatrix} 0.2588 & 0.8365 & -0.4830 \\ -0.8365 & 0.4441 & 0.3209 \\ 0.4830 & 0.3209 & 0.8147 \end{bmatrix} \quad \triangleleft$$

By definition $R_{2,2} = \ell_{YY}$, so $\beta_{YY} = \cos^{-1}(0.4441) = 51.74° \quad \triangleleft$

Exercise 3.20



Given bars ABC & CD lie in XY plane in initial position where $\theta = \phi = 0$, final position is $\phi = -70°$ & $\theta = 120°$

Find $\Delta \bar{r}_D$

Solution: Attach $x'y'z'$ to bar ABC to evaluate $\Delta \bar{r}_C$ & attach xyz to bar CD.

Transformations are $[x'\,y'\,z']^T = [R_x(\theta)][X\,Y\,Z]^T$ and

$[x\,y\,z]^T = [R_x(\theta)][R_z(-50°)][x'\,y'\,z']^T = [R][X\,Y\,Z]^T$

where $[R] = [R_x(\phi)][R_z(-50°)][R_x(\theta)]$

Point C is fixed relative to $x'y'z'$, so

$$(\bar{r}_{C/A})_0 = (\bar{r}_{C/A})_f = (400 - 400\cos 50°)\bar{i}' + 400\sin 50°\bar{j}'$$

$$= 142.9\bar{i}' + 306.4\bar{j}' \Rightarrow (\Delta\bar{r}_C)_{x'y'z'} = \bar{0}, \text{ Also } \Delta\bar{r}_A = \bar{0}.$$

$$\begin{Bmatrix} \Delta\bar{r}_C\cdot\bar{I} \\ \Delta\bar{r}_C\cdot\bar{J} \\ \Delta\bar{r}_C\cdot\bar{K} \end{Bmatrix} = \left[[R_x(\theta)] - [U]\right]\begin{Bmatrix} 142.9 \\ 306.4 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -459.6 \\ 265.4 \end{Bmatrix}$$

To evaluate $\Delta\bar{r}_D$, set $\phi_0 = \theta_0 = 0$, $\phi_f = -70°$, & $\theta_f = 120°$, so

$$[R]_0 = [R_z(-50°)],\ [R]_f = [R_x(120°)][R_z(-30°)][R_x(120°)]$$

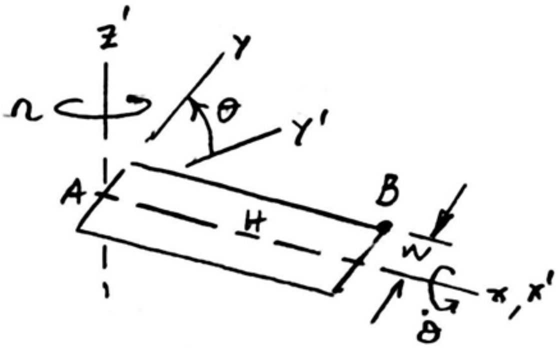
Point D is fixed relative to xyz, so

$$(\bar{r}_{D/C})_0 = (\bar{r}_{D/C})_f = 200\bar{j} \Rightarrow (\Delta\bar{r}_D)_{xyz} = \bar{0}$$

$$\begin{Bmatrix} \Delta\bar{r}_D\cdot\bar{I} \\ \Delta\bar{r}_D\cdot\bar{J} \\ \Delta\bar{r}_D\cdot\bar{K} \end{Bmatrix} = \begin{Bmatrix} \Delta\bar{r}_C\cdot\bar{I} \\ \Delta\bar{r}_C\cdot\bar{J} \\ \Delta\bar{r}_C\cdot\bar{K} \end{Bmatrix} + \left[[R]_f^T - [R]_0^T\right]\begin{Bmatrix} 0 \\ 200 \\ 0 \end{Bmatrix}$$

or $\Delta\bar{r}_D = -100.8\bar{I} - 447.4\bar{J} + 397.4\bar{K}$ mm ◁

Exercise 3.37



Given constant Ω & arbitrary $\theta(t)$.

Find $\bar{a}_{B/A}$.

Solution: Attach xyz to the panel, let $x'y'z'$ rotate at Ω only

$\bar{\omega} = \Omega\bar{k}' + \dot{\theta}\bar{i}'$, $\bar{\Omega}_1 = \bar{\Omega}_2 = \Omega\bar{k}'$

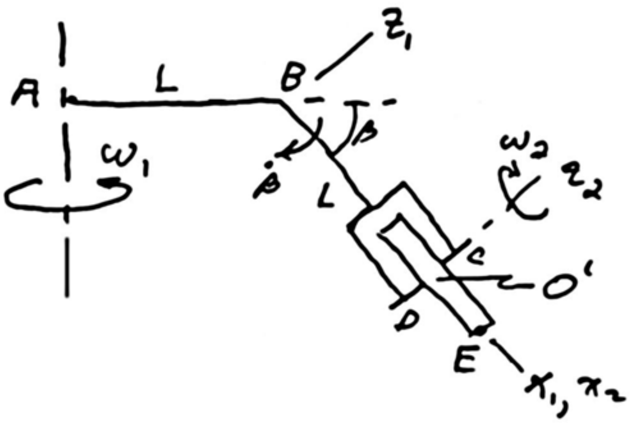
Use xyz as the global system $\Rightarrow \bar{\alpha} = \dot{\theta}(\Omega\bar{k}')\times\bar{i}' = \Omega\dot{\theta}\bar{j}'$

$(\bar{v}_B)_{xyz} = (\bar{a}_B)_{xyz} = \bar{0}$, $\bar{r}_{B/A} = H\bar{i}' + w(\cos\theta\,\bar{j}' + \sin\theta\,\bar{k}')$

$\bar{a}_{B/A} = \bar{\alpha}\times\bar{r}_{B/A} + \bar{\omega}\times(\bar{\omega}\times\bar{r}_{B/A})$

$= [2\Omega\dot{\theta}w\sin\theta - H\Omega^2]\bar{i}' - (\Omega^2+\dot{\theta}^2)w\cos\theta\,\bar{j}' - w\dot{\theta}^2\sin\theta\,\bar{k}'$ $\triangleleft$

Exercise 3.38



Given $\omega_2 = 1200\left(\frac{2\pi}{60}\right)$ rad/s, $\omega_1 = 20$ rad/s, $\beta = 60°$

Find $\bar{v}_E$ and $\bar{a}_E$ using (a) $x_1 y_1 z_1$ attached to BCD, (b) $x_2 y_2 z_2$ attached to the disk

Solution (a) Use $x_1 y_1 z_1$

Origin B $\Rightarrow$ circular motion $\bar{v}_B = L\omega_1 \bar{j}_1$

$$\bar{a}_B = L\omega_1^2 \bar{e}_{A/B} = L\omega_1^2(-\cos\beta\,\bar{i}_1 - \sin\beta\,\bar{k}_1)$$

Angular motion: $\bar{\omega} = \omega_1 \bar{e}_1, \bar{\alpha} = \bar{0}, \bar{e}_1 = -\sin\beta\,\bar{i}_1 + \cos\beta\,\bar{k}_1$

Relative motion: $\bar{r}_{E/B} = (L+R)\,\bar{i}_1$

Point E is at low points, so $(\bar{v}_E)_{x_1y_1z_1} = -R\omega_2\bar{j}_1$,

$$(\bar{a}_E)_{x_1y_1z} = R\omega_2^2(-\bar{i}_1)$$

Thus $\bar{v}_E = L\omega_1\bar{j}_1 - R\omega_2\bar{j}_1 + \omega_1(-\sin\beta\,\bar{i}_1 + \cos\beta\,\bar{k}_1)\times(L+R)\bar{i}_1$

$$= [\omega_1 L + \omega_1(L+R)\cos\beta - \omega_2 R]\,\bar{j}$$

$$\bar{a}_E = -L\omega_1^2(\cos\beta\,\bar{i}_1 + \sin\beta\,\bar{k}_1) + (-R\omega_2^2\,\bar{i}_1)$$
$$+ \omega_1(-\sin\beta\,\bar{i}_1 + \cos\beta\,\bar{k}_1)\times[\omega_1(-\sin\beta\,\bar{i}_1 + \cos\beta\,\bar{k}_1)\times(L+R)\bar{i}_1]$$
$$+ 2\omega_1(-\sin\beta\,\bar{i}_1 + \cos\beta\,\bar{k}_1)\times(-R\omega_2\,\bar{j}_1)$$
$$= [-L\omega_1^2 - (L+R)\omega_1^2\cos\beta + 2\omega_1\omega_2 R](\cos\beta\,\bar{i}_1 + \sin\beta\,\bar{k}_1)$$
$$-R\omega_2^2\,\bar{i}_1$$

Solution (b) Use $x_2 y_2 z_2$ parallel to $x_1 y_1 z_1$ at instant

Origin O' $\Rightarrow$ circular motion: $\bar{v}_{O'} = L(1+\cos\beta)\,\omega_1\bar{j}_2$

$$\bar{a}_{O'} = (L+R)\omega_1^2(-\cos\beta\,\bar{i}_2 - \sin\beta\,\bar{k}_2)$$

Exercise 3.38 (cont.)

No relative motion: $\bar{r}_{E/O'} = R\,\bar{i}_2$, $(\bar{v}_E)_{x_2y_2z_2} = (\bar{a}_E)_{x_2y_2z_2} = \bar{0}$

$$\bar{\omega} = \omega_1\bar{e}_1 + \omega_2\bar{e}_2,\quad \bar{\Omega}_2 = \omega_1\bar{e}_1,\quad \bar{\alpha} = \omega_2(\omega_1\bar{e}_1\times\bar{e}_2)$$

Set $\bar{e}_1 = -\sin\beta\,\bar{i}_2 + \cos\beta\,\bar{k}_2$, $\bar{e}_2 = -\bar{k}_2$

$$\bar{\omega} = -\omega_1\sin\beta\,\bar{i}_2 + (\omega_1\cos\beta - \omega_2)\,\bar{k}_2,\quad \bar{\alpha} = -\omega_1\omega_2\sin\beta\,\bar{j}_2$$

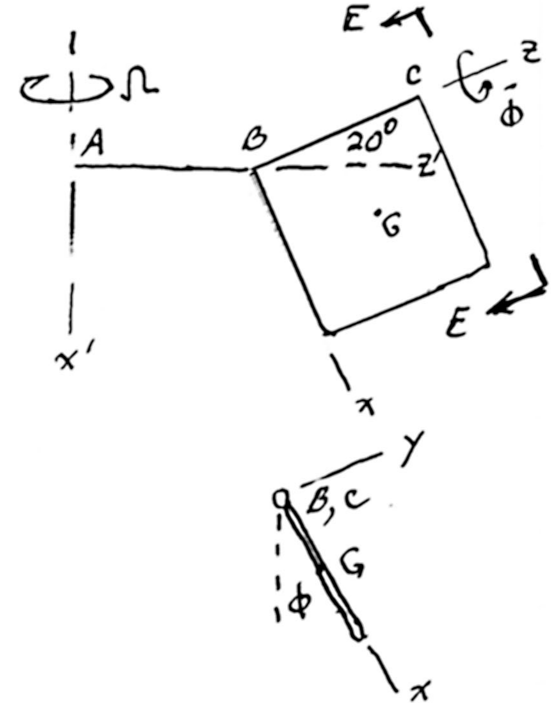
Then $\bar{v}_E = L(1+\cos\beta)\,\omega_1\,\bar{j}_2 + [-\omega_1\sin\beta\,\bar{i}_2 + (\omega_1\cos\beta - \omega_2)\,\bar{k}_2]\times R\,\bar{i}_2$

$$= [\omega_1 L(1+\cos\beta) + \omega_1 R\cos\beta - \omega_2 R]\,\bar{j}_2 \quad \triangleleft$$

$$\bar{a}_E = (L+R)\,\omega_1^2(-\cos\beta\,\bar{i}_2 - \sin\beta\,\bar{k}_2) + (-\omega_1\omega_2\sin\beta\,\bar{j}_2)\times R\,\bar{i}_2$$
$$+ [-\omega_1\sin\beta\,\bar{i}_2 + (\omega_1\cos\beta - \omega_2)\,\bar{k}_2]\times\big([-\omega_1\sin\beta\,\bar{i}_2$$
$$+ (\omega_1\cos\beta - \omega_2)\,\bar{k}_2]\times R\,\bar{i}_2\big)$$
$$= [-\omega_1^2 L\cos\beta\,(1+\cos\beta) - R(\omega_1\cos\beta - \omega_2)^2]\,\bar{i}_2$$
$$+ [\omega_1^2\{-L\sin\beta(1+\cos\beta) - R\sin\beta\cos\beta\} + 2R\omega_1\omega_2\sin\beta]\,\bar{k}_2 \quad \triangleleft$$

Results for $\bar{v}_E$ & $\bar{a}_E$ are equivalent

Exercise 3.52



Given constant Ω, arbitrary $\phi(t)$, body-fixed xyz

Find (a) $\bar\omega$ & $\bar\alpha$, (b) $\bar v_G$ & $\bar a_G$

Solution: Define $x'y'z'$ attached to vertical shaft

$$\bar\omega = \Omega\bar i' + \dot\phi\bar k,\quad \bar\omega' = \Omega\bar i'$$

$$\bar\alpha = \ddot\phi\bar k + \dot\phi(\Omega\bar i'\times\bar k)$$

Also $\bar v_B = -L\Omega\bar j'$, $\bar a_B = -L\Omega^2\bar k'$

$\bar r_{G/B} = \frac{L}{2}(\bar i + \bar k)$, $(\bar v_G)_{xyz} = 0$, $(\bar a_G)_{xyz} = \bar 0$

Transformation: $[x\ y\ z]^T = [R_z(\phi)][R_y(-20^\circ)][x'\ y'\ z']^T$

$$\text{so } [R] = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos 20^\circ & 0 & \sin 20^\circ \\ 0 & 1 & 0 \\ -\sin 20^\circ & 0 & \cos 20^\circ \end{bmatrix}$$

$$= \begin{bmatrix} 0.9397\cos\phi & \sin\phi & 0.3420\cos\phi \\ -0.9397\sin\phi & \cos\phi & -0.3420\sin\phi \\ -0.3420 & 0 & 0.9397 \end{bmatrix}$$

Then $[\bar i'\ \bar j'\ \bar k']^T = [R]^T[\bar i\ \bar j\ \bar k]^T$

so $\bar i' = 0.9397\cos\phi\,\bar i - 0.9397\sin\phi\,\bar j - 0.3420\bar k$, etc.

$$\bar\omega = 0.9397\Omega\cos\phi\,\bar i - 0.9397\Omega\sin\phi\,\bar j + (-0.3420\Omega + \dot\phi)\bar k \quad \triangleleft$$

$$\bar\alpha = -0.9397\Omega\dot\phi\sin\phi\,\bar i - 0.9397\Omega\dot\phi\cos\phi\,\bar j + \ddot\phi\bar k \quad \triangleleft$$

$$\bar v_G = \bar v_B + \bar\omega\times\bar r_{G/B},\quad \bar a_G = \bar a_B + \bar\alpha\times\bar r_{G/B} + \bar\omega\times(\bar\omega\times\bar r_{G/B})$$

$$\bar v_G = -1.47L\Omega\sin\phi\,\bar i + \{-(0.171 + 1.47\cos\phi)L\Omega + 0.5L\dot\phi\}\bar j$$

$$+ 0.4698L\Omega\sin\phi\,\bar k \quad \triangleleft$$

Exercise 3.52 (cont.)

$$\bar{a}_6 = L[(-0.5 - 0.5027\cos\phi + 0.4415(\cos\phi)^2)\Omega^2 + 0.342\Omega\dot{\phi} - 0.5\dot{\phi}^2]\bar{i}$$
$$+ L[0.5\ddot{\phi} + (0.5027\sin\phi - 0.2208\sin 2\phi)\Omega^2]\bar{j} + L[(-1.381$$
$$-0.1607\cos\phi)\Omega^2 + 0.9397\dot{\phi}\Omega\cos\phi]\bar{k} \quad \triangleleft$$