MCHE 513: Intermediate Dynamics
Fall 2018 – Homework 1

Assigned:	Tuesday, August 20th
Due:	Friday, August 31st, 5pm

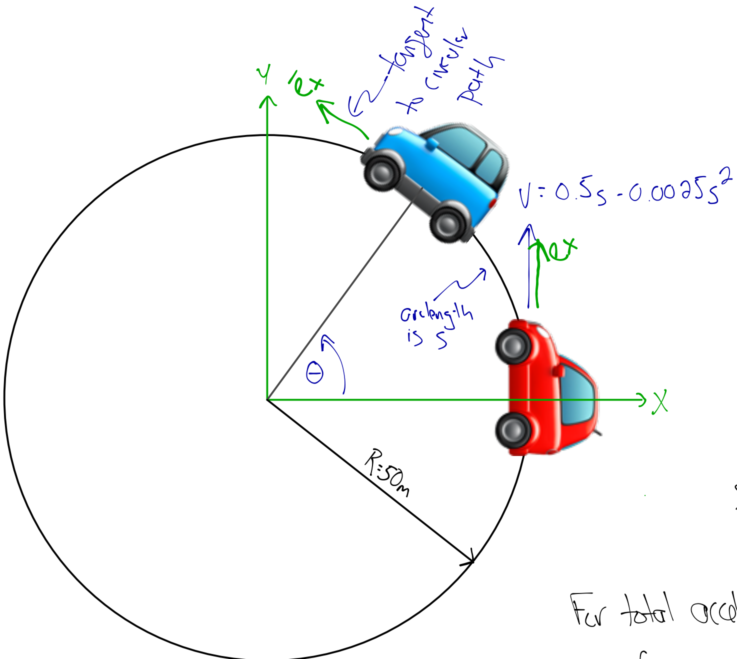
Assignment: From "Engineering Dynamics" by Jerry Ginsberg, problems:
2.2, 2.6, 2.19, 2.24, 2.31, 2.34

Submission: Emailed *single* pdf document:

- to `joshua.vaughan@louisiana.edu`
- with subject line and filename `ULID-MCHE513-HW1`, where `ULID` is your ULID
- *Note:* Submissions with incorrect filenames or submitted as multiple images/pdfs will be rejected.

Problem 2.2

EXERCISE 2.2 An automobile follows a circular road whose radius is 50 m. Let x and y respectively denote the eastern and northern directions, with origin at the center of the circle. Suppose the vehicle starts from rest at $x = 50$ m heading north, and its speed depends on the distance s it travels according to $v = 0.5s - 0.0025s^2$, where s is measured in meters and v is in meters per second. It is known that the tires will begin to skid when the total acceleration of the vehicle is $0.6g$. Where will the automobile be and how fast will it be going when it begins to skid? Describe the position in terms of the angle of the radial line relative to the x axis.



- car at initial position
- car some time later

We know that

$$\bar{a}_P = \dot{v}\bar{e}_t + \frac{v^2}{\rho}\bar{e}_n$$

In this case:

$$\rho = 50\text{m} \qquad v = 0.5s - 0.0025s^2$$

So

$$\dot{v} = v\frac{dv}{ds}$$

For total accel, we can look at $|\bar{a}| = \sqrt{\bar{a}\cdot\bar{a}}$

$$|\bar{a}| = \left[\dot{v}^2 + \left(\frac{v^2}{\rho}\right)^2\right]^{1/2} = 0.6g$$

or

$$\dot{v}^2 + \left(\frac{v^2}{\rho}\right)^2 = (0.6g)^2$$

$$\dot{v} = v\frac{dv}{ds} = v(0.5 - 0.005s)$$

$$v^2(0.5 - 0.005s)^2 + \frac{v^4}{\rho^4} = (0.6g)^2$$

Now, solve this equation for s (see Jupyter Notebook for my solution)

Find $s = 32.39\text{m}$.

We know that

$$\boxed{\begin{gathered}\theta = \frac{s}{\rho} = \frac{32.39}{50} \approx 37^\circ \\ v = 0.5(32.39) + 0.0025(32.39)^2 \approx 13.6\,\text{m/s}\end{gathered}}$$

Problem 2.6

EXERCISE 2.6 A particle follows a planar path defined by $x = k\xi$, $y = 2k[1 - \exp(\xi)]$, such that its speed is $v = \beta\xi$, where k and β are constants. Determine the velocity and acceleration at $\xi = 0.5$.

$\bar{r}$ is given by $\quad k\xi\bar{i} + 2k[1-e^{\xi}]\bar{j}$

$\bar{e}_t = \dfrac{\bar{r}'}{s'}$ where $'$ indicates derivative with respect to ξ

$\bar{r}' = k\bar{i} - 2ke^{\xi}\bar{j}$

we know $|\bar{e}_t| = 1 \rightarrow \dfrac{\bar{r}'\cdot\bar{r}'}{(s')^2} = 1 \qquad \dfrac{k^2+4ke^{2\xi}}{(s')^2} = 1 \longrightarrow s' = \left(k^2+4ke^{2\xi}\right)^{1/2}$

so $\bar{e}_t = \dfrac{k\xi\bar{i} + 2k[1-e^{\xi}]\bar{j}}{\left(k^2+4ke^{2\xi}\right)^{1/2}}$

$$\frac{1}{\rho}\bar{e}_n = \frac{\bar{e}_t'}{s'} = \frac{1}{k\left(1+4e^{2\xi}\right)^{1/2}}\left[\frac{-2e^{\xi}}{\left(1+4e^{2\xi}\right)^{1/2}}\bar{j} - \frac{1}{2}\left(\bar{i} - 2e^{2\xi}\bar{j}\right)\frac{8e^{2\xi}}{\left(1+4e^{2\xi}\right)^{3/2}}\right]$$

substituting $\xi = 0.5$, we find

$\bar{e}_t \hat{=} 0.29\bar{i} - 0.96\bar{j} \qquad \dfrac{1}{\rho}\bar{e}_n = \dfrac{-0.05}{k}\bar{i} + \dfrac{0.12}{k}\bar{j}$

$s' = 3.45k$

$v = \beta\xi,\ \xi = 0.5$

$$\boxed{\bar{v} = v\bar{e}_t = (0.5\beta)\left(0.29\bar{i} - 0.96\bar{j}\right)}$$

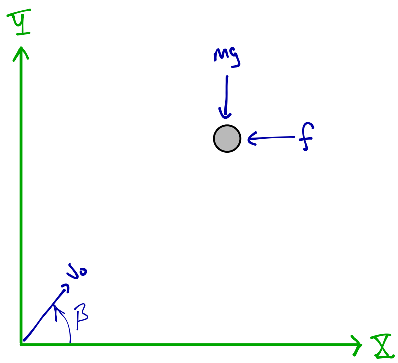
$\bar{a} = \dot{v}\bar{e}_t + \dfrac{v^2}{\rho}\bar{e}_n$

$\dot{v} = v\dfrac{v'}{s'} = v\dfrac{\beta}{\left(k^2+4ke^{2\xi}\right)^{1/2}} \leftarrow$ at $\xi = 0.5 \quad \dot{v} \hat{=} 0.15\dfrac{\beta^2}{k}$

$$\boxed{\bar{a} = \left[0.03\bar{i} - 0.17\bar{j}\right]\frac{\beta^2}{k}}$$

Problem 2.19

EXERCISE 2.19 A 200-g ball is thrown from the ground with the initial velocity $v_0 = 20$ m/s at an angle of elevation β. In addition to its weight, there is a headwind that generates a horizontal resistance of 0.5 N. (a) For the case in which $\beta = 30^\circ$ find the horizontal distance at which the ball returns to the elevation from which it was thrown. Also find the velocity of the ball at that location. (b) Find the value of β that maximizes the range for a specified value of v_0.



The acceleration of the ball is

$$\bar{a} = \ddot{x}\bar{I} + \ddot{y}\bar{J}$$

So

$$\sum \bar{F} = m\bar{a} = (-f)\bar{I} + (-mg)\bar{J} = m(\ddot{x}\bar{I} + \ddot{y}\bar{J})$$

$$m\ddot{x} = -f \rightarrow \ddot{x} = f/m = \frac{0.5N}{200g} = -2.5\, m/s^2$$

$$m\ddot{y} = -mg \rightarrow \ddot{y} = -g$$

The initial conditions are:

$$x(0) = 0, \quad y(0) = 0, \quad \bar{V}_0 = (V_0\cos\beta)\bar{I} + (V_0\sin\beta)\bar{J}$$

Because there is constant accel. in all directions, we know (from high-school physics)

$$x(t) = \frac{1}{2}a_x t^2 + v_{x0}t + x_0 \quad \text{and} \quad y(t) = \frac{1}{2}a_y t^2 + V_{y0}t + y_0$$

$$x(t) = \frac{1}{2}(-2.5)t^2 + (V_0\cos\beta)t \quad \text{and} \quad y(t) = -\frac{1}{2}gt^2 + (V_0\sin\beta)t$$

Now, we can use these two equations for both a) and b)

a) Find when $y(t) = 0$ for $t > 0$

$$0 = -\frac{1}{2}gt^2 + (V_0\sin\beta)t \rightarrow 0 = t\left(-\frac{1}{2}gt + V_0\sin\beta\right) \rightarrow t = \frac{2V_0\sin\beta}{g}$$

Substitute this time into the $x(t)$ equation to find

$$x\left(\frac{2V_0\sin\beta}{g}\right) = \frac{1}{2}(-2.5)\left(\frac{2V_0\sin\beta}{g}\right)^2 + (V_0\cos\beta)\left(\frac{2V_0\sin\beta}{g}\right) \rightarrow x_f = -2.5\left(\frac{V_0\sin\beta}{g}\right)^2 + \frac{2V_0^2\sin\beta\cos\beta}{g}$$

for $\beta = 30^\circ \rightarrow \boxed{x(t_f) \approx 32.7m}$

Problem 2.19 (cont.)

b) In part a), we saw that the final position is:

$$x_f = -2.5\left(\frac{v_0 \sin\beta}{g}\right)^2 + \frac{2v_0^2 \sin\beta\cos\beta}{g}$$

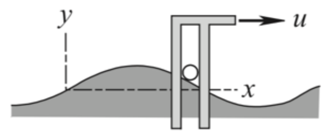
To optimize distance, find β where $\frac{\partial x_f}{\partial \beta} = 0$

See the Jupyter Notebook for my solution to this

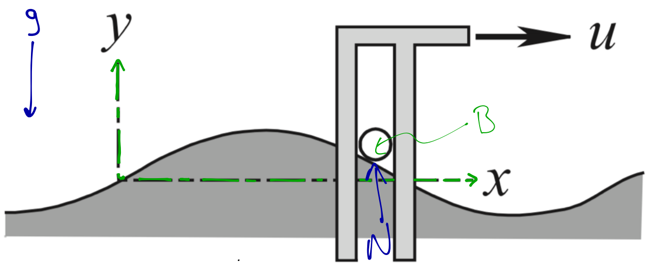
Find that $\boxed{\beta \approx 37.9^\circ \text{ and } x_f = 35.6\text{m for this } \beta}$

Problem 2.24

EXERCISE 2.24 The diagram shows a small ball that is pushed in the vertical plane along a hill whose elevation is $y = H\sin(\pi x/L)$. The motion is actuated by an angle arm that translates horizontally at constant speed u. It may be assumed that the ball remains in contact with the hill. (a) Derive expressions for the velocity and acceleration of the ball as functions of its horizontal distance x from the origin. (b) Determine the maximum speed v of the ball and the value(s) of x at which it occurs. (c) Determine the maximum acceleration magnitude of the ball and the value(s) of x at which it occurs. (d) What is the largest value of u for which the ball will remain in contact with the hill when $x = L/2$? Friction is negligible, but gravity is not.



Exercise 2.24



Exercise 2.24

$$\bar{v}_B = \dot{x}\,\bar{i} + \dot{y}\,\bar{j}$$

$$\bar{a}_B = \ddot{x}\,\bar{i} + \ddot{y}\,\bar{j}$$

$$\dot{x} = u \qquad \dot{y} = \dot{x}\frac{dy}{dx} = \dot{x}\left(\frac{-H\pi}{L}\cos(\pi x/L)\right) = \frac{-uH\pi}{L}\cos\left(\frac{\pi x}{L}\right)$$

$$\ddot{x} = \dot{u} = 0 \qquad \ddot{y} = \dot{x}\frac{d\dot{y}}{dx} = \dot{x}\left(\frac{-uH\pi^2}{L^2}\sin(\pi x/L)\right) = \frac{-u^2H\pi^2}{L^2}\sin\left(\frac{\pi x}{L}\right)$$

a) $$\boxed{\bar{v}_B = u\,\bar{i} + \left[\frac{-uH\pi}{L}\cos\left(\frac{\pi x}{L}\right)\right]\bar{j} \qquad \bar{a}_B = -\frac{u^2H\pi^2}{L^2}\sin\left(\frac{\pi x}{L}\right)\bar{j}}$$

b) $$\text{speed} = |v_B| = |\bar{v}_B\cdot\bar{v}_B| = \left(u^2 + \frac{u^2H^2\pi^2}{L^2}\cos^2\left(\frac{\pi x}{L}\right)\right)^{1/2} = u\left[1 + \frac{H^2\pi^2}{L^2}\cos^2\left(\frac{\pi x}{L}\right)\right]^{1/2}$$

This is ± 1 at limits of amp
$+1$ at $\pi x/L = 0, \pi, 2\pi, \ldots$

So, the max velocity occurs when

$$\boxed{x = nL,\quad n = 0, 1, 2, \ldots \quad \text{and is } |\bar{v}_B| = u\left[1 + \frac{H^2\pi^2}{L^2}\right]^{1/2}}$$

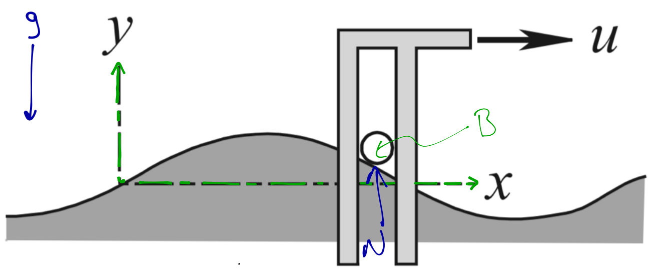
c) $$|\bar{a}_B| = |\bar{a}_B\cdot\bar{a}_B| = \frac{u^2H\pi^2}{L^2}\left|\sin\left(\frac{\pi x}{L}\right)\right|$$ ← This is maximized when $\sin\left(\frac{\pi x}{L}\right) = \pm 1$ ← happens when $\frac{\pi x}{L} = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

So, max accel occurs when

$$\boxed{x = \left(\frac{2n-1}{2}\right)L,\quad n = 0, 1, 2, \ldots \quad \text{and is } |\bar{a}_B| = \frac{u^2H\pi^2}{L^2}}$$

Problem 2.24 (cont.)

d) If the ball remains in contact with the ground, the normal force is nonzero.



Exercise 2.24

at $L/2$ $\quad \bar{a}_B = \frac{-u^2 H \pi^2}{L} \bar{J}$

$$\sum \bar{F} \cdot \bar{J} = N - mg = m\bar{a}_B \cdot \bar{J}$$

So

$$N = m a_B \cdot \bar{J} + mg$$

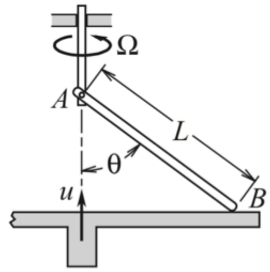
For $N > 0$ $\quad m a_B > mg \rightarrow a_B > g$

So need

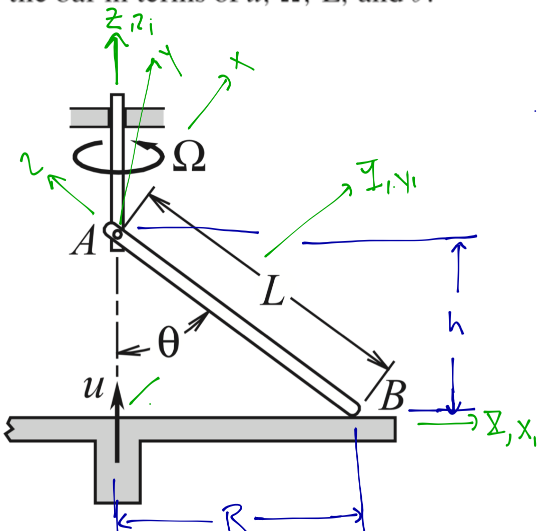
$$-\frac{u^2 H \pi^2}{L^2} > g \rightarrow \boxed{u < \sqrt{\frac{g L^2}{H \pi^2}}}$$

Problem 2.31

EXERCISE 2.31 The vertical shaft rotates at the constant rate Ω, and the elevation of pin A is constant. End B of the bar slides over the base table, which translates upward at the constant speed u. Describe the velocity and acceleration of end B of the bar in terms of u, Ω, L, and θ.



Exercise 2.31



Exercise 2.31

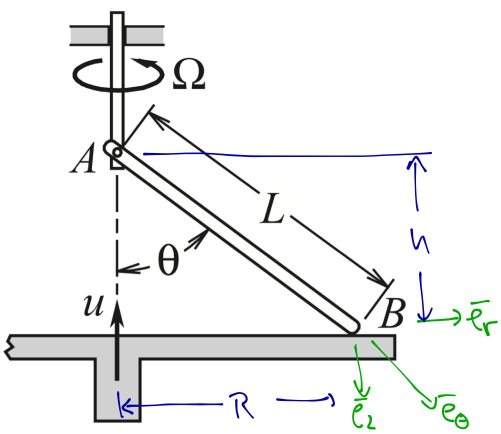
XYZ - fixed with Z up

$x_1y_1z_1$ - fixed to vertical rod (aligned to XYZ as drawn here)

xyz - fixed to rod AB

Using these, we can easily work this problem using methods from Ch. 3

Or... we can use cylindrical coords (look at pages 53-54 of the book)



Exercise 2.31

$R = L\sin\theta \qquad h = L\cos\theta \rightarrow \dot{h} = -L\dot{\theta}\sin\theta = -u$

$\dot{R} = L\dot{\theta}\cos\theta = u\dfrac{\cos\theta}{\sin\theta} \leftarrow \text{sub } \dot{\theta} \leftarrow u = L\dot{\theta}\sin\theta \rightarrow \dot{\theta} = \dfrac{u}{L\sin\theta}$

$\ddot{R} = -u\dot{\theta}\dfrac{1}{\sin^2\theta} \leftarrow \text{sub } \dot{\theta} \qquad u \equiv \text{const. so } \ddot{h} = 0$

$= -\dfrac{u^2}{L}\dfrac{1}{\sin^3\theta}$

In cylindrical coords, we know

$\bar{v}_B = \dot{R}\bar{e}_r + R(-\Omega)\bar{e}_\theta - u\bar{e}_z \quad \leftarrow -\Omega$ due to $\bar{e}_z$ being down

$$\boxed{\bar{v}_B = (u\cot\theta)\bar{e}_r - L\Omega\sin\theta\,\bar{e}_\theta - u\bar{e}_z}$$

and

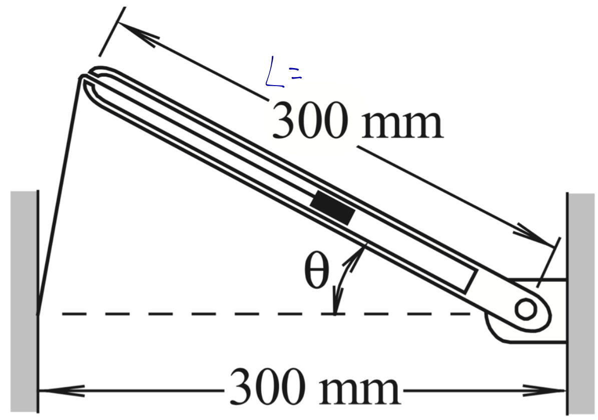
$\bar{a}_B = (\ddot{R} - R\Omega^2)\bar{e}_r + 2\dot{R}(-\Omega)\bar{e}_\theta \quad$ - due to direction of $\bar{e}_z$

plug in terms to find:

$$\boxed{\bar{a}_B = \left[-\frac{u^2}{L}\frac{1}{\sin^3\theta} - L\Omega^2\sin\theta\right]\bar{e}_r + \left[-2u\Omega\cot\theta\right]\bar{e}_\theta}$$

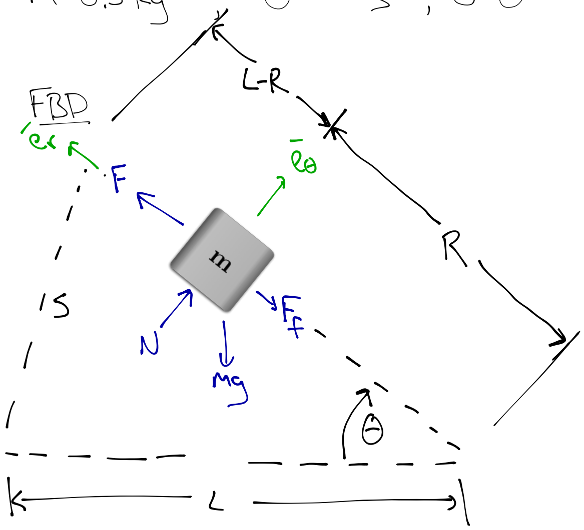
Problem 2.34

EXERCISE 2.34 The cable, whose length is 300 mm, is fastened to the 500-g block. Clockwise rotation of the arm at a constant angular speed of 5 rad/s causes the block to slide outward. The motion occurs in the vertical plane, and the coefficient of sliding friction is 0.4. Determine the tensile force in the cable and the force exerted by the block on the walls of the groove when $\theta = 53.1301^\circ$.



Exercise 2.34

$m = 0.5\,kg \qquad \dot{\theta} = 5\,\frac{rad}{s}\,, \quad \ddot{\theta} = 0$



cable length $\rightarrow \quad s + (L-R) = 0.3\,m$

$R = (s+L) - 0.3$ $\qquad$ } $L \doteq const$

$\dot{R} = \dot{s} \qquad \ddot{R} = \ddot{s}$

also know that $\quad s = 2\left(L\sin\frac{\theta}{2}\right)$

$\dot{s} = L\dot{\theta}\cos\frac{\theta}{2}\;, \quad \ddot{s} = -\frac{L}{2}\dot{\theta}^2\sin\frac{\theta}{2}$

At $\theta = 53.1301^\circ \doteq 0.93\,rad \rightarrow s \doteq 0.27,\ \dot{s} \doteq 1.34,\ \ddot{s} \approx -1.68,\ R \doteq 0.27,\ \dot{R} = \dot{s},\ \ddot{R} = \ddot{s}$

$$\bar{a} = \left(\ddot{R} - R\dot{\theta}^2\right)\bar{e}_r + \left(\cancel{R\ddot{\theta}}^{\,0} + 2\dot{R}\dot{\theta}\right)\bar{e}_\theta$$

Now, sum forces on the mass

$$\Sigma\bar{F} = m\bar{a}$$

$$\Sigma\bar{F} = \left(F - F_f - mg\sin\theta\right)\bar{e}_r + \left(N - mg\cos\theta\right)\bar{e}_\theta$$

In $\bar{e}_\theta$

$$N - mg\cos\theta = m\left(2\dot{R}\dot{\theta}\right) \rightarrow \boxed{N = 2m\dot{R}\dot{\theta} + mg\cos\theta \approx 4.28N}$$

In $\bar{e}_r$

$$F - F_f - mg\sin\theta = m\left(\ddot{R} - R\dot{\theta}^2\right) \rightarrow F - \mu N - mg\sin\theta = m\left(\ddot{R} - R\dot{\theta}^2\right)$$

$$\boxed{F = m\left(\ddot{R} - R\dot{\theta}^2\right) + \mu N + mg\sin\theta \approx 1.44N}$$