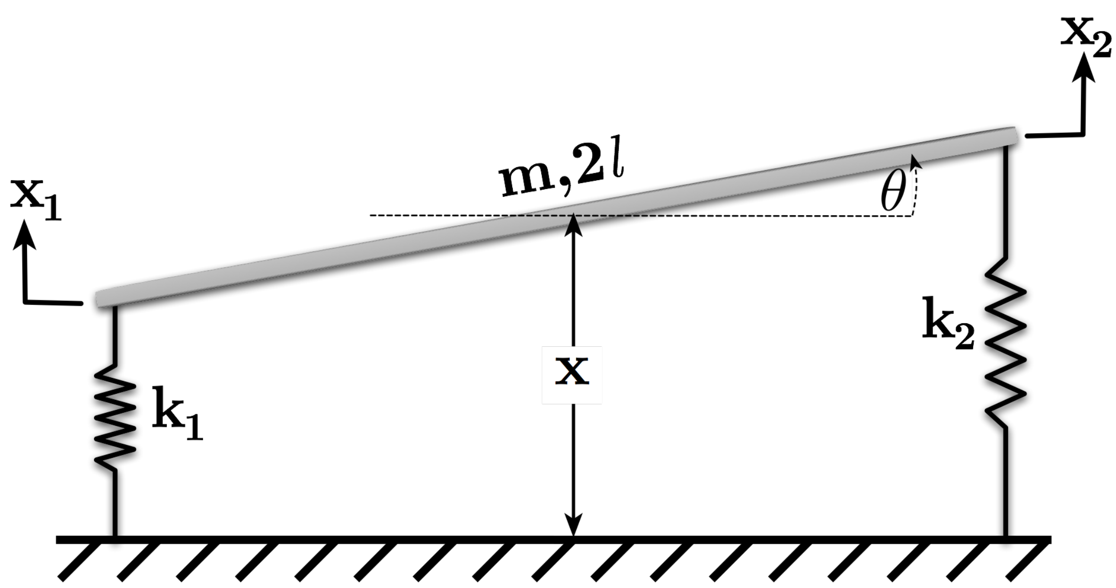
Repeated and Zero Frequencies (Sec. 4.14)



$$\begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} k_1+k_2 & (k_2-k_1)l \\ (k_2-k_1)l & (k_1+k_2)l^2 \end{bmatrix} \begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Let $k_1 = k_2 = k$

$$\begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} 2k & 0 \\ 0 & 2kl^2 \end{bmatrix} \begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Q: What do you notice about these equations?

They're decoupled!!! <-- This means that x and θ are normal coordinates.
(We also called these modal coordinates)

Q: What does this mean?

Vibration in one mode is purely in x. The other mode is purely in θ.

We can find the eigenvalues to be:

$$\omega_1^2 = \frac{2k}{m} \quad \text{and} \quad \omega_2^2 = \frac{2kl^2}{I}$$

Q: What happens if the two eigenvalues (natural freq.) are equal?

$$\omega_1^2 = \omega_2^2 \quad \left(\frac{2k}{m} = \frac{2kl^2}{I} \right)$$

For the eigenvalue problem, we get:

$$\left. \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{X} \\ \bar{\Theta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\} \text{The eignvectors are indeterminate}$$

Repeated and Zero Frequencies (cont.)

Q: What does this mean physically?

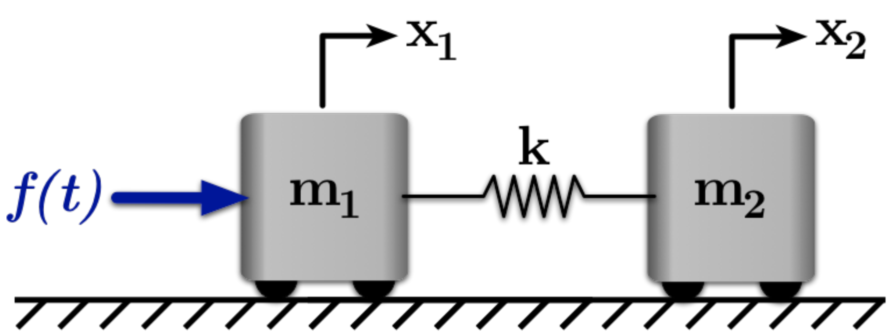
Any excitation will cause the system to oscillate at same frequency!

Q: So, how should we choose the eigenvectors for repeated eigenvalues?

Choose them independent of one another (and other non-repeated eigenvalue/eigenvector pairs)

Use some physical intuition to pick. Here, $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ make sense ← This matches what we found when $\omega_1 \neq \omega_2$

Zero Frequency?

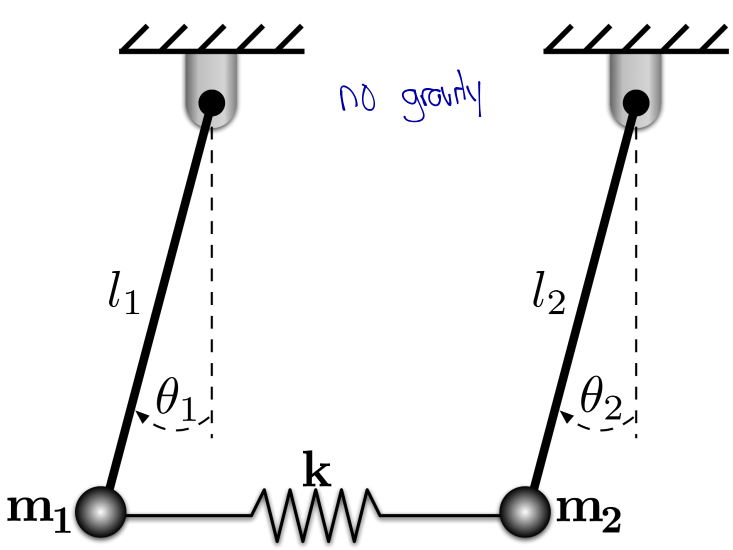


$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix}$$

If we solve this eigenvalue problem, we find $\omega_1^2 = 0$

This is called the rigid-body mode, because the system behaves like a rigid body (like k=infinity).

Q: Do you expect this system to have a rigid-body mode?



Yes. Displace the pendulums identically and it will remain in the new location (There is no restoring force.).