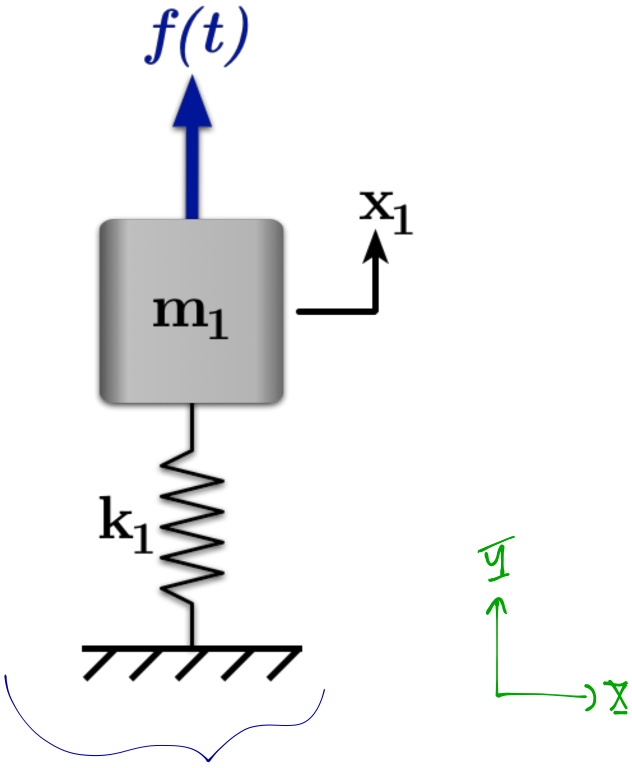
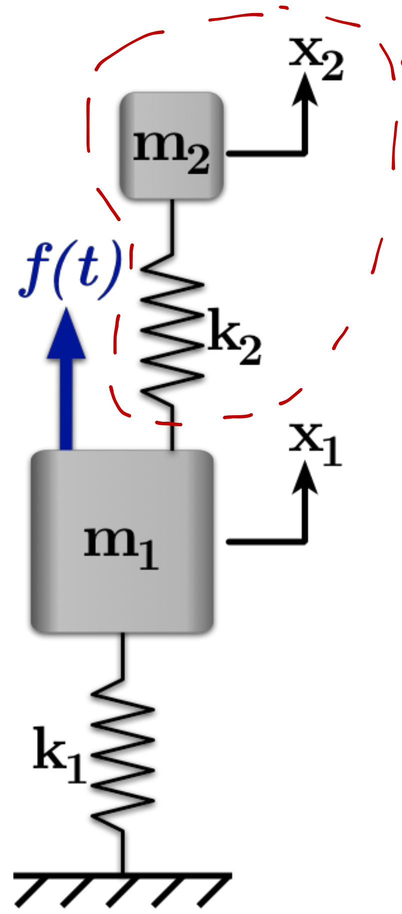
Vibration Absorbers without Damping (Sec. 4.4)

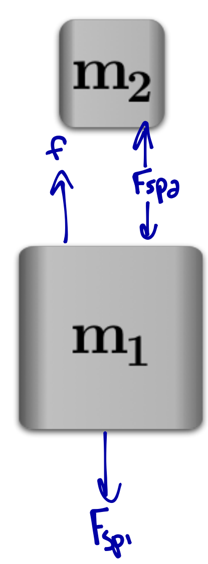


Q: What happens if we operate this near $\sqrt{\frac{k_1}{m_1}}$?



Q: Can we design this subsystem (by picking k2 and m2) to minimize the response of x1 to f(t)?

FBD



$F_{sp2} = k_2(x_1 - x_2)$

$F_{sp1} = k_1 x_1$

$m_1\ddot{x}_1 = -k_1x_1 - k_2(x_1 - x_2) + f$ $\qquad$ $m_2\ddot{x}_2 = k_1(x_1 - x_2)$

$m_1\ddot{x}_1 + (k_1 + k_2)x_1 - k_2x_2 = f$ $\qquad$ $m_2\ddot{x}_2 + k_1x_2 - k_1x_1 = 0$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix}$$

$\leftarrow$ assume $f(t) = \bar{f}e^{i\omega t}$, so $F = \begin{bmatrix} \bar{f} \\ 0 \end{bmatrix} e^{i\omega t}$

$F = \bar{F}e^{i\omega t}$

Assume $x(t)$ to match the form of $f(t) \Rightarrow X(t) = \bar{X}e^{i\omega t}$

substitute into the eq of motion and collect terms

$$[K - \omega^2 M]\bar{X} = \bar{F} \longrightarrow \bar{X} = [K - \omega^2 M]^{-1}\bar{F}$$

$$\bar{X} = \begin{bmatrix} k_1+k_2-\omega^2 m_1 & -k_2 \\ -k_2 & k_2-\omega^2 m_2 \end{bmatrix}^{-1}\begin{bmatrix} \bar{f} \\ 0 \end{bmatrix} = \frac{1}{\Delta(\omega)}\begin{bmatrix} k_2-\omega^2 m_2 & k_2 \\ k_2 & k_1+k_2-\omega^2 m_1 \end{bmatrix}\begin{bmatrix} \bar{f} \\ 0 \end{bmatrix}$$

where $\Delta(\omega) = (k_1+k_2-\omega^2 m_1)(k_2-\omega^2 m_2) - k^2$

so $\bar{X}_1 = \frac{(k_2-\omega^2 m_2)\bar{f}}{\Delta(\omega)}$ and $\bar{X}_2 = \frac{k_2\bar{f}}{\Delta(\omega)}$

Q: Notice anything?

We can make $\bar{X}_1$ (the amp of $x_1(t)$) $= 0$ if $k_2 - \omega^2 m_2 = 0 \leftarrow \omega = \sqrt{\frac{k_2}{m_2}}$

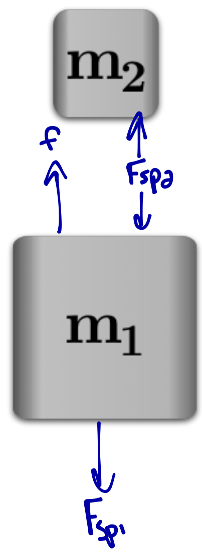
Vibration Absorbers without Damping (cont.)

Q: How is this possible?

Look at the FBD.

If F_{sp2} is exactly opposite of $f(t)$ for all time, then there are no external forces on m_1 (assume $x_1 = 0$)

FBD



Let's look at the equations of motion:

Assume $f(t) = \bar{f}\cos(\omega_n t)$ where $\omega_n = \sqrt{\frac{k_1}{m_1}} = \sqrt{\frac{k_2}{m_2}}$

$$\bar{X}_2 = \frac{k_2 \bar{f}}{\Delta(\omega)} \rightarrow \Delta(\omega) = \left(k_1 + k_2 - \frac{k_1}{m_1}(m_1)\right)\left(k_2 - \left(\frac{k_2}{m_2}\right)m_2\right) - k_2^2 = (k_2)(0) - k_2^2$$

$$\bar{X}_2 = \frac{k_2 \bar{f}}{-k_2^2} \rightarrow \text{ so } x_2(t) = \frac{-\bar{f}}{k_2}\cos(\omega_n t)$$

Q: What's the resulting spring force? (assuming $x_1 = 0$)

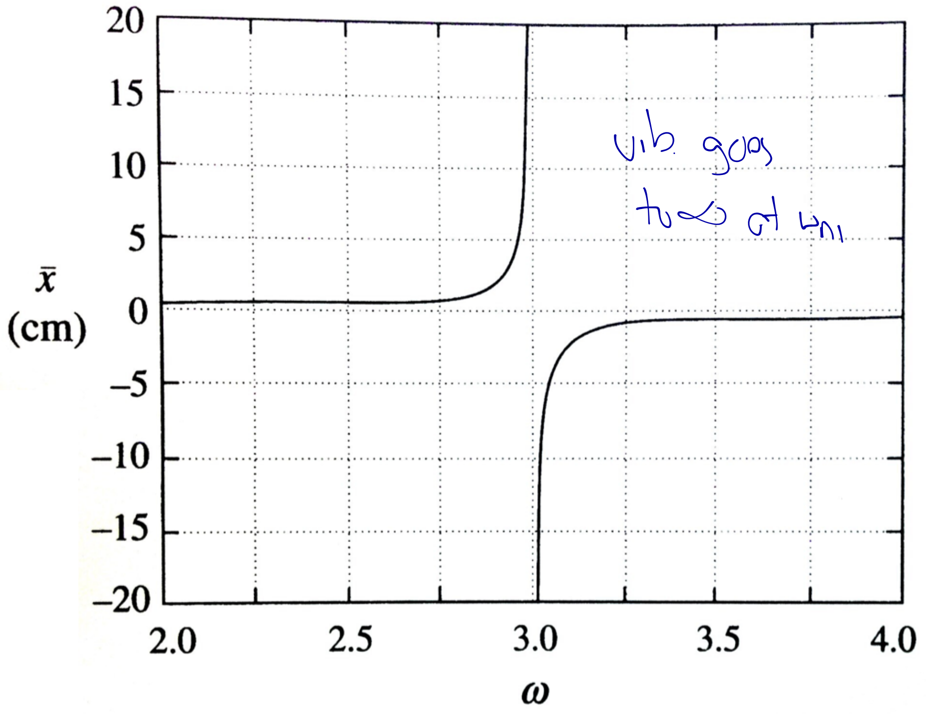
$$F_{sp2} = k_2(x_2) = k_2\left(\frac{-\bar{f}}{k_2}\cos(\omega_n t)\right) = \underbrace{-\bar{f}\cos(\omega_n t)}$$

The resulting spring force **exactly** cancels the excitation force.

Vibration Absorber Design

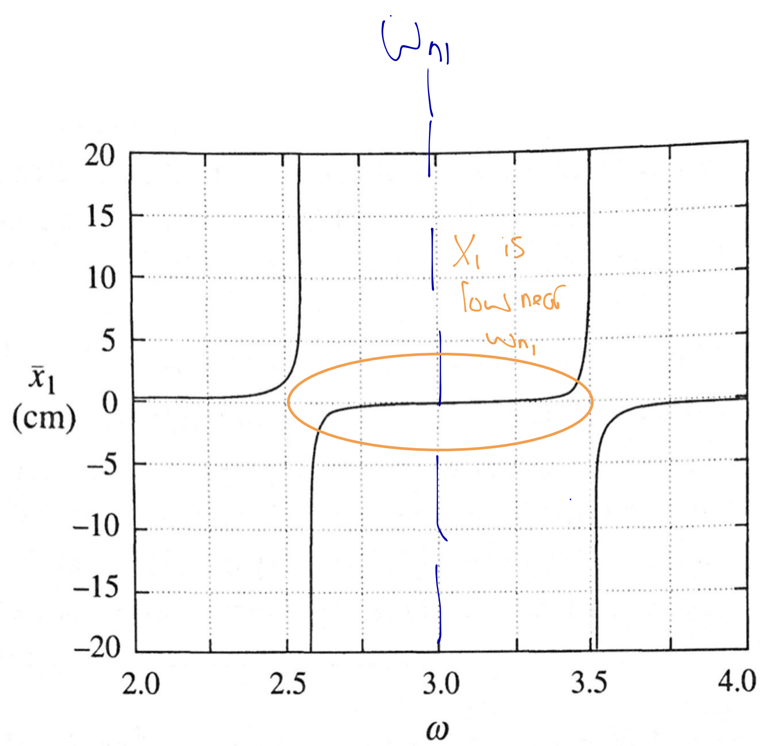
Let $k_1 = 900 \, N/m$, $m_1 = 100 \, kg$, $\bar{F} = 1N$, and $\omega = 3 \frac{rad}{s}$ $\left(= \omega_n = \sqrt{\frac{k_1}{m_1}} \right)$

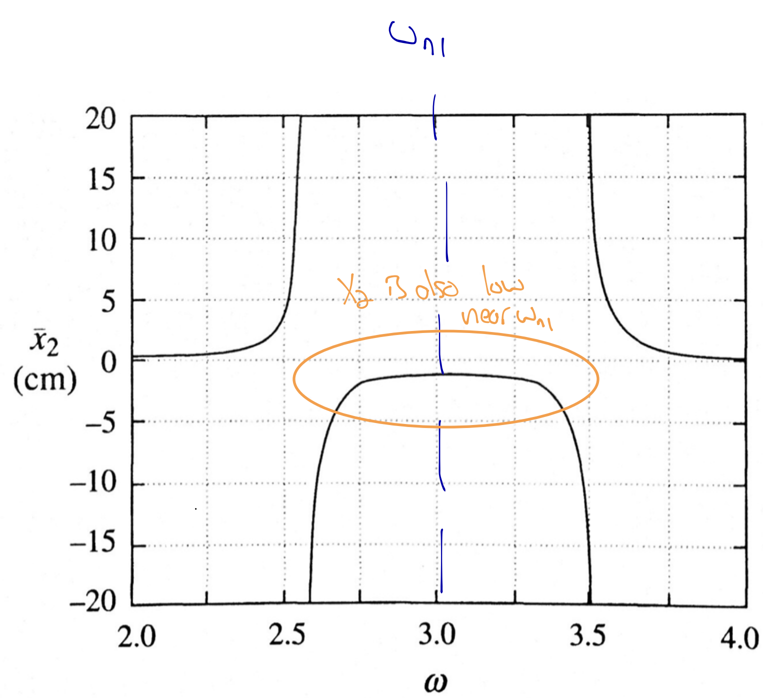
Q: What's the response without a vibration absorber?



To design a vibration absorber, choose k_2 and m_2 such that $\frac{k_2}{m_2} = \frac{k_1}{m_1}$ (to match $\omega_{n1} = \omega_{n2}$)

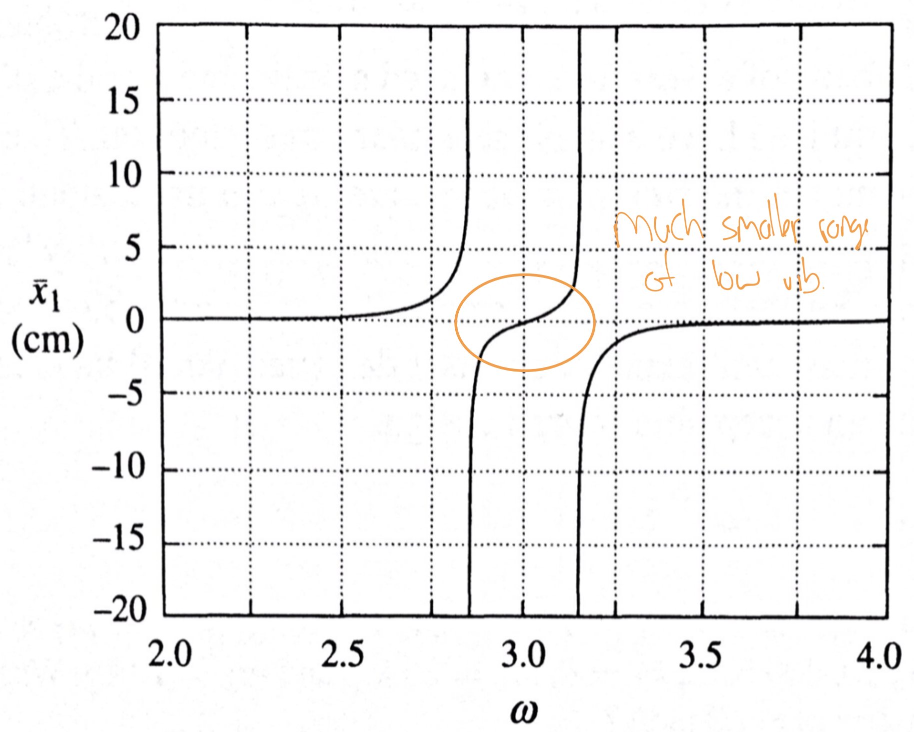
choose $m_2 = 10 \, kg$ (10% of m_1) and $k_2 = 90 \, N/m$ (to match ω_{n1})

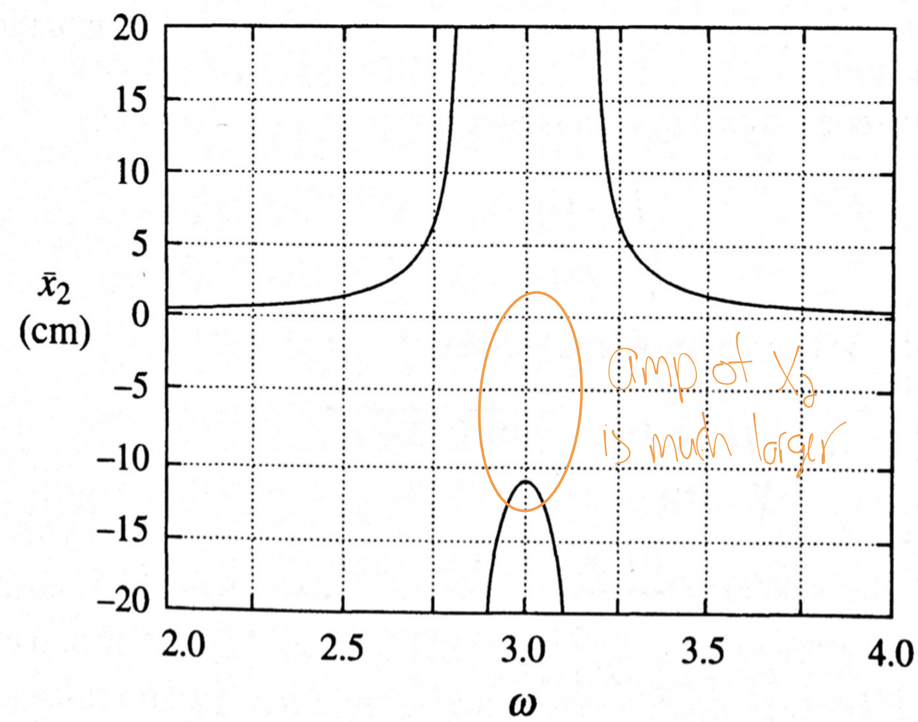




Vibration Absorber Design (cont.)

Q: What happens if we want/need a smaller m_2?
(we just need $\frac{k_1}{m_1} = \frac{k_2}{m_2}$, right?)





Q: Why?

1) If we decrease m_2, we need to decrease k_2 to keep $\frac{k_1}{m_1} = \frac{k_2}{m_2}$
2) The spring force balancing $f(t)$ prop. to k_2
3) So, if k_2 decreases, X_2 must increase to generate the same cancelling force.

Fundamental Compromise in Vibration Absorber Design:

- Large 2nd mass leads to:
  - low amplitude m2 motion
  - robustness around the desired freq.
  - at the cost of increased mass (and likely volume)

- Small 2nd mass leads to:
  - higher amplitude m2 motion
  - less robustness to changes in frequency
  - benefit is a lighter/smaller system