MCHE 485: Mechanical Vibrations
Spring 2020 – System Identification Review

Assigned:	Sunday, April 26, 2020
Due:	Will not be collected, but solutions will be posted at 5pm on Wednesday, April 29

Assignment: Answer the two attached problems, making sure to clearly indicate and support your answers.

Submission: N/A

Problem 1

You've been given the plot in Figure 1.1 and the data in Table 1.1 and asked to determine some system properties.

a. Estimate the natural frequency and damping ratio.

b. Does the system appear to be linear? Explain why or why not, supporting your answer with data.

c. An arrogant graduate student insists that the frequency response in Figure 1.2 is from the same system as the response in Figure 1.1. Is this student right? If not, explain why the two responses are not from the same system. Given this student's attitude, it will take an argument supported by data to convince him. (If you believe the student is correct, also explain why you believe so.)

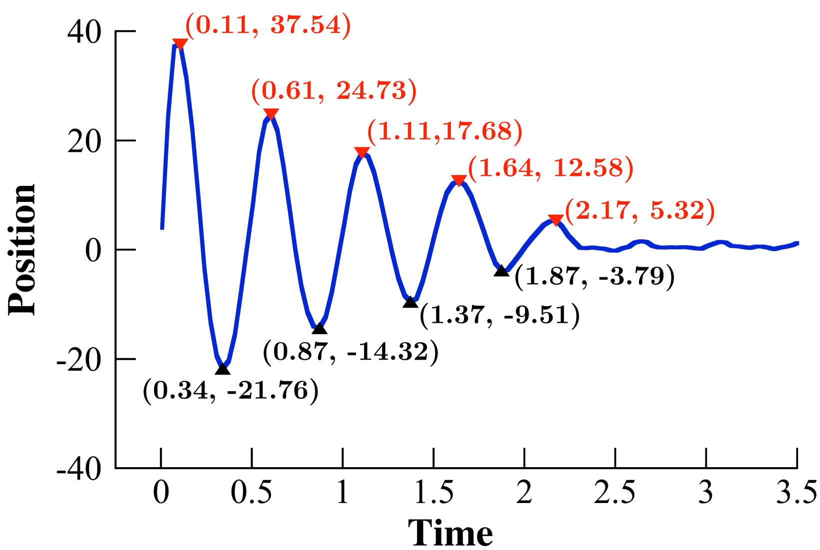

Figure 1.1: Response Data

Table 1.1: Zero Crossings

Zero Crossing Time (s)
0.2233
0.4567
0.7233
0.9900
1.2567
1.4900
1.8233
1.9900
2.4567
2.5233

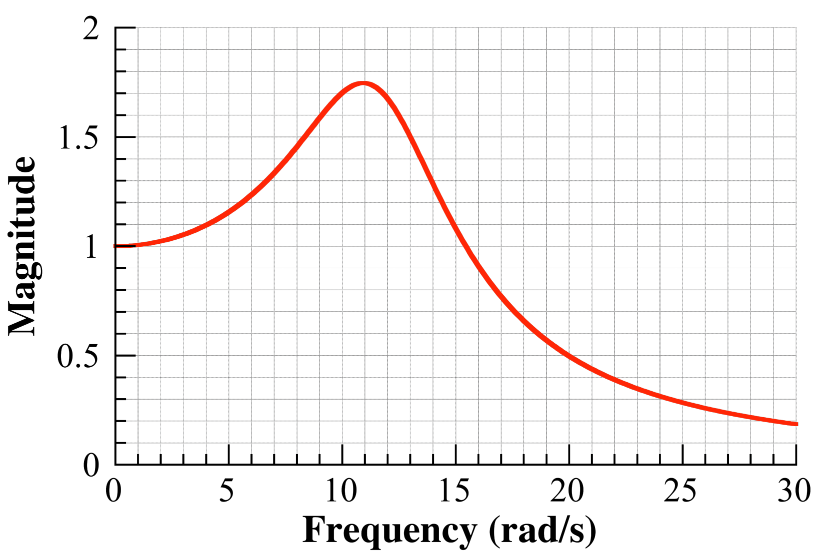

Figure 1.2: Graduate Student Frequency Response Data

Problem 1

Estimating Frequency from Zero Crossings

Zero Crossing Time	Frequency Estimate	
	Negative-Slope Crossings	Positive-Slope Crossings
0.2233		
0.4567	2.00	
0.7233		1.88
0.9900	1.87	
1.2567		2.00
1.4900	1.76	
1.8233		2.00
1.9900		
Over Entire Response	1.88	1.96

Estimate frequency by measuring the period via zero crossings. Be sure to count full periods (every other crossing). Measuring over multiple periods is usually advised.

Here, the estimated frequency is ~1.92 Hz.

Use log. dec. to estimate damping ratio.

Log Decrement looking at positive amplitude peaks

Time (s)	Max Amp	σ - adjacent peaks	ζ - adjacent peaks	σ - Entire Response	ζ - Entire Response
0.11	37.54				
0.61	24.73	0.4174	0.0666		
1.11	17.68	0.3356	0.0535		
1.64	12.58	0.3407	0.0543		
2.17	5.32	0.8605	0.1383	0.4886	0.0780
			Excluding last point	0.3646	0.0581

Log Decrement looking at negative amplitude peaks

Time (s)	Min Amp	σ - adjacent peaks	ζ - adjacent peaks	σ - Entire Response	ζ - Entire Response
0.34	-21.76				
0.87	-14.33	0.4182	0.0667		
1.37	-9.51	0.4096	0.0653		
1.87	-3.79	0.9188	0.1478	0.5822	0.0931
			Excluding last point	0.4139	0.0660

For this response, ignoring the last peaks gives the best estimate of damping. It is $\zeta \approx 0.06$.

Problem 1 (cont.)

b) Looking at the data on the previous page, the damping ratio remains fairly constant between cycles, with the exception of the last peak. This suggests linear, viscous damping. The estimated frequency is also relatively constant between cycles, suggesting linearity as well. Given this, the system can be said to be linear in the range of this response.

c) Looking at the location of the response peak, the natural frequency appears to be close to the estimated value. However, the damping ratio in the frequency response plot appears to much greater than 0.06. We can estimate the damping ratio from this frequency response by:

- comparing peak and static response amplitudes
- using 1/2 power points

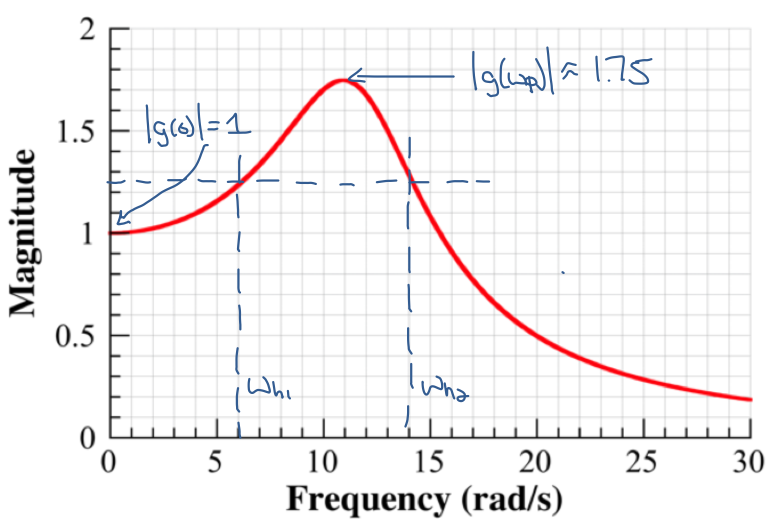

Figure 4.2: Graduate Student Frequency Response Data

We know:

$$\frac{|g(\omega_p)|}{|g(0)|} \approx \frac{1}{2\zeta}$$

$$1.75 = \frac{1}{2\zeta}$$

$$0.29 = \zeta$$

Note: This is the $\zeta \ll 1$ approx.

Both these estimates are much higher than the damping ratio for the system in the free resp.

If we use the half-power points:

$$\frac{1.75}{\sqrt{2}} = 1.24 \qquad \omega_{h1} \cong 6\ \frac{rad}{s} \qquad \omega_{h2} \cong 14\ \frac{rad}{s}$$

$$\delta_h = \omega_{h2} - \omega_{h1} = 8\ \frac{rad}{s}$$

$$\zeta = \frac{\delta_h}{2\omega_n} = \frac{8}{2(1.92 * 2\pi)} \cong 0.33$$

Aside: The actual damping used to generate this plot was $\zeta = 0.3$.

Problem 2

You've been given the plot in Figure 2.1 and the data in Table 2.1 and asked to determine some system properties.

a. Estimate the natural frequency and damping ratio.
b. Does the system appear to be linear? Explain why or why not, supporting your answer with data.
c. Your boss insists that you model the system according to Figure 2.2. Is this a wise choice? Explain why or why not, citing the properties of the model and the response you were given as support. *Hint:* Writing the equations of motion for this system should provide some insight and support for this answer.

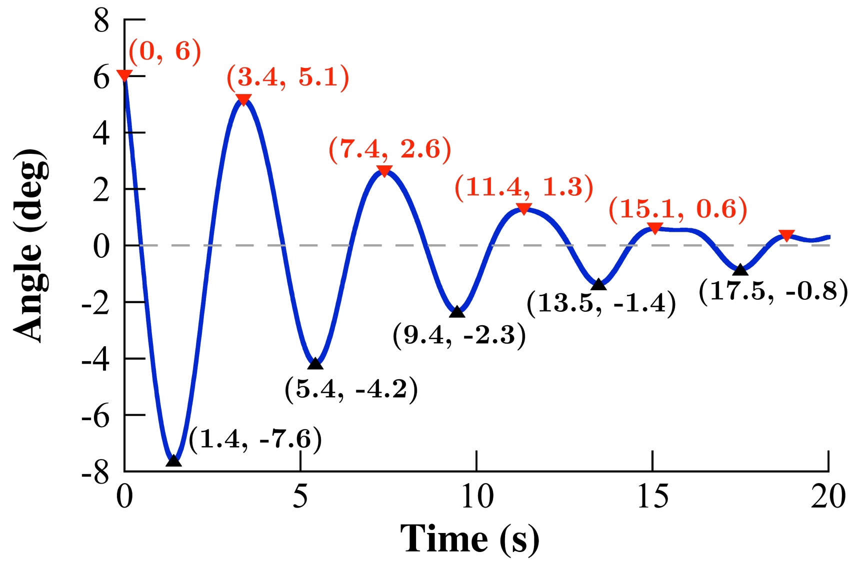

Figure 2.1: Response Data

Table 2.1: Zero Crossings

Zero Crossing Times (s)
0.4705
2.4324
4.5145
6.4364
8.5586
10.4204
12.6226
14.3644
16.7267
18.2883

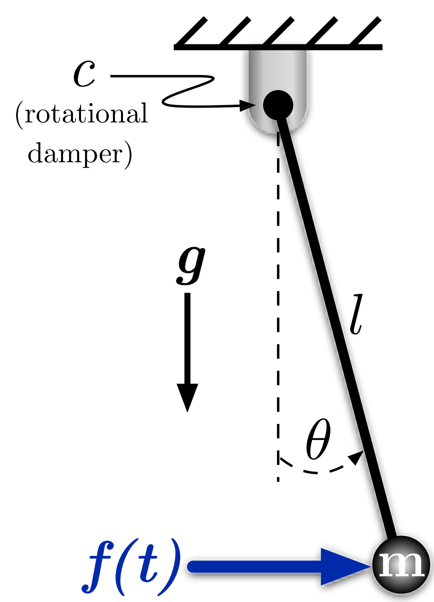

Figure 2.2: Your Boss's Idea for a Model

Problem 2

a) Use the zero crossings to estimate frequency

Zero Crossing Time	Frequency Estimate (Hz)	
	Positive-Slope Crossings	Negative-Slope Crossings
0.4705		
2.4324	0.2473	
4.5145		0.2497
6.4364	0.2473	
8.5586		0.2510
10.4204	0.2461	
12.6226		0.2536
14.3644	0.2437	
16.7267		0.2548
18.2883		
Over Entire Response	0.2461	0.2523

Use log dec to estimate damping

$$\sigma = \frac{1}{N} ln\left(\frac{x(0)}{x(Nt_p)}\right) \rightarrow \zeta = \frac{\sigma}{\sqrt{4\pi^2 + \sigma^2}}$$

where N is the number of periods between peaks

From positive peaks

Time (s)	Max Amp	σ - adjacent peaks	ζ - adjacent peaks	σ - Entire Response	ζ - Entire Response
0.00	6.00				
3.40	5.10	0.1625	0.0259		
7.40	2.60	0.6737	0.1079		
11.40	1.30	0.6931	0.1110		
15.10	0.60	0.7732	0.1240	0.5756	0.0920

From negative peaks

Time (s)	Min Amp	σ - adjacent peaks	ζ - adjacent peaks	σ - Entire Response	ζ - Entire Response
1.40	-7.60				
5.40	-4.20	0.5931	0.0948		
9.40	-2.30	0.6022	0.0963		
13.50	-1.40	0.4964	0.0793		
17.50	-0.80	0.5596	0.0894	0.5628	0.0899

Problem 2 (cont.)

From these, estimate $\omega_n = 0.25Hz$ and $\zeta = 0.09$

Note: The actual freq. used to generate the response was 0.25Hz. The actual damping ratio was 0.10. There was simply noise in the form of a second sine wave added.

b) The system does appear to be fairly linear. The frequency and damping ratios calculated over several different periods are fairly constant.

However, the system may not be "purely" linear, or may not be a single DOF system, as the response is not a pure sinusoid.

c) Your boss's suggestion is a good one. The pendulum system has one dominant mode, like the response. It also has a rotational damper that will cause its response to decay to zero.