

## Possibly Useful Equations

$$\bar{f} = m\bar{a}$$

$$I_0\bar{a} = \sum \bar{M}_0$$

$$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$$

$$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$$

$$e^{\pm i\omega t} = \cos(\omega t) \pm i \sin(\omega t)$$

$$x(t) = ae^{i\omega_n t} + be^{-i\omega_n t}$$

$$x(t) = a \cos \omega_n t + b \sin \omega_n t$$

$$x(t) = e^{-\zeta\omega_n t} [a \cos(\omega_d t) + b \sin(\omega_d t)]$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\int u dv = uv - \int v du$$

$$\delta_{oc}V = \nabla \sum$$

$$x(t) = \frac{\omega_n^2 \bar{y}}{\omega_n^2 - \omega^2} \sin(\omega t)$$

$$\frac{d}{dt} \left( \frac{\delta L}{\delta \dot{q}_i} \right) + \frac{\delta R D}{\delta \dot{q}_i} - \frac{\delta L}{\delta q_i} = Q_i, \quad i = 1, \dots, n$$

$$M\ddot{X} + C\dot{X} + KX = F$$

$$\det(K - \omega^2 M) = 0 \quad [K - \omega^2 M] \bar{X} = 0$$

$$2 + 2 = 2 \times 2 = 2^2 = 0b0010 = 0x2$$

$$i \equiv \sqrt{-1}$$

$$x(t) = \int_0^t f(\tau)h(t - \tau)d\tau$$

$$x(t) = \int_0^t f(t - \tau)h(\tau)d\tau$$

$$f(t) = \sum_{n=0}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

$$a_n = \frac{\omega_0}{\pi} \int_0^{\frac{2\pi}{\omega_0}} f(t) \cos(n\omega_0 t) dt$$

$$b_n = \frac{\omega_0}{\pi} \int_0^{\frac{2\pi}{\omega_0}} f(t) \sin(n\omega_0 t) dt$$

$$a_0 = \frac{\omega_0}{2\pi} \int_0^{\frac{2\pi}{\omega_0}} f(t) dt$$

$$V(\omega, \zeta) = e^{-\zeta\omega t_n} \sqrt{[C(\omega, \zeta)]^2 + [S(\omega, \zeta)]^2}$$

$$\tilde{X}_i = \frac{1}{\sqrt{X_i^T M X_i}} X_i$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$x(t) = c + e^{-\zeta\omega_n t} [a \cos(\omega_d t) + b \sin(\omega_d t)]$$

$$\sigma = \frac{1}{N} \ln \left( \frac{x(0)}{x(Nt_p)} \right)$$

$$\sigma = \ln \left( \frac{x(0)}{x(t_p)} \right)$$

$$\zeta = \frac{\sigma}{\sqrt{4\pi^2 + \sigma^2}}$$

$$\zeta = \frac{\sigma}{2\pi}$$

$$\omega_p = \omega_n \sqrt{1 - 2\zeta^2}$$

$$\zeta \approx \frac{\delta_h}{2\omega_n}$$

$$E = mc^2$$

$$A = \begin{bmatrix} 0 & -K \\ -K & -C \end{bmatrix} \quad B = \begin{bmatrix} -K & 0 \\ 0 & M \end{bmatrix}$$

$$[U^T M U] \ddot{H} + [U^T K U] H = U^T F \cos \omega t$$

## Possibly Useful Equations

$$\begin{aligned}
 x(t) &= -\frac{(2\zeta\omega\omega_n)\frac{\bar{f}}{m}}{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega\omega_n)^2} \cos \omega t + \frac{(\omega_n^2 - \omega)\frac{\bar{f}}{m}}{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega\omega_n)^2} \sin \omega t \\
 &= \left[ \frac{\bar{f}}{m} \frac{1}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega\omega_n)^2}} \right] \cos(\omega t - \phi), \quad \text{where } \phi = \tan^{-1} \left( \frac{2\zeta\omega\omega_n}{\omega_n^2 - \omega^2} \right)
 \end{aligned}$$

$$\begin{aligned}
 g(\omega) &= \sqrt{\frac{\omega_n^4 + (2\zeta\omega\omega_n)^2}{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega\omega_n)^2}} e^{i\phi}, \quad \text{where} \\
 \phi &= \phi_1 - \phi_2, \quad \text{where } \phi_1 = \tan^{-1} \left( \frac{2\zeta\omega}{\omega_n} \right) \text{ and } \phi_2 = \tan^{-1} \left( \frac{2\zeta\omega\omega_n}{\omega_n^2 - \omega^2} \right)
 \end{aligned}$$

$$\begin{aligned}
 g(\Omega) &= \sqrt{\frac{1 + (2\zeta\Omega)^2}{(1 - \Omega^2)^2 + (2\zeta\Omega)^2}} e^{i\phi}, \quad \text{where} \\
 \phi &= \tan^{-1}(2\zeta\Omega) - \tan^{-1} \left( \frac{2\zeta\Omega}{1 - \Omega^2} \right)
 \end{aligned}$$

$$\begin{aligned}
 g(\omega) &= \frac{1}{m\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega\omega_n)^2}} e^{-i\phi}, \quad \text{where} \\
 \phi &= \tan^{-1} \left( \frac{2\zeta\omega\omega_n}{\omega_n^2 - \omega^2} \right)
 \end{aligned}$$

$$\begin{aligned}
 x(t) &= \frac{e\beta\omega^2}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega\omega_n)^2}} \cos(\omega t - \phi), \\
 &\text{where } \phi = \tan^{-1} \left( \frac{2\zeta\omega\omega_n}{\omega_n^2 - \omega^2} \right)
 \end{aligned}$$

$$\begin{aligned}
 x(t) &= \frac{e\beta\Omega^2}{\sqrt{(1 - \Omega^2)^2 + (2\zeta\Omega)^2}} \cos(\omega t - \phi), \quad \text{where} \\
 \phi &= \tan^{-1} \left( \frac{2\zeta\Omega}{1 - \Omega^2} \right)
 \end{aligned}$$