MCHE 485: Mechanical Vibrations

Spring 2015 – Final Exam

Thursday, May 7

Name: ______________________________ **CLID:** ____________

Directions: Complete the attached problems making sure to clearly indicate your answer, show your work, and list any assumptions that you have made (with justification for them, if necessary). If you need extra space for any question, you may attach additional sheets of paper.

Academic Honesty (just a reminder):

An essential rule in every class of the University is that all work for which a student will receive a grade or credit be entirely his or her own or be properly documented to indicate sources. When a student does not follow this rule, s/he is dishonest and s/he defeats the purpose of the course and undermines the goals of the University.

______ Problem 1 – 15 Points

The system in Figure 1 consists of a mass, m, connected to input y through spring k and damper c.

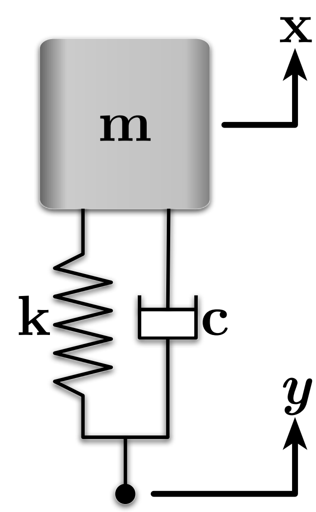

Figure 1: A Mass-Spring System

a. Write the equations of motion for this system.

b. What is the natural frequency?

c. Assume a unit step input in $y(t)$. Write the response, $x(t)$, to this step input.

d. Plot the response, $x(t)$, to the unit step input for damping ratios of $\zeta = 0.0$, $\zeta = 0.2$, and $\zeta = 0.7$. Be sure to clearly label the axes and differentiate between the responses. For the $\zeta = 0.2$ case, label the rise time, settling time, and peak overshoot. You may replot the response on a separate set of axes, if needed, to properly label these quantities.

e. Now, assume a harmonic input in $y(t)$. Write the transfer function from the amplitude of the input to the amplitude of the output.

f. Sketch the approximate frequency response for $\zeta = 0.0$, $\zeta = 0.2$, and $\zeta = 0.7$. For each damping ratio, be sure to indicate:

   i. Magnitude and phase as ω approaches 0.

   ii. Magnitude and phase as ω approaches infinity.

   iii. Magnitude when ω equals the natural frequency of the system, ω_n.

______ Problem 2 – 25 Points

The system in Figure 2 is a common model used for rotating machinery. It consists of a mass, m_1, connected to ground through a spring k, and a damper, c. A second mass, m_2, is offset from center by eccentricity, e, and is rotating at an angular velocity of ω. There is a secondary mass, m_3, and spring, k_2, that will be added in for parts d.–g. of this problem.

Ignoring the secondary, k_2–m_3, subsystem:

a. Write the equations of motion describing the motion of m_1.

b. What is the natural frequency?

c. Sketch the approximate frequency response. Be sure to indicate:

   i. Magnitude as ω approaches 0.

   ii. Magnitude as ω approaches infinity.

   iii. Magnitude when ω equals the natural frequency of the system, ω_n.

Now, the secondary subsystem consisting of k_2 and m_2 is added.

d. How should this subsystem be designed such that m_1 remains stationary, or nearly stationary, over a range of input frequencies near the natural frequency of the original system. Give values for k_2 and m_3 in terms of k_1, m_1, m_2, and e.

e. What is the fundamental compromise in the design for part d. ?

f. Write the equations of motion for the new system, including the secondary, k_2–m_3, subsystem.

g. Sketch the approximate frequency response for x and x_3 for this new system.

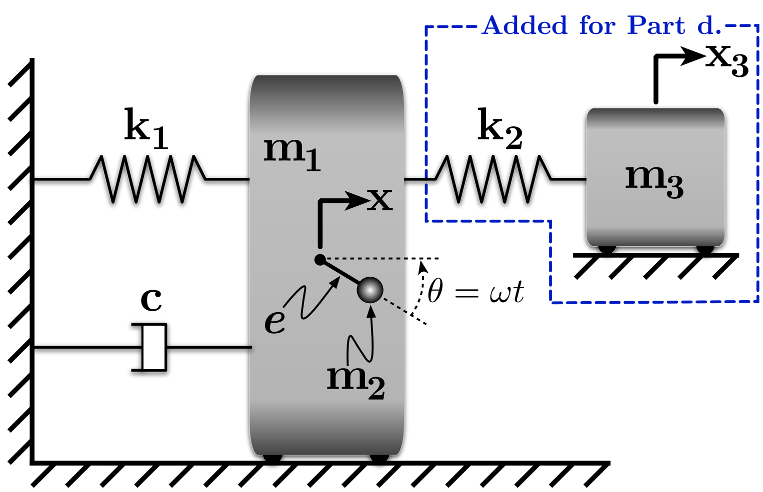

Figure 2: A Rotating Imbalance System

______ **Problem 3 – 25 Points**

The system in Figure 3 is a mass-spring system moving along the ground. Figure 4 is a similar system with friction. The coefficient of friction between the mass and ground in this system is μ.

a. Write the equations of motion for these two systems.

b. For both systems, plot the response, $x(t)$, to initial conditions:

$$x(0) = x_0 \quad \text{and} \quad \dot{x}(0) = -v_0$$

For the system in Figure 3, plot the response for a damping ratio $\zeta = 0.1$. For the system in Figure 4, use a coefficient of friction that would result in approximately the same settling time. Be sure to clearly label the axes and differentiate between the responses. Indicate what is different between the responses for two energy dissipation models.

A coworker suggests attaching the system in Figure 5 to the main system mass. He claims that this proposed sensor can measure acceleration.

c. For the proposed sensor in Figure 5, write the transfer function between the sensor motion, $y(t)$, and the measurement, $x(t)$.

d. Explain how the sensor can be used to measure acceleration. (*Hint*: Using the transfer function from part c., write the relationship between $x(t)$ and $\ddot{y}(t)$.)

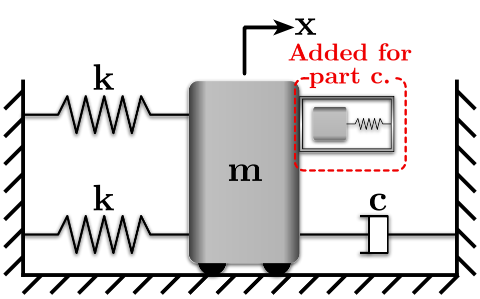

Figure 3: A Mass-Spring-Damper System

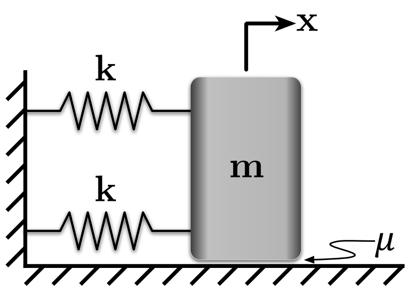

Figure 4: A Mass-Spring System with Friction

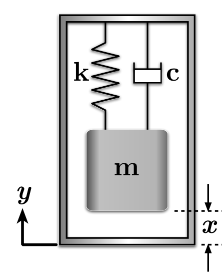

Figure 5: The Proposed Sensor

______ **Problem 4 – 20 Points**

The absentminded professor from Mid-Term 1 is still spilling his coffee on the way from the faculty lounge to his office. The recommendations you gave him based on single-mode model you previously developed failed to capture the complete dynamics. Extend the 1 degree-of-freedom model by modeling the sloshing using the two modes of horizontal motion, x_1 and x_2, of the coffee mass, m_1 and m_2, connected by a springs, k, and dampers, c. The mug wall is considered to be rigid. A figure of this simple model is shown in Figure 6. The professor's unique stroll creates a purely-horizontal harmonic input to the mug, $y(t) = \bar{y}e^{i\omega t}$.

a. Write the equations of motion for this simplified model of the coffee mug system.

b. Ignoring the dampers, set up and outline the solution procedure to determine the natural frequencies and mode shapes of this system. Include all information necessary up to the point of the actual algebra needed solve the problem. Be sure to fully describe the steps necessary.

c. Again ignoring the dampers, if $m_1 = m_2$, what to you expect the mode shapes to be? Why?

d. Still ignoring the dampers, sketch the approximate frequency response. Be sure to indicate:

   i. Magnitude ω approaches 0.

   ii. Magnitude ω approaches infinity.

   iii. Magnitude when ω equals the natural frequencies of the system, ω_1 and ω_2.

e. Now, consider the dampers. Do you expect the mode shapes to change from those found by the steps you outlined in part b. Why or why not?

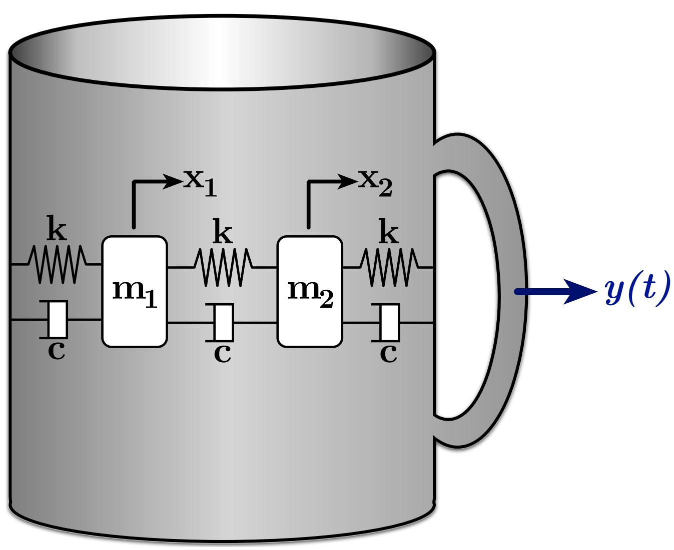

Figure 6: Simplified, Multi-mode Coffee Sloshing Model

______ Problem 5 – 15 Points

The system in Figure 7 represents a pair of links connected by a spring, k. Ignore gravity and consider the linkages of length l to be massless and inflexible. There is a pure torque, T, at the upper joint in the figure. There are point masses, both of mass m, at the ends of the links. You may assume that the spring is at equilibrium with both angles, θ and ϕ, are zero (*Note*: The figure doesn't reflect this equilibrium.).

a. Write the linearized equations of motion for this system.

b. Assuming small angles of θ and ϕ, will there be a rigid-body mode? Why or why not?

c. Set up and outline the solution procedure to determine the response the torque, T. Include all information necessary up to the point of the actual algebra needed solve the problem. Be sure to fully describe the steps necessary.

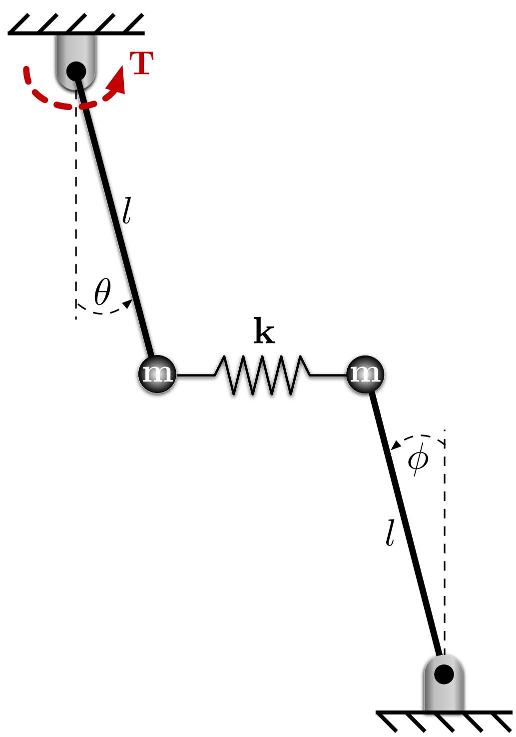

Figure 7: Two-Link-Spring System

Possibly Useful Equations

$$\bar{f} = m\bar{a}$$

$$I_0\bar{\alpha} = \sum \bar{M}_0$$

$$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$$

$$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$$

$$e^{\pm i\omega t} = \cos(\omega t) \pm i\sin(\omega t)$$

$$x(t) = ae^{i\omega_n t} + be^{-i\omega_n t}$$

$$x(t) = a\cos\omega_n t + b\sin\omega_n t$$

$$x(t) = e^{-\zeta\omega_n t}\left[a\cos(\omega_d t) + b\sin(\omega_d t)\right]$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\int u\ dv = uv - \int v\ du$$

$$\delta_{oc}V = \forall \sum$$

$$x(t) = \frac{\omega_n^2\bar{y}}{\omega_n^2 - \omega^2}\sin(\omega t)$$

$$\frac{d}{dt}\left(\frac{\delta L}{\delta\dot{q}_i}\right) + \frac{\delta RD}{\delta\dot{q}_i} - \frac{\delta L}{\delta q_i} = Q_i, \quad i = 1, \ldots, n$$

$$M\ddot{X} + C\dot{X} + KX = F$$

$$\det\left(K - \omega^2 M\right) = 0 \qquad \left[K - \omega^2 M\right]\bar{X} = 0$$

$$2 + 2 = 2 \times 2 = 2^2 = 0\mathtt{b}0100 = 0\mathtt{x}2$$

$$i \equiv \sqrt{-1}$$

$$x(t) = \int_0^t f(\tau)h(t-\tau)d\tau$$

$$x(t) = \int_0^t f(t-\tau)h(\tau)d\tau$$

$$f(t) = \sum_{n=0}^{\infty} a_n\cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n\sin(n\omega_0 t)$$

$$a_n = \frac{\omega_0}{\pi}\int_0^{\frac{2\pi}{\omega_0}} f(t)\cos(n\omega_0 t)dt$$

$$b_n = \frac{\omega_0}{\pi}\int_0^{\frac{2\pi}{\omega_0}} f(t)\sin(n\omega_0 t)dt$$

$$a_0 = \frac{\omega_0}{2\pi}\int_0^{\frac{2\pi}{\omega_0}} f(t)dt$$

$$V(\omega,\zeta) = e^{-\zeta\omega t_n}\sqrt{[C(\omega,\zeta)]^2 + [S(\omega,\zeta)]^2}$$

$$\tilde{X}_i = \frac{1}{\sqrt{X_i^T M X_i}}X_i$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$x(t) = c + e^{-\zeta\omega_n t}\left[a\cos(\omega_d t) + b\sin(\omega_d t)\right]$$

$$\sigma = \frac{1}{N}ln\left(\frac{x(0)}{x(Nt_p)}\right)$$

$$\sigma = ln\left(\frac{x(0)}{x(t_p)}\right)$$

$$\zeta = \frac{\sigma}{\sqrt{4\pi^2 + \sigma^2}}$$

$$\zeta = \frac{\sigma}{2\pi}$$

$$\omega_p = \omega_n\sqrt{1 - 2\zeta^2}$$

$$\zeta \approx \frac{\delta_h}{2\omega_n}$$

$$E = mc^2$$

$$A = \begin{bmatrix} 0 & -K \\ -K & -C \end{bmatrix} \qquad B = \begin{bmatrix} -K & 0 \\ 0 & M \end{bmatrix}$$

$$\left[U^T M U\right]\ddot{H} + \left[U^T K U\right]H = U^T F\cos\omega t$$

Possibly Useful Equations

$$
\begin{aligned}
x(t) &= -\frac{(2\zeta\omega\omega_n)\frac{\bar{f}}{m}}{(\omega_n^2-\omega^2)^2+(2\zeta\omega\omega_n)^2}\cos\omega t + \frac{(\omega_n^2-\omega)\frac{\bar{f}}{m}}{(\omega_n^2-\omega^2)^2+(2\zeta\omega\omega_n)^2}\sin\omega t \\
&= \left[\frac{\bar{f}}{m}\frac{1}{\sqrt{(\omega_n^2-\omega^2)^2+(2\zeta\omega\omega_n)^2}}\right]\cos\left(\omega t-\phi\right), \quad \text{where} \quad \phi = \tan^{-1}\left(\frac{2\zeta\omega\omega_n}{\omega_n^2-\omega^2}\right)
\end{aligned}
$$

$$
\begin{aligned}
g(\omega) &= \sqrt{\frac{\omega_n^4+(2\zeta\omega\omega_n)^2}{(\omega_n^2-\omega^2)^2+(2\zeta\omega\omega_n)^2}}e^{i\phi}, \quad \text{where} \\
\phi &= \phi_1-\phi_2, \quad \text{where } \phi_1 = \tan^{-1}\left(\frac{2\zeta\omega}{\omega_n}\right) \text{ and } \phi_2 = \tan^{-1}\left(\frac{2\zeta\omega\omega_n}{\omega_n^2-\omega^2}\right)
\end{aligned}
$$

$$
\begin{aligned}
g(\Omega) &= \sqrt{\frac{1+(2\zeta\Omega)^2}{(1-\Omega^2)^2+(2\zeta\Omega)^2}}e^{i\phi}, \quad \text{where} \\
\phi &= \tan^{-1}\left(2\zeta\Omega\right)-\tan^{-1}\left(\frac{2\zeta\Omega}{1-\Omega^2}\right)
\end{aligned}
$$

$$
\begin{aligned}
g(\omega) &= \frac{1}{m\sqrt{(\omega_n^2-\omega^2)^2+(2\zeta\omega\omega_n)^2}}e^{-i\phi}, \quad \text{where} \\
\phi &= \tan^{-1}\left(\frac{2\zeta\omega\omega_n}{\omega_n^2-\omega^2}\right)
\end{aligned}
$$

$$
\begin{aligned}
x(t) &= \frac{e\beta\omega^2}{\sqrt{(\omega_n^2-\omega^2)^2+(2\zeta\omega\omega_n)^2}}\cos(\omega t-\phi), \\
&\quad \text{where} \quad \phi = \tan^{-1}\left(\frac{2\zeta\omega\omega_n}{\omega_n^2-\omega^2}\right)
\end{aligned}
$$

$$
\begin{aligned}
x(t) &= \frac{e\beta\Omega^2}{\sqrt{(1-\Omega^2)^2+(2\zeta\Omega)^2}}\cos(\omega t-\phi), \quad \text{where} \\
\phi &= \tan^{-1}\left(\frac{2\zeta\Omega}{1-\Omega^2}\right)
\end{aligned}
$$