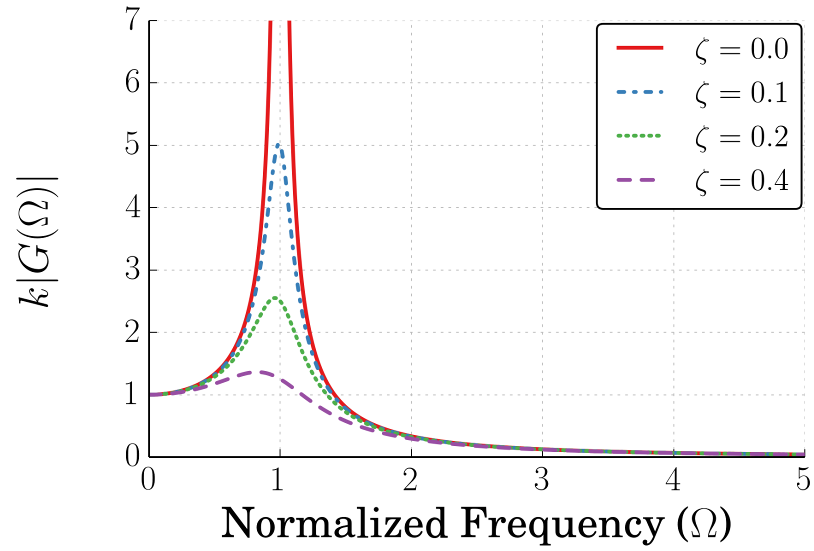
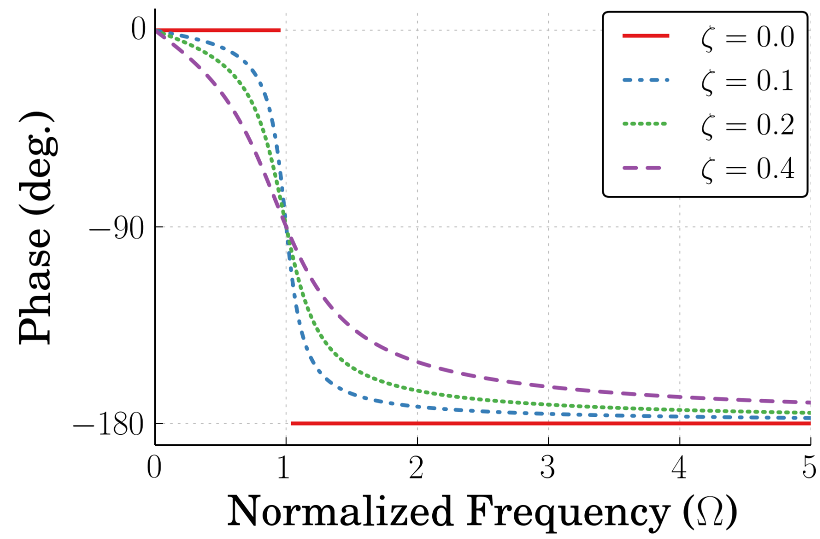
System Identification from Forced Responses

Q: How can we estimate system properties from forced responses?





Q: How can we (experimentally) generate plots like those above?

1. Select the frequency of our harmonic input, measure the amplitude of response.

2. Repeat over a range of frequencies.

Q: How can we estimate frequency?

Directly from frequency location of the peak
(Okay for lightly damped systems, $\zeta \ll 1$)

See the book for derivation

Q: What about damping?

1. Look at the ratio of the peak amplitude to the static reponse (ω =0)

Remember that for a direct-forced response:

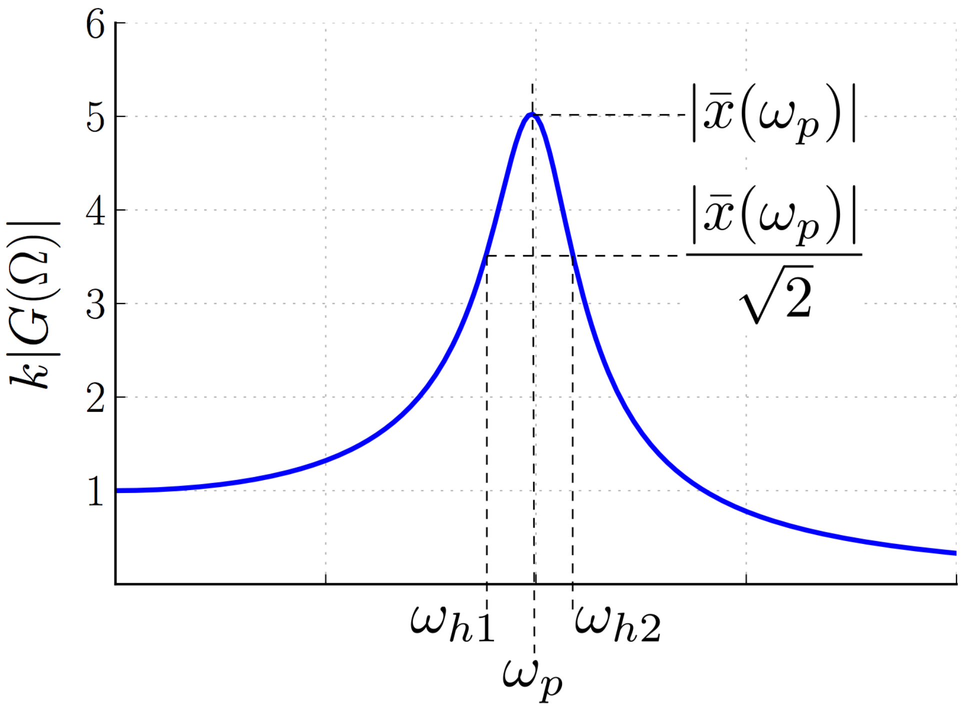
$$|X| = \frac{F}{m}\frac{1}{\sqrt{(\omega_n^2-\omega^2)^2+(2\zeta\omega\omega_n)^2}}$$ so this ratio is:

$$\frac{|G(\omega_p)|}{|G(0)|} = \frac{\frac{F}{m}\frac{1}{\sqrt{(\omega_n^2-\omega_p^2)^2+(2\zeta\omega_p\omega_n)^2}}}{\frac{F}{m\omega_n^2}} \Longrightarrow \frac{1}{2\zeta\sqrt{1-\zeta^2}} \approx \frac{1}{2\zeta} \text{ when } \zeta \ll 1$$

2. Look at half-power points

2. Look at half-power points (cont.)

See the book for derivation



Define $\delta_h = \omega_{h2} - \omega_{h1}$

We can estimate:

$$\zeta = \frac{\delta_h}{2\omega_n}$$

Q: See any problems with this?

- for low damping, peak is narrow. It can be difficult to find $1/\sqrt{2}$ points
- for high damping, not much difference between the peak and half-power amps

System Identification Summary

- There are many methods to estimate natural freq. and damping ratio.
- The ones we looked at so far are for 1DOF systems (or systems with 1 dominant mode)
- We often need to filter the data before these calculations (Real data is noisy.)
- Often preferable to combine methods.

Problem 2.45

2.45. Figure P2.45 shows the steady state response (transients are ignored) of a direct force excited, SDOF system. The mass is equal to 2 kg. What other system parameters can you determine?

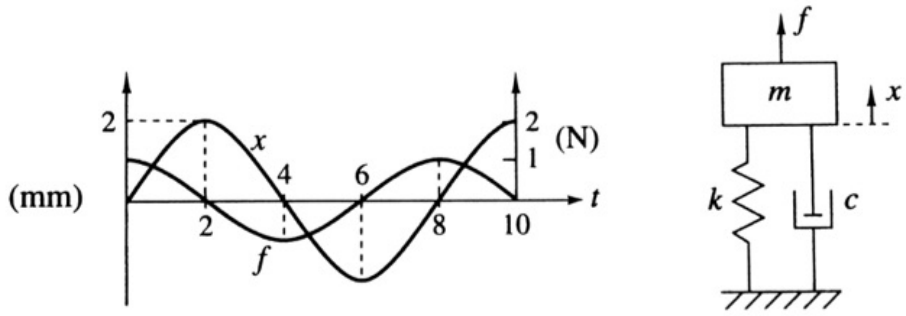


Figure P2.45

We know the transfer function for this system is:

$$\frac{|\bar{X}|}{|\bar{F}|} = \frac{1}{m\sqrt{(\omega_n^2-\omega^2)^2+(2\zeta\omega\omega_n)^2}}$$

We can use this in an examination of the relative amplitudes

and we can write the phase shift as:

$$\phi = \tan^{-1}\left(\frac{2\zeta\omega\omega_n}{\omega_n^2-\omega^2}\right)$$

The timing of the response peaks (or zero crossings) to get phase-shift info

So $$\frac{2}{1} = \underbrace{\frac{1}{m\sqrt{(\omega_n^2-\omega^2)^2+(2\zeta\omega\omega_n)^2}}}_{\text{Defining } m \text{ and } \omega \rightarrow \text{ This equation has 2 unknowns}}$$

We know m=2kg from the problem description

We can determine $\boxed{\omega = \frac{2\pi}{8} = \frac{\pi}{4}\ \frac{\text{rad}}{\text{s}}}$

and

The response lags the input by 2s. This represents 1/4 of a period of the input.

This represents a $\boxed{\text{phase lag of } \frac{1}{4}(2\pi) = \frac{-\pi}{2} \text{ radians} \rightarrow -90 \text{ degrees}}$ ← - sign because it lags the input

So $$\underbrace{\tan^{-1}\left(\frac{2\zeta\omega\omega_n}{\omega_n^2-\omega^2}\right) = \frac{\pi}{2}}_{\text{This equation has 2 unknowns (the same unknowns as above)}}$$

$\boxed{\text{So, we have two equations and two unknowns, and we can solve for } \omega_n \text{ and } \zeta.}$

$\boxed{\text{From } \omega_n, \zeta, \text{ and } m, \text{ we can find } k \text{ and } c.}$

See IPython Notebook for this solution.

Problem 2.68

2.68. The plot in Figure P2.68*a* shows amplitude response of the system illustrated in Figure P2.68*b* a centrifuge as a function of frequency *with* a specimen, m_2, in the machine. The mass of the centrifuge is m_1 and it's restrained by a spring of stiffness k. Given that the radius of the spinning chamber is 0.1 m and that the mass m_1 is 100 kg, find the mass of the specimen as well as the spring constant k.

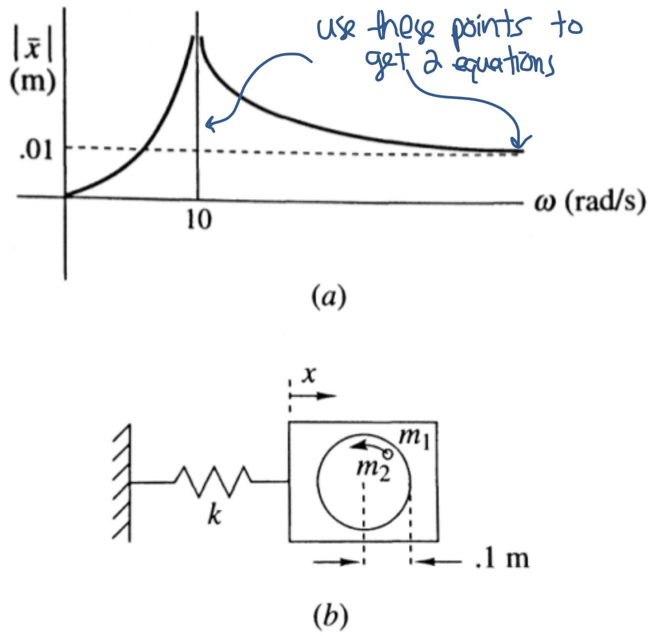


Figure P2.68

We know (Eq. 2.9.6 in the book) that

$$\bar{X} = -\frac{m_2 e\omega^2}{(m_1+m_2)\omega^2 + k + i\omega c}$$

in this case, we're given:

$e = 0.1\text{m}$
$m_1 = 100\text{ kg}$

to get k, use $\omega_n = 10 \frac{\text{rad}}{\text{s}}$ (location of peak response)

$$\omega_n = \sqrt{\frac{k}{m_1+m_2}} = 10 \frac{\text{rad}}{\text{s}}$$

as $\omega \to \infty$, $|\bar{X}| \to e\beta$ where $\beta = \frac{m_2}{m_1+m_2}$

$$e\beta = \frac{em_2}{m_1+m_2} = 0.01\text{m}$$

2 equations with 2 unknowns:

$$\sqrt{\frac{k}{m_1+m_2}} = 10 \rightarrow \frac{k}{m_1+m_2} = 100 \rightarrow k = 100(m_1+m_2) = 100\left(\tfrac{10}{9}m_1\right) \Rightarrow \boxed{k = 11111 \tfrac{N}{m}}$$

$$\frac{em_2}{m_1+m_2} = 0.01 \rightarrow 0.1m_2 = 0.01(m_1+m_2) \rightarrow 0.09m_2 = 0.01m_1$$

$$\boxed{m_2 = \tfrac{1}{9}m_1}$$

Problem 2.71

2.71. Assume that most of the wet clothes in a dryer (Figure P2.71) are uniformly distributed around the drum (mass = 10 kg). In addition to these clothes, a single .8 kg lump of clothing also lies against the drum's surface. How will these mass terms enter the equation of motion (2.9.5) for the system?

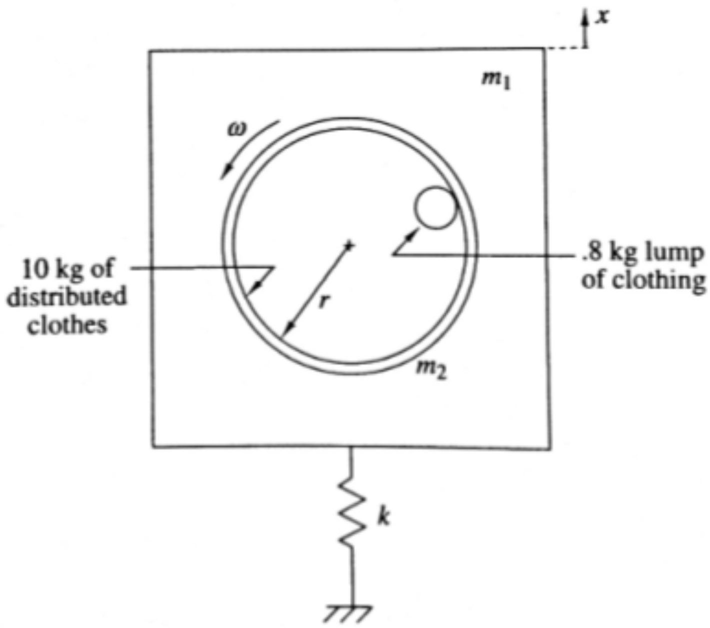


Figure P2.71

Here, only the 0.8kg portion of the clothes mass is the imbalance. The balanced portion of the clothes, 10kg and drum itself, m2, may be grouped with the "body" mass, m1.

So, the equation becomes:

$$\boxed{(m_1 + m_2 + 10.8)\ddot{x} + kx = 0.8 r\omega^2 \cos(\omega t)}$$

Problem 2.74

2.74. A simplified model of a rotating shaft within its bearings is shown in Figure P2.74. If the shaft is imbalanced, it will experience a time-varying force due to the rotating mass. The operating frequency is 60 rad/s, the support stiffness k_1 is 170,000 N/m, and the total rotating mass m is 100 kg. Determine the amplitude of vibration, given that at high frequencies ($\omega \to \infty$) the oscillation amplitude is equal to .001 m. You'll have to figure out l from the given data.

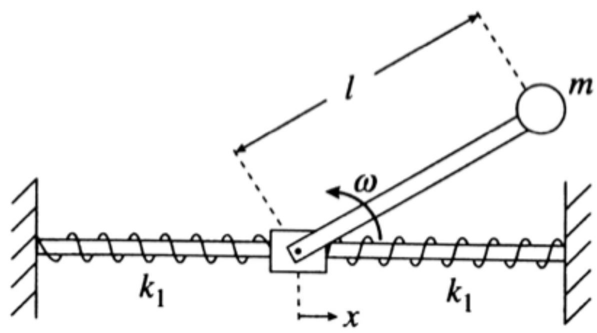
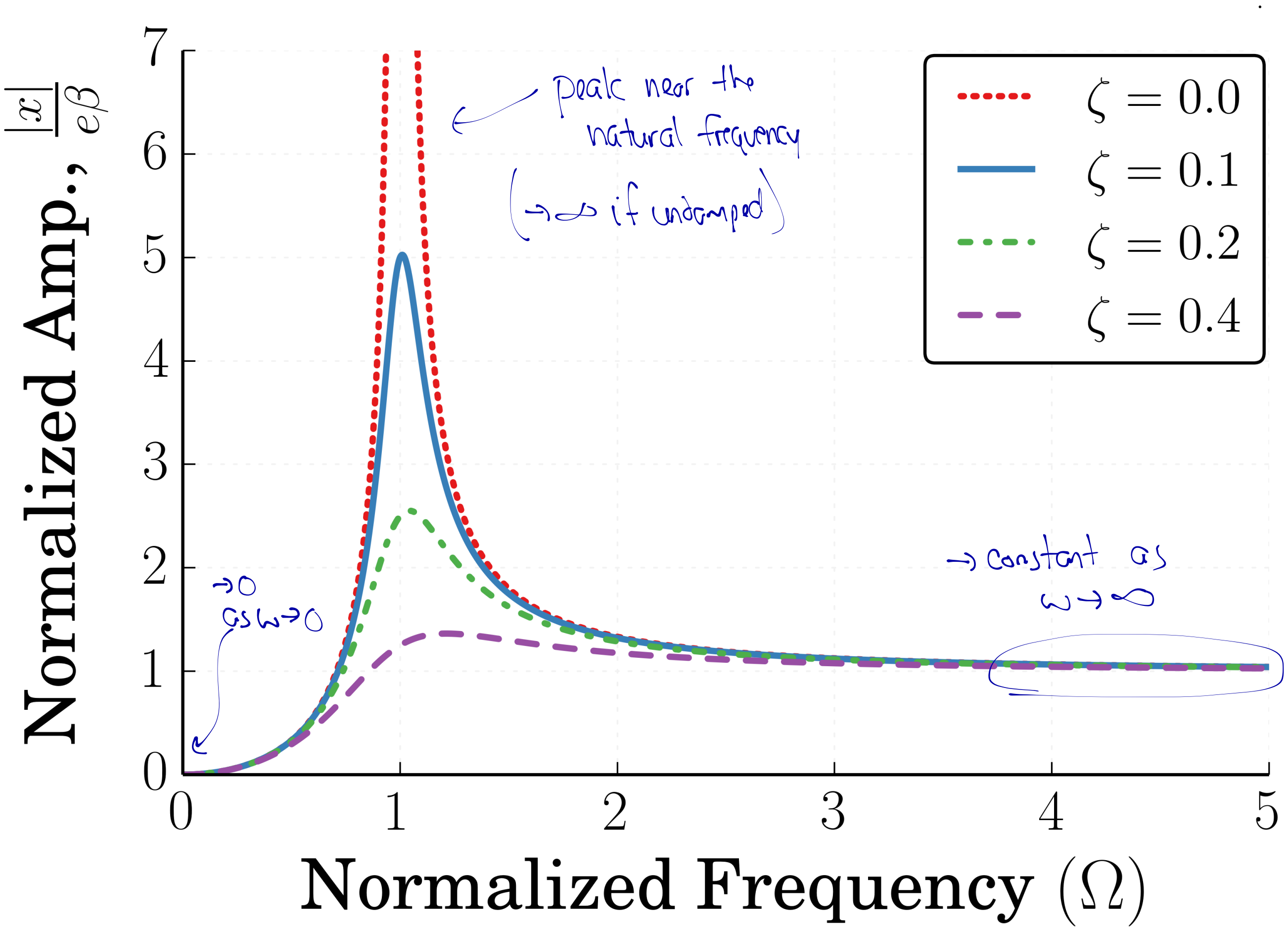


Figure P2.74

We know that the frequency response for a rotating imbalance system looks like:



} This value = $l\beta$ for the non-normalized value.

So

The problem tells us that $|x| = 0.001\text{m}$ as $\omega \to \infty$. $l\beta = 0.001\text{m}$

$\beta = \frac{m_2}{m_1+m_2} = ?$ ← Assume the collar is massless, so that $\beta = 1$ and $l = 0.001\text{m}$

We also know that the natural freq for these systems is $\omega_n = \sqrt{\frac{k}{m_1+m_2}}$, which reduces to $\omega_n = \sqrt{\frac{k}{m_2}} = \sqrt{\frac{170000}{100}}$

$\omega_n = 41.23\ \frac{\text{rad}}{\text{s}}$

The transfer function for a rotating imbalance simplifies to:

$$\frac{l\beta\omega^2}{\omega_n^2 - \omega^2}$$

Substituting $\omega = 60\ \frac{\text{rad}}{\text{s}}$ and ω_n, the magnitude is: $\boxed{|-0.0019|\text{m}}$

Problem 2.78

2.78. Estimate the size of the peak amplitude response for

$$3\ddot{x} + 17.5\dot{x} + 4000x = 15\sin(\omega t)$$

without solving the equation exactly.

Look at our "normal" form of equations of motion

$$m\ddot{x} + c\dot{x} + kx = f(t)$$

or

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \frac{f(t)}{m} \longleftarrow \ddot{x} + \underbrace{\frac{17.5}{3}}_{2\zeta\omega_n}\dot{x} + \underbrace{\frac{4000}{3}}_{\omega_n^2}x = \frac{15}{3}\sin(\omega t)$$

So $\omega_n^2 = \frac{4000}{3} \rightarrow \omega_n = 1333\ \frac{rad}{s}$

$2\zeta\omega_n = \frac{17.5}{3} \rightarrow \zeta = \frac{17.5}{6\omega_n} = 0.0022$ } This suggests we should get a large peak

Also, the peak will occur very close to ω_n

So, use $\omega = \omega_n$ and estimate amp there.

We know that

$$x(t) = \frac{1}{\sqrt{(\omega_n^2-\omega^2)^2 + (2\zeta\omega_n\omega)^2}}\left(\frac{15}{3}\sin\omega t\right)$$

$$= \underbrace{\left[\frac{1}{2\zeta\omega_n^2}\,5\right]}_{\text{This is the magnitude at } \omega=\omega_n}\sin\omega t$$

So: $\boxed{|X_{max}| = \frac{5}{2\zeta\omega_n^2} = 0.0235}$