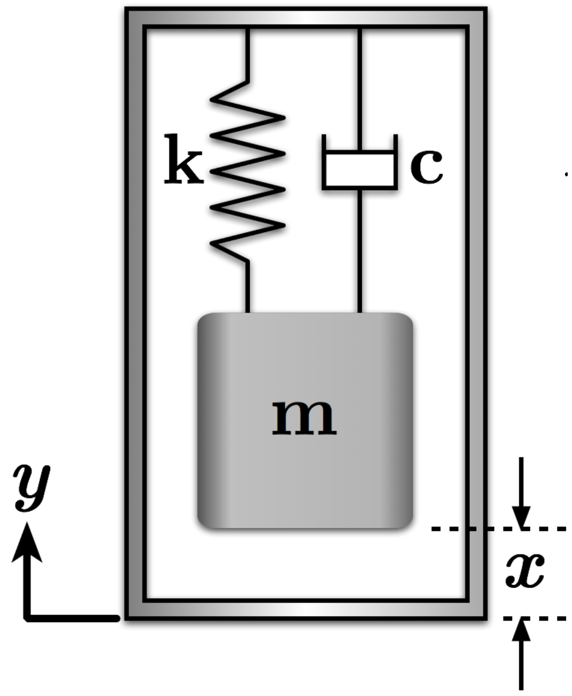
Accelerometers and Seismomters (Sec 2.12)

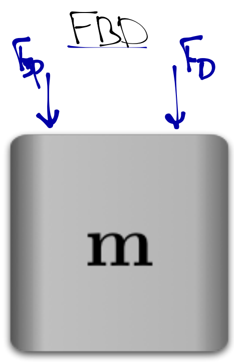


$y \equiv$ displacement of the device (what we're trying to measure)

$x =$ relative displacement of the mass

For accelerometers, x is usually measured with piezoelectrics (charge $\propto$ deformation)

We can measure x, but want information about y → We need eq. of motion



Q: What are F_{sp} and F_D?

Remember that x is the relative displacement → $F_{sp} = kx$ and $F_D = c\dot{x}$

Q: What is the total accel of m? (Remember Newton-Euler needs total accel)

$\ddot{y} + \ddot{x}$ → accel of housing plus accel of m relative to the housing

So, $m\bar{a} = \sum F \to m(\ddot{x}+\ddot{y}) = -kx - c\dot{x} \longrightarrow m\ddot{x} + c\dot{x} + kx = m\ddot{y}$

We can divide by m to write: $\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \ddot{y}$

Assume:

$y(t) = \bar{y}e^{i\omega t}$ and $x(t) = \bar{x}e^{i\omega t}$ (take derivs and sub into eq. of motion)

$$(\omega_n^2 - \omega^2 + 2i\zeta\omega\omega_n)\bar{x} = \omega^2\bar{y} \longrightarrow \bar{x} = \frac{\omega^2\bar{y}}{(\omega_n^2-\omega^2+2i\zeta\omega\omega_n)}$$

So, $x(t) = \dfrac{\omega^2}{\omega_n^2-\omega^2+2i\zeta\omega\omega_n}\bar{y}e^{i\omega t}$

This is $-\ddot{y}$ $\longrightarrow$ $x(t) = \dfrac{-\ddot{y}(t)}{(\omega_n^2-\omega^2+2i\zeta\omega\omega_n)}$

Accelerometers and Seismomters (cont.)

Accelerometer

Look at the case when $\omega \ll \omega_n$

(excitation freq. is much lower than the device natural freq.)

$$x(t) \simeq \frac{-1}{\omega_n^2}\ddot{y}(t) \longrightarrow \ddot{y}(t) = -\omega_n^2 x(t)$$

To measure accel. with this device, measure $x(t)$ then mult by $-\omega_n^2$

Seismometer

Look at the case when $\omega \gg \omega_n$

(excitation freq. is much higher than the device natural freq.

$$x(t) = \frac{\omega^2}{(\omega_n^2 - \omega^2 + 2\zeta i\omega\omega_n)}\bar{Y}e^{i\omega t} \longrightarrow x(t) = -\bar{Y}e^{i\omega t}$$

To measure $y(t)$, measure $x(t)$

Q: What do these cases tell us about how we should design these devices?

For accel.

- need to ensure $\omega \ll \omega_n$ $\leftarrow$ want high ω_n

$\therefore$ want high k and/or low m $\longrightarrow$ small device

For seis

- need to ensure $\omega \gg \omega_n$ $\leftarrow$ want low ω_n

$\therefore$ low k and/or high m $\longrightarrow$ big device

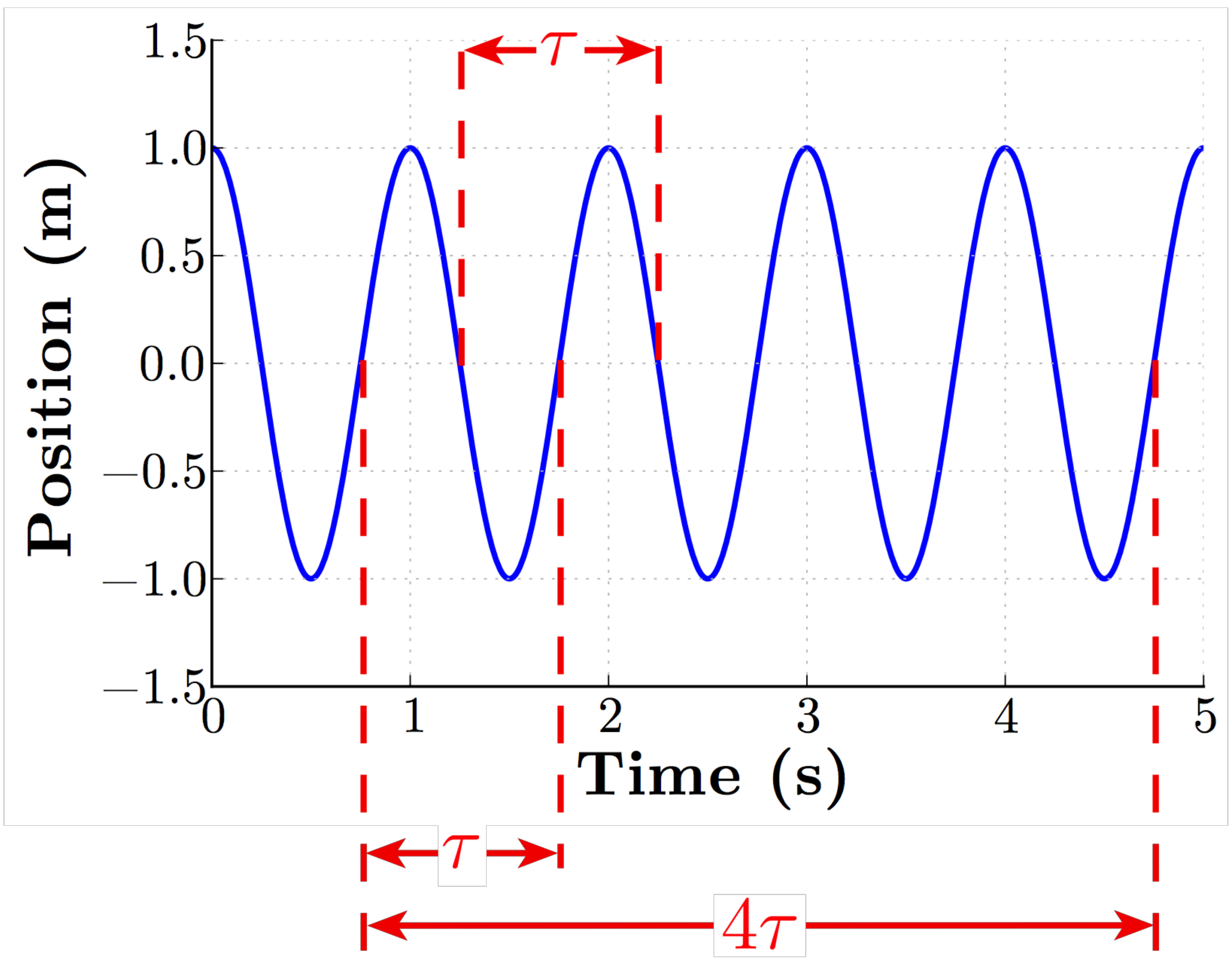
System Identification (Sec 2.10)

Given some vibrating system, how can we accurately estimate natural frequency and damping ratio?

Let's start with systems with 1 dominant mode (1 DOF is important, like all the systems we've studied so far).

Q: For undamped systems, how could we estimate natural frequency?

- Time between "zero" crossings
- It's better to count N periods and divide
- We can also get an idea about the linearity of a system:
  Is the time between zeros always (approx) equal?
  Yes -> System is linear
  No --> System may be nonlinear



Some key points:

- We're really counting about equil. If the system has a nonzero equil., it's often best to remove this offset.
- Be sure to count entire periods
- You can do this by looking at the slope (i.e. count from one positive-slope zero-intersection to the next positive-slope intersection
- It's usually best to get multiple estimates, the compare and average (or otherwise decide which is the best to use)

System Identification (cont.)

Q: What about damping?

Changes the freq of oscillation to $\omega_d = \omega_n\sqrt{1-\zeta^2}$ $\leftarrow$ if $\zeta \ll 1$ $\omega_d \approx \omega_n$

But, remember that strictly speaking you will be calculating the damped natural frequency, not the "pure" natural frequency

Q: How could we estimate the damping itself?

Remember that the response is $x(t) = \underbrace{e^{-\zeta\omega_n t}}_{\text{We can get an estimate of damping from this term}}(a\cos\omega_d t + b\sin\omega_d t)$

We can get an estimate of damping from this term

It sets the decay rate of $x(t)$, so look at that envelope to estimate ζ

Log Decrement (To estimate damping ratio from free responses)

$x(t) = e^{-\zeta\omega_n t}(a\cos\omega_d t + b\sin\omega_d t)$ define $x(0) = x_0$ and $\dot{x}(0) = 0 \longrightarrow a = x_0$ and $b = 0$

$x(t) = e^{-\zeta\omega_n t}(x_0\cos\omega_d t)$ $\leftarrow$ Now, let's look at the response N-periods later

Each period $\tau = \frac{2\pi}{\omega_d}$ so N periods $= N\tau = \frac{2N\pi}{\omega_d}$

$$x\left(\frac{2N\pi}{\omega_d}\right) = \exp\left[-\zeta\omega_n\frac{2N\pi}{\omega_d}\right]\left[x_0\underbrace{\cos\left(\omega_d\frac{2N\pi}{\omega_d}\right)}_{=1\ \forall N}\right] = x_0\exp\left[\frac{-2N\pi\zeta}{\sqrt{1-\zeta^2}}\right]$$

Now, let's look at the ratio between the 1st peak $(x(0) = x_0)$ and the N^{th} peak:

$$\frac{x(0)}{x(N\tau)} = \frac{x(0)}{x\left(\frac{2N\pi}{\omega_d}\right)} = \frac{\cancel{x_0}}{\cancel{x_0}\exp\left[\frac{-2N\pi\zeta}{\sqrt{1-\zeta^2}}\right]} = \exp\left[\frac{2N\pi\zeta}{\sqrt{1-\zeta^2}}\right]$$

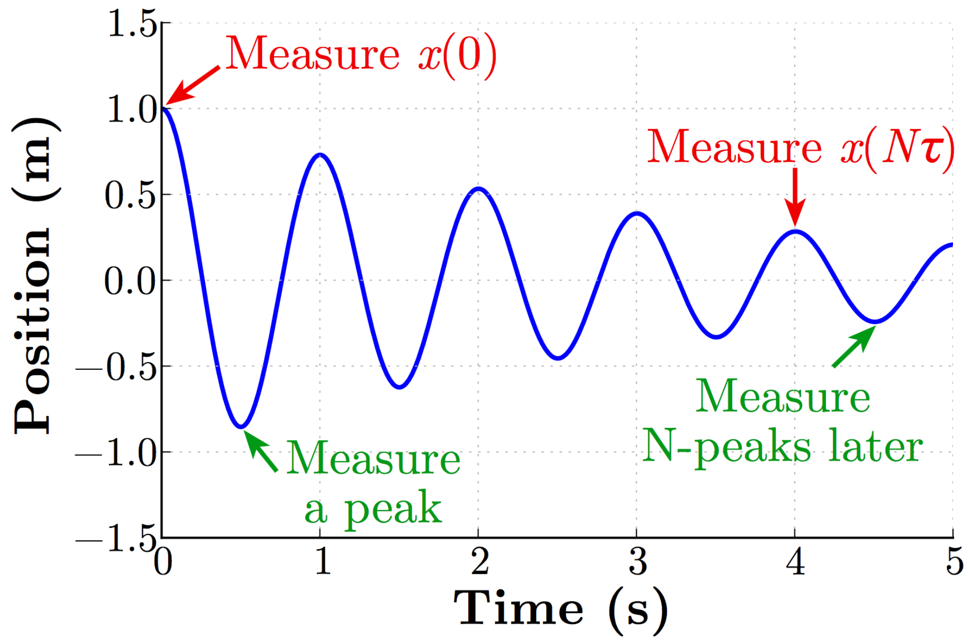
We can measure x_0 and $x\left(\frac{2N\pi}{\omega_d}\right)$ from our experimental response. To get ζ, we just need to solve this equation for it.

$$\ln\left(\frac{x(0)}{x(N\tau)}\right) = \frac{2N\pi\zeta}{\sqrt{1-\zeta^2}}$$

define $\sigma = \frac{1}{N}\ln\left(\frac{x(0)}{x(N\tau)}\right)$ then $\zeta = \frac{\sigma}{\sqrt{4\pi^2+\sigma^2}}$ $\left[\text{for } \zeta \ll 1,\ \zeta \approx \frac{\sigma}{2\pi}\right]$

Log Dec procedure summary

1. Measure x(0) and $x(N\tau)$ from the response.



2. Plug these into $\sigma = \frac{1}{N} \ln\left[\frac{x(0)}{x(N\tau)}\right]$ to get σ.

3. Use $\zeta = \frac{\sigma}{\sqrt{4\pi^2+\sigma^2}}$ to estimate ζ. ← If $\zeta \ll 1$, we can use $\zeta \approx \frac{\sigma}{2\pi}$.

We can also use this to help determine linearity:

- Estimate damping for various ranges (different "Ns")
- If linear, the damping should be (approx.) equal for all