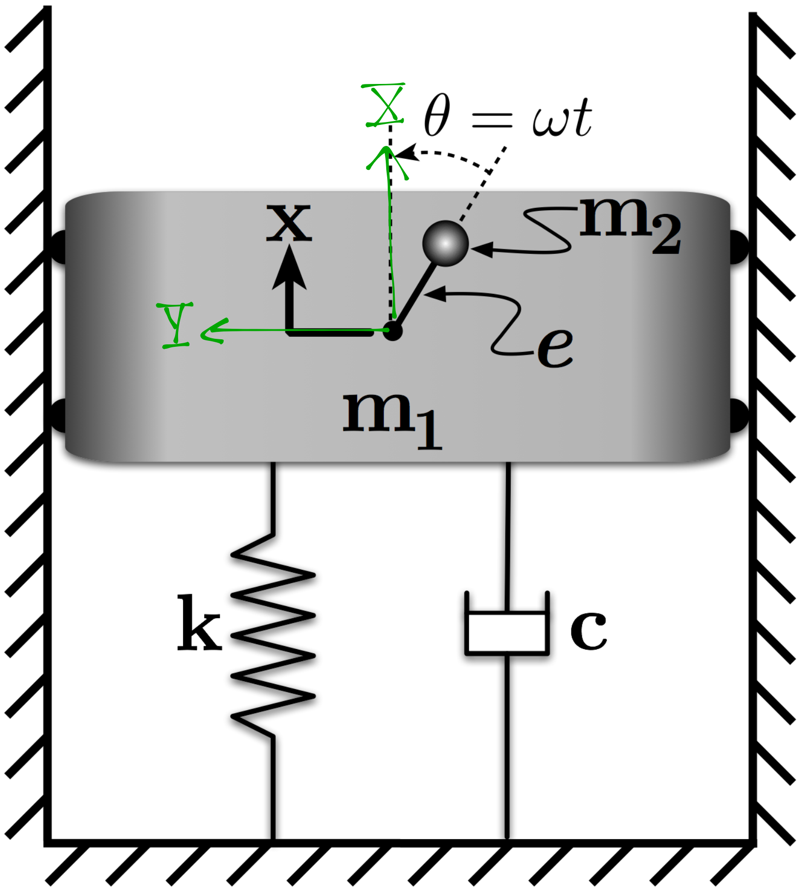
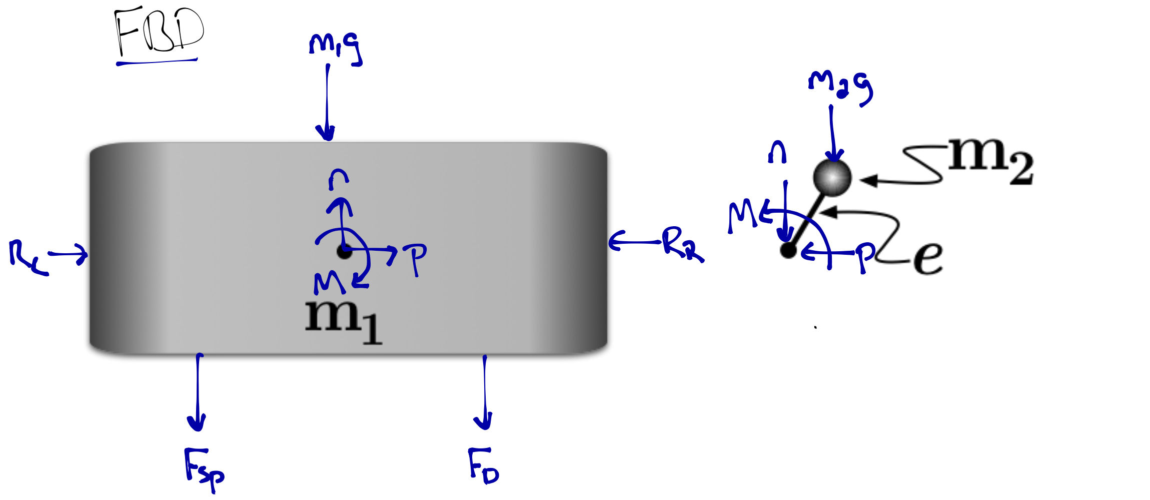
Rotating Imbalance (Sec. 2.9)



* Mass m1 is constrained to move vertically
* Mass m2 is rotating at a constant rate
* Mass m2 is offset by e

FBD



Assume we're operating around equil → we can ignore gravity

For m_1

$$m_1 \ddot{x} = -F_{sp} - F_D + n$$

$$m_1 \ddot{x} = -kx - c\dot{x} + n$$

For m_2 (let's only look at x-direction only)

$$m_2 a_x = -n \quad \leftarrow \quad a_x = \text{accel of } m_2 \text{ in the x-direction} \leftarrow$$

We need to find n in order to get the response of m_1

$$\bar{r}_{m_2/o} = \left[(x + e\cos\omega t)\bar{I} + (e\sin\omega t)\bar{J}\right]$$

$$\bar{V}_{m_2} = \left[(\dot{x} - e\omega\sin\omega t)\bar{I} + (e\omega\cos\omega t)\bar{J}\right]$$

$$\bar{a}_{m_2} = \left[\underbrace{(\ddot{x} - e\omega^2\cos\omega t)}_{a_x}\bar{I} + (-e\omega^2\sin\omega t)\bar{J}\right]$$

$$m_1\ddot{x} + c\dot{x} + kx = -m_2(\ddot{x} - e\omega^2\cos\omega t) \xleftarrow{\text{plug into } m_1 \text{ eq.}} m_2(\ddot{x} - e\omega^2\cos\omega t) = -n$$

$$\boxed{(m_1 + m_2)\ddot{x} + c\dot{x} + kx = m_2 e\omega^2\cos\omega t}$$

Rotating Imbalance (cont.)

Let's write the cos "input" in complex form ← Remember it's cosine, so we'll take the real part later

$$(m_1+m_2)\ddot{x}+c\dot{x}+kx=m_2 e\omega^2 e^{i\omega t}$$

Now, assume a solution to match the form of the input – $x(t)=\bar{X}e^{i\omega t}$ (plug into the eq. of motion and solve for $\bar{X}$)

$$\bar{X}=\frac{m_2 e\omega^2}{-(m_1+m_2)\omega^2+k+i\omega c}$$

Q: What happens as $\omega \to 0$? — response goes to 0

Q: What happens as $\omega \to \infty$ — $\bar{X} \to \frac{-m_2 e}{m_1+m_2}$

Notice that both of these trends are different than what we've seen so far.

Let's look at the parameterized version of this response (in terms of ω_n^2 and ζ)

divide by (m_1+m_2)

$$\ddot{x}+2\zeta\omega_n\dot{x}+\omega_n^2 x=e\beta\omega^2\cos\omega t \quad \text{where } \beta=\frac{m_2}{m_1+m_2},\ \omega_n^2=\frac{k}{m_1+m_2},\ \text{and } 2\zeta\omega_n=\frac{c}{m_1+m_2}$$

Again writing in complex form, assuming $x(t)=\bar{X}e^{i\omega t}$, and solving for $\bar{X}$, we find:

$$\bar{X}=\frac{e\beta\omega^2}{(\omega_n^2-\omega^2)+2i\zeta\omega\omega_n}$$

So,

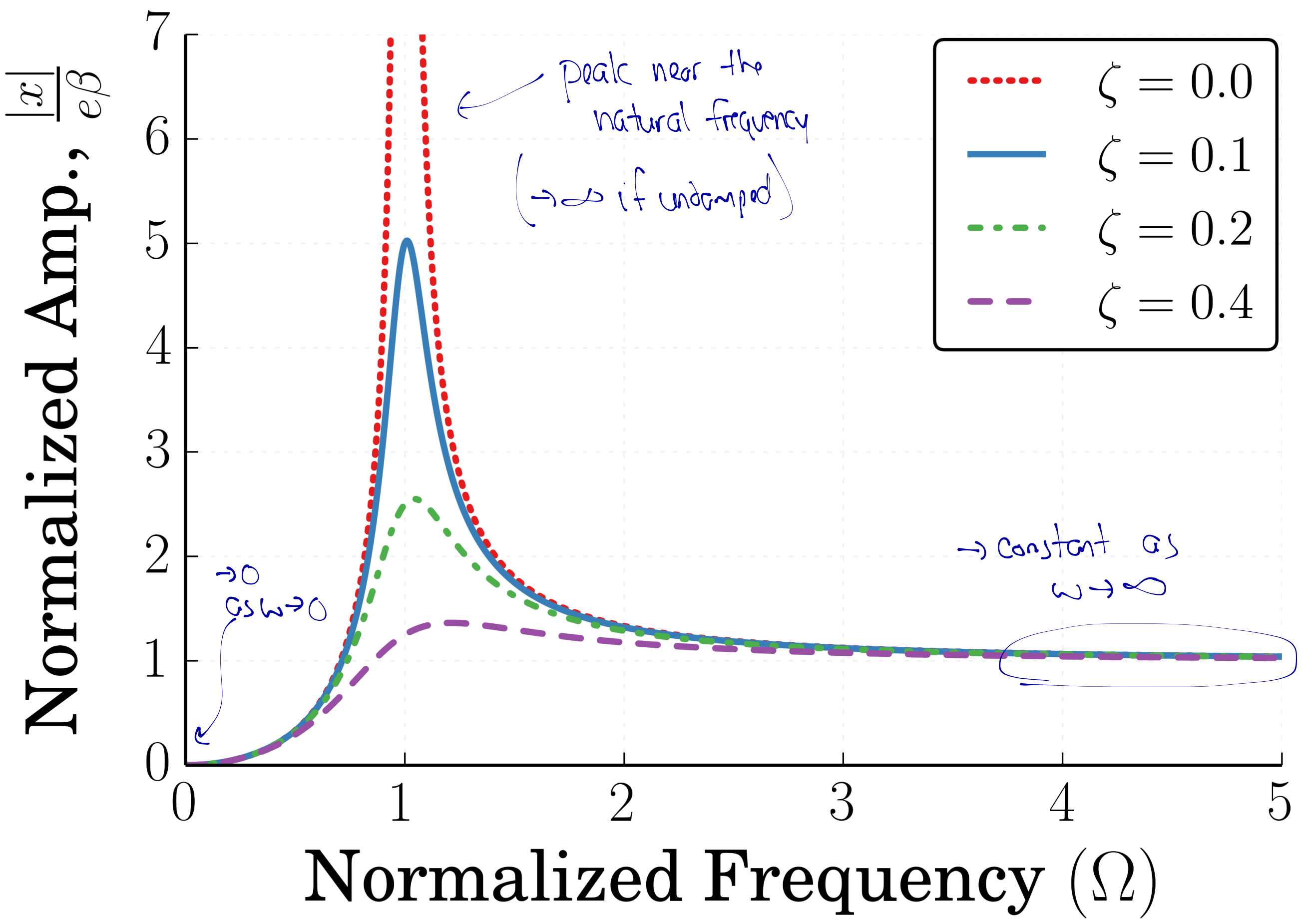
$$x(t)=\frac{e\beta\omega^2}{(\omega_n^2-\omega^2)+2i\zeta\omega\omega_n}e^{i\omega t}$$

we want the real part of this complex response

$$\boxed{x(t)=\frac{e\beta\omega^2}{\sqrt{(\omega_n^2-\omega^2)^2+(2\zeta\omega\omega_n)^2}}\cos(\omega t-\phi) \quad \text{where} \quad \phi=\tan^{-1}\left(\frac{2\zeta\omega\omega_n}{\omega_n^2-\omega^2}\right)}$$

Note: We could also normalize by freq – $\Omega=\frac{\omega}{\omega_n}$ to find $\bar{X}=\frac{e\beta\Omega^2}{(1-\Omega^2)^2+2i\zeta\Omega}$

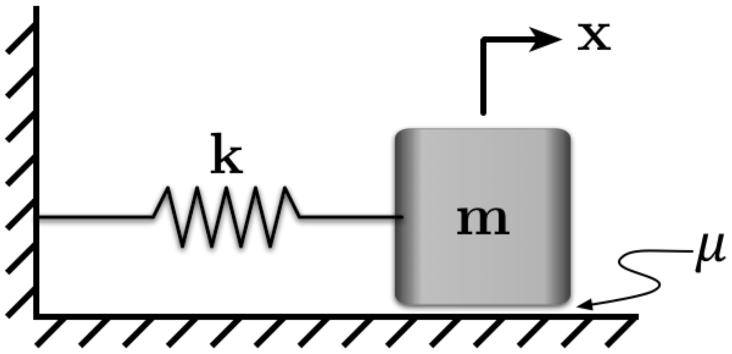
Rotating Imbalance (cont.)

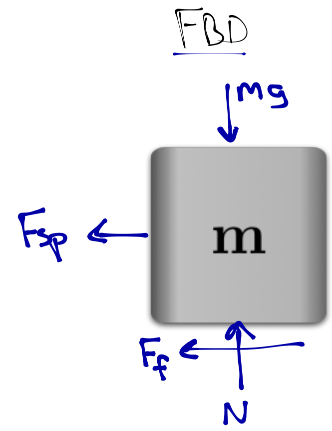


Note: We're skipping Section 2.10 for now. We'll come back to it.

Other Types of Damping (Sec. 2.11)

Dry Friction (Coulomb Damping)





Q: What is F_f (the force from friction)?

$$F_f = \mu N \, \text{sgn}(\dot{x})$$

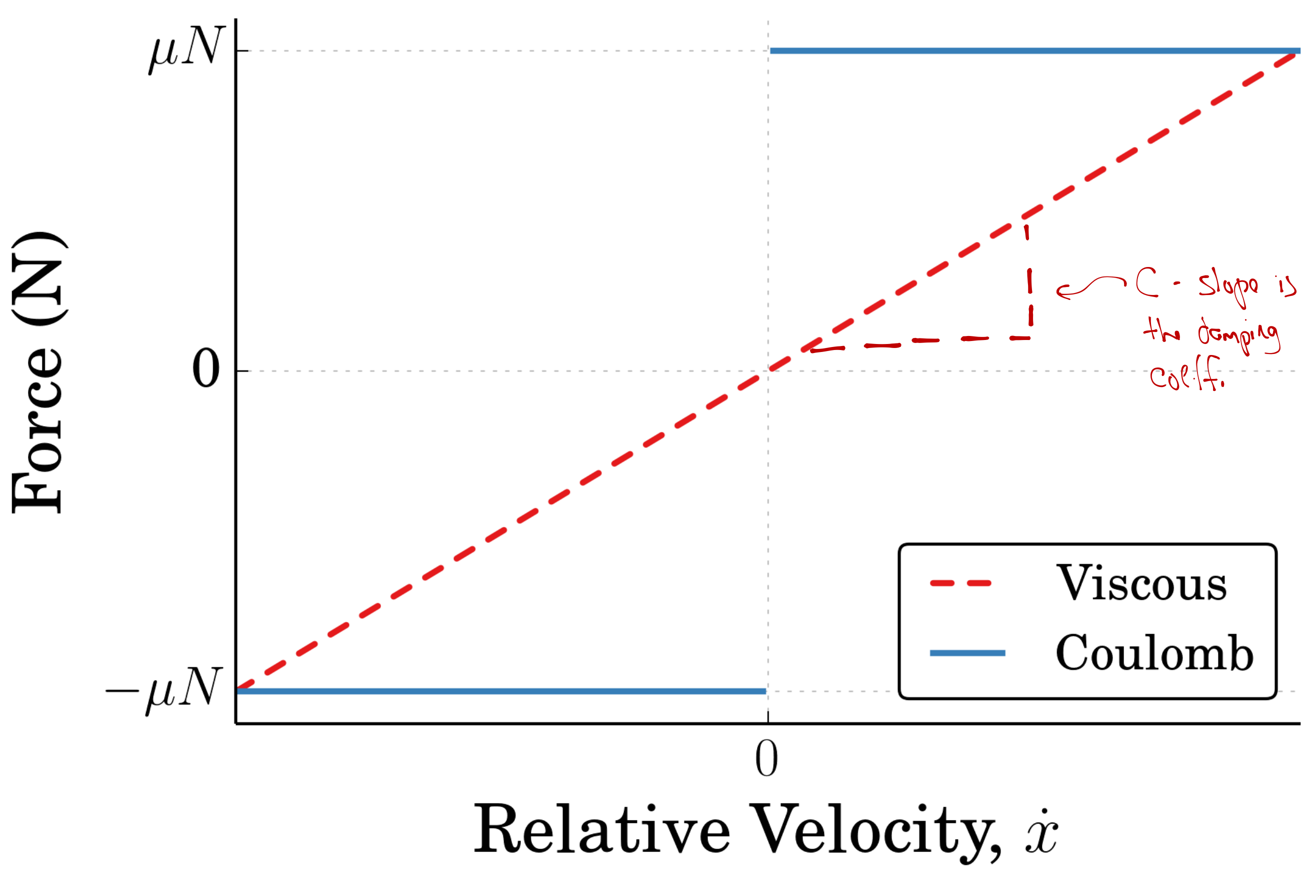
μ: coeff of friction

N: Normal Force

$\text{sgn}(\dot{x})$: sign of relative velocity

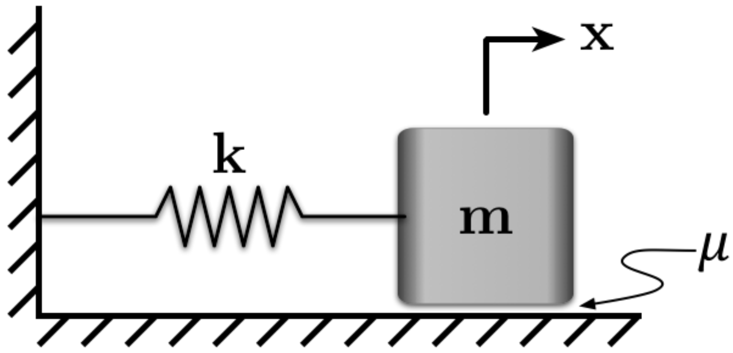
Note: The book writes this as $N = mg$. This is not strictly/always true.

Q: What does this force "look" like?



Dry Friction/Coulomb Damping (cont.)

Q: How does this difference affect the response? (Is it still linear?)



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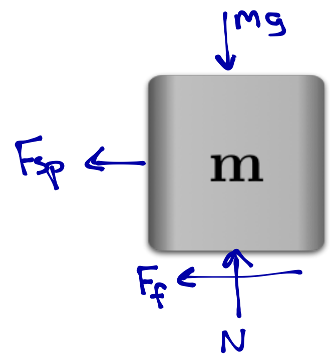


We saw that $F_f = \mu N\,\mathrm{sgn}(\dot{x})$, so we need to find N

(for nontrivial cases, you'll likely need to solve $\sum \vec{F} = m\vec{a}$ in all directions to know)

Here: $\sum \vec{F}\cdot\vec{J} = m\ddot{y} \longrightarrow \quad m\ddot{y} = -mg + N \leftarrow$ we know $\ddot{y}=0$ so $N = mg$

So,

$$\sum \vec{F}\cdot\vec{I} = m\ddot{x} \rightarrow \quad m\ddot{x} = -F_{sp} - F_F = -kx - \mu N\,\mathrm{sgn}(\dot{x})$$

$$m\ddot{x} + \underbrace{\mu N\,\mathrm{sgn}(\dot{x})} + kx = 0$$

remember that the magnitude of this is constant

Rewrite as piecewise:

$m\ddot{x} + kx = -\mu N, \quad \dot{x} > 0$

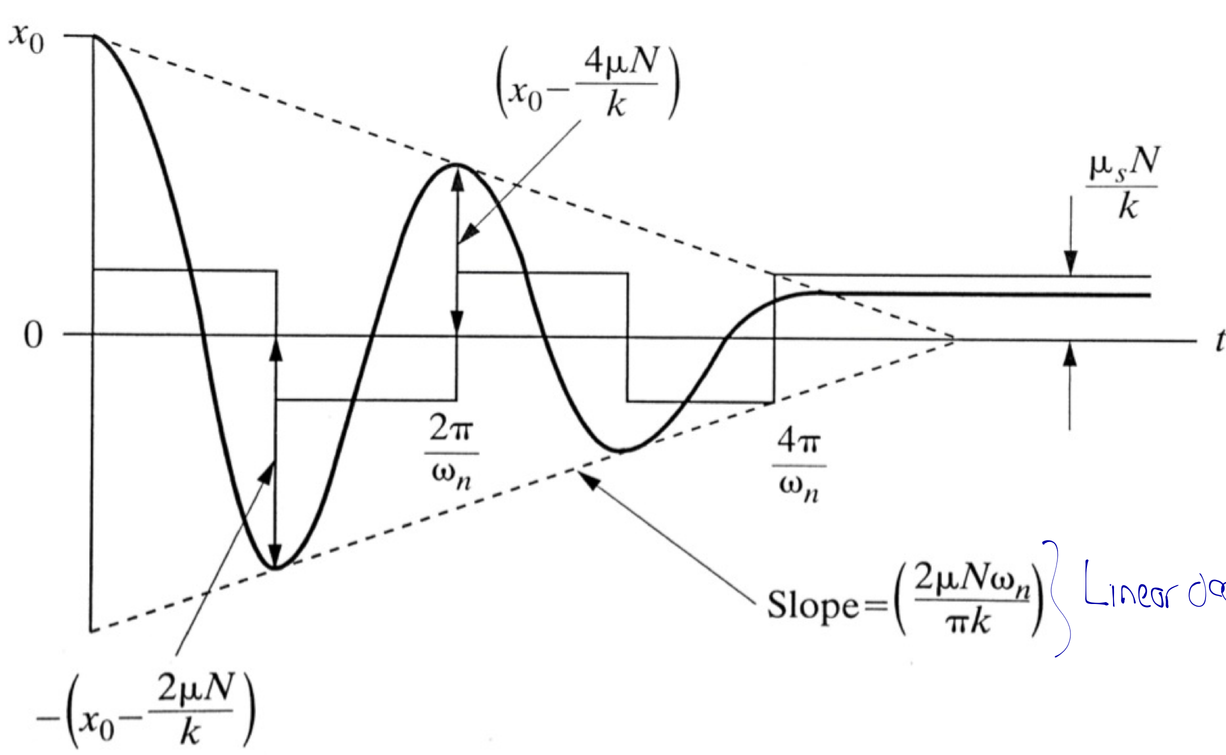
$m\ddot{x} + kx = \mu N, \quad \dot{x} < 0$

This is piecewise linear. To solve, solve the first segment, then use its final values as the initial cond. for the next segment....

Q: So what does the response look like?

Hint: How does the relationship between the spring and friction forces change?

(F_{sp} is linear with x. F_f is not. $|F_{sp}|$ decreases with decay. $|F_f|$ does not.)



Not guaranteed to settle to zero.

Linear decay envelope

Dry Friction/Coulomb Damping (cont.)

Key points:
* Decay envelope is linear
* Friction doesn't affect natural freq.
* Stops after finite time
* Time and position of stop is dependent on initial conditions, system parameters, etc.

Q: When does it stop?

$F_{sp} \le F_f$ (spring force cannot overcome friction)

Structural Damping

* Approximates energy dissipation by loading/unloading of materials
* p. 121-125 of the book
* We won't cover it (or be tested on it), but do look at it.