Rotating Imbalance (Sec. 2.9)

- Mass $m_1$ is constrained to move vertically
- Mass $m_2$ is rotating at a constant rate
- Mass $m_2$ is offset by $e$

Assume $m_1$ is moving around $m_2$, so we can ignore gravity.

For $m_1$:

\[ m_1 \ddot{x} = -F_p - F_D + n \]
\[ m_1 \ddot{x} = -kx - cx + n \]

For $m_2$, (let's only look at $x$-direction only):

\[ m_2 \ddot{x} = -n \quad \Rightarrow \quad \ddot{x} = \text{accel of } m_2 \text{ in the } x\text{-direction} \]

\[ \bar{m}_2 \ddot{y} = \left[ (x + e \cos \omega t) \bar{I} + (e \sin \omega t) \bar{J} \right] \]

\[ \bar{F}_{mg} = \left[ (x - e \omega^2 \cos \omega t) \bar{I} + (e \omega \sin \omega t) \bar{J} \right] \]

\[ \bar{F}_m = \left[ \bar{m}_2 \ddot{y} + \bar{F}_{mg} \right] \]

\[ \bar{a}_m = \left[ \bar{m}_2 \ddot{y} + \bar{F}_{mg} \right] \]

\[ \bar{a}_x \]

Plug into $m_2$:

\[ m_2 (x - e \omega^2 \cos \omega t) = -n \]

\[ \frac{m_1 + m_2}{m_1 + m_2} \ddot{x} + cx + kx = m_2 e \omega^2 \cos \omega t \]
Let's write the cos 'input' in complex form. Remember it's cosine, so we'll take the real part later.

\[(m_1 + m_2)\dot{x} + c \ddot{x} + kx = m_2 e^{\omega t}\]

Now, assume a solution to match the form of the input — \(x(t) = e^{\omega t}\). (Plug into the eq of motion and solve for \(x\))

\[x = \frac{m_2 e^{\omega t}}{-(m_1 + m_2)\omega^2 + k}\]

1. What happens as \(\omega \to 0\)? — response goes to 0.
2. What happens as \(\omega \to \infty\)? \(x \to \frac{m_2 e^{\omega t}}{m_1 m_2}\)

Notice that both of these trends are different than what we've seen so far.

Let's look at the parametrical version of this response (in terms of \(m_1\) and \(k\)).

Divide by \((m_1 m_2)\)

\[x + 2\delta\omega x + \omega^2 x = e^{\beta t} \cos \phi \text{ constant} \]

where \(\beta = \frac{m_2}{m_1 m_2} \quad \text{and} \quad \omega_0 = \sqrt{\frac{k}{m_1 m_2}} \quad \text{and} \quad \delta = \frac{c}{m_1 m_2}\)

Again writing in complex form, assuming \(x(t) = \bar{x} e^{\omega t}\), and solving for \(\bar{x}\), we find

\[\bar{x} = \frac{e^{\beta t}}{(\omega_0^2 t^2) + 2i\delta \omega_0}\]

So,

\[x(t) = \frac{e^{\beta t}}{(\omega_0^2 t^2) + 2i\delta \omega_0} e^{\omega t}\]

We used the real part of this complex version.

\[x(t) = \frac{e^{\beta t}}{\sqrt{(\omega_0^2 t^2) + 2i\delta \omega_0}} \cos(\omega t - \phi) \quad \text{where} \quad \phi = \tan^{-1} \left( \frac{2i\delta \omega_0}{\omega_0^2 t^2} \right)\]

Note we could also rewrite by freq. - selection to find \(x = \bar{x} e^{\beta t}\).
Rotating Imbalance (cont.)

- Peak near the natural frequency
- $\zeta = 0.0$
- $\zeta = 0.1$
- $\zeta = 0.2$
- $\zeta = 0.4$

Normalized Amp., $|x|/e_\beta$

Normalized Frequency ($\Omega$)
Other Types of Damping (Sec. 2.11)

Dry Friction (Coulomb Damping)

\[ F_f = \mu N \text{sign}(\dot{x}) \]

Q: What is \( F_f \) (the force from friction)?

Q: What does this force "look like?"
Q: How does this affect the system? (Is it still linear?)

We saw that $F_x = \mu N \text{sign}(x)$, so we need to find $N$.

For normal case, you likely need to solve $\mathcal{E} = \mathcal{W}$ in all continuity to know.

Here: $\mathcal{E} = \dot{\mathcal{W}} \implies m \dot{x} = -mg + N \implies m \dot{x} = -mg$ since $N = mg$.

So,

$$\mathcal{E} = \dot{\mathcal{W}} = m \dot{x} = -mg - F_x = -mg - kx - \mu N \text{sign}(x)$$

Remember that the magnitude of the friction is constant.

$$m \dot{x} + kx + \mu N \text{sign}(x) = 0$$

Rearrange as pressure

$$m \dot{x} + kx = -\mu N, \quad x > 0$$

$$m \dot{x} + kx = \mu N, \quad x < 0$$

This is pressure linear. To solve, solve the first segment, then use its final value as the initial condition for the next segment...

Q: So what does the response look like?

Hint: How does the relationship between the spring and friction force change?

($F_x$ is linear with $x$, $F_f$ is not. $|F_x|$ increases with decay, $|F_f|$ can not.)

Not guaranteed to settle to zero.

Linear decay envelope.

Slope $= \left(\frac{2\mu N \omega_n}{\pi k}\right)$
Dry Friction/Coulomb Damping (cont.)

Key points:
* Decay envelope is linear
* Friction doesn't affect natural freq.
* Stops after finite time
* Time and position of stop is dependent on initial conditions, system parameters, etc.

Q: When does it stop?

\[ F_{sp} \leq F_f \quad \text{(spring force cannot overcome friction)} \]

Structural Damping
* Approximates energy dissipation by loading/unloading of materials
* p. 121-125 of the book
* We won't cover it (or be tested on it), but do look at it.