Dynamics Review

Newton's Laws

1) $\sum \bar{F} = 0$ then $\bar{v} = \text{constant}$ ← Objects in motion, ...

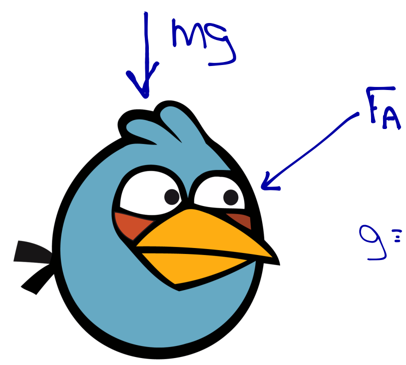
2) $\sum \bar{F} = m\bar{a}$

3) $\bar{F}_{21} = \bar{F}_{12}$ ← Equal and opposite

Just using these...

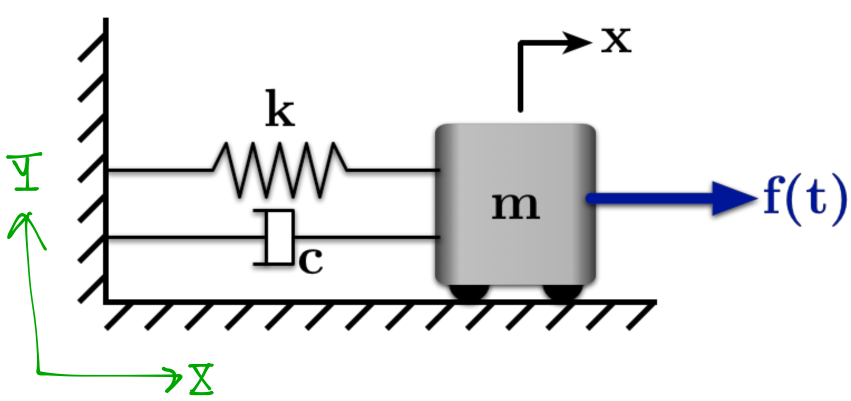
Q: What are the forces on the Angry Bird?

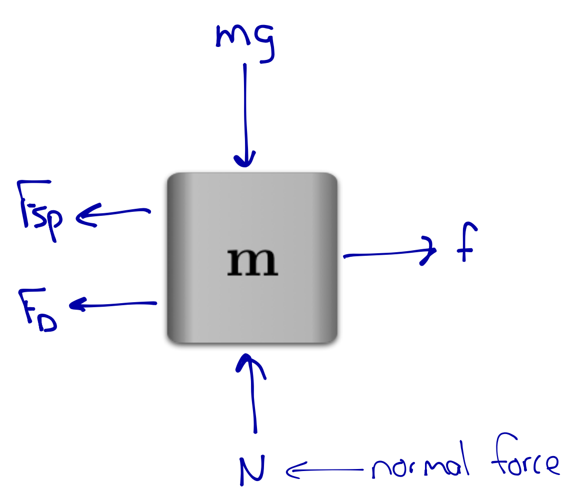
Draw a Free Body Diagram



g = accel due to gravity

Q: What are the forces on this system?





Newton (actually Newton-Euler) says that:

$\sum \bar{F} = m\bar{a}$ ← $\bar{F}$ and $\bar{a}$ are vectors, so they have magnitude and direction

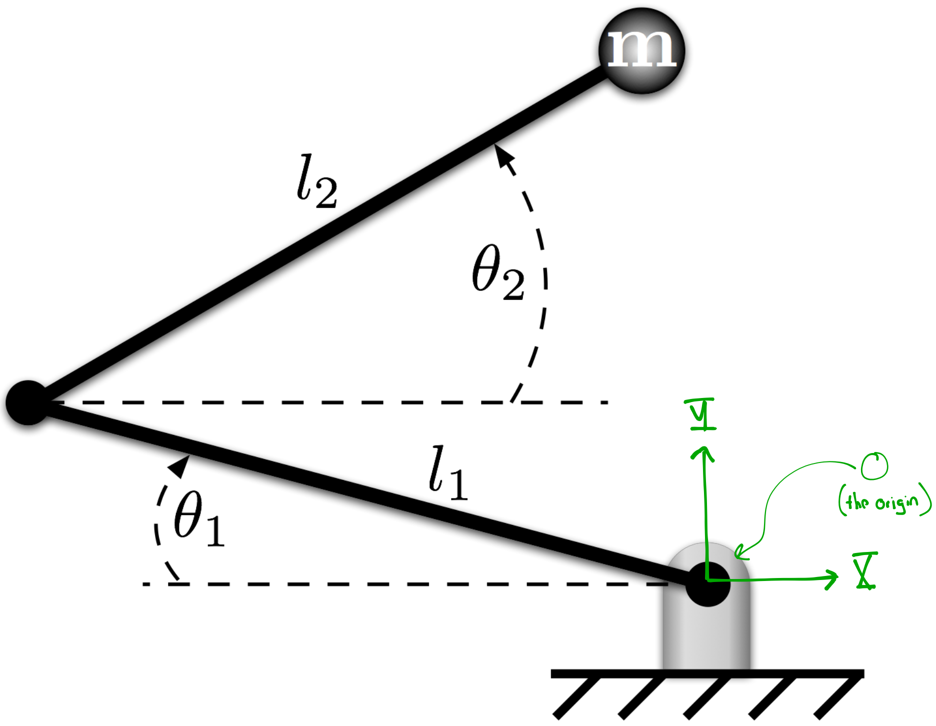
Q: How do we find $\bar{F}$ for the system above?

Sum the forces in each direction

$$\sum \bar{F} = (-F_{sp} - F_D + f)\underset{\uparrow \atop X}{\bar{I}} + (N - mg)\underset{\uparrow \atop Y}{\bar{J}}$$

Kinematics

Describe the motion without consideration for what's causing it.



$\bar{r}_{m/o} \equiv$ position of m relative to O

$= \left(-l_1 \cos\theta_1 + l_2 \cos\theta_2\right)\bar{I} + \left(l_1 \sin\theta_1 + l_2 \sin\theta_2\right)\bar{J}$

$\bar{V}_m \equiv$ velocity of $m = \underbrace{\frac{d}{dt}\left(\bar{r}_{m/o}\right) = \dot{\bar{r}}_{m/o}}_{\text{remember that it's a vector}}$

$= \left(l_1\dot{\theta}_1 \sin\theta_1 - l_2\dot{\theta}_2 \sin\theta_2\right)\bar{I}$

$+ \left(l_1\dot{\theta}_1 \cos\theta_1 + l_2\dot{\theta}_2 \cos\theta_2\right)\bar{J}$

Newtonian (Newton/Euler) Kinetics

Describes motion *and* it's causes (forces and torques)

Linear Motion

$\sum \bar{F} = m\bar{a}$

or

$\sum \bar{F} = \dot{\bar{L}} = \frac{d}{dt}\left(m\bar{V}\right)$ linear momentum

Q: When is the 2nd form necessary?

When m is changing.
Rockets are an example.

Rotational Motion

$\sum \bar{M}_A = m\bar{r}_{c/A} \times \bar{a}_A + \dot{\bar{H}}_A$ Change in angular momentum $\bar{H}_A$

$\dot{\bar{H}}_A = \frac{d}{dt}\left(I\bar{\omega}\right)$

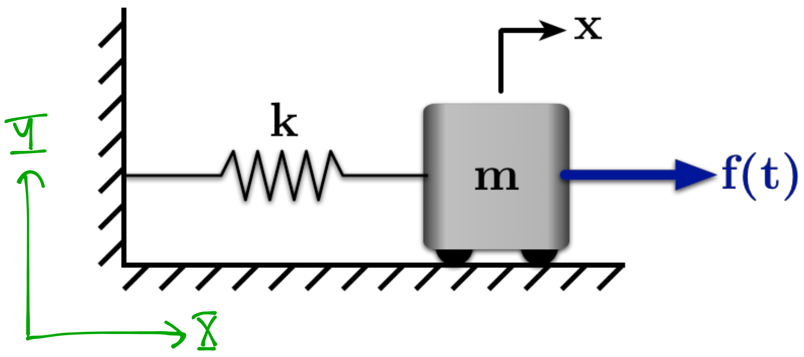
$\sum \bar{M}_A = \dot{\bar{H}}_A$ iff $\bar{r}_{c/A} \times \bar{a}_A = 0$

1) point A is the COM $\left(r_{c/A} = 0\right)$

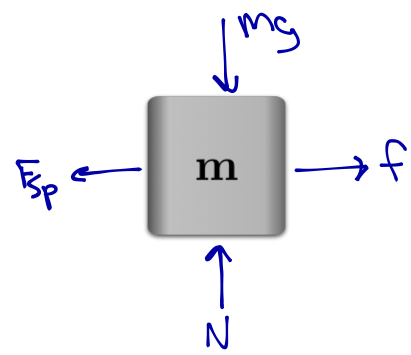
2) point A has constant velocity $\left(\bar{a}_A = 0\right)$

3) point A is accelerating directly toward or away from the center of mass (cross-product = 0)

Linear Example



Free body diagram



$$\Sigma \bar{F} = (-F_{sp} + f)\bar{I} + (N - mg)\bar{J}$$

Q: What is F_{sp}?

Hooke's Law (for linear springs) — $F_{sp} = k\delta$ where δ is the deflection of the spring from its equilibrium length

Q: What sign should F_{sp} be?

It depends on your choice of coordinate frame.

The spring always wants to return to its equilibrium length.

$$\Sigma \bar{F} = m\bar{a}$$

$$\Sigma (F_x \bar{I} + F_y \bar{J}) = m(\ddot{x}\bar{I} + \ddot{y}\bar{J})$$

$F_x = m\ddot{x}$ ← Scalar version representing $\bar{I}$ direction

Q: What is $\ddot{y}$ for this problem?

$\ddot{y} = 0$, so $N = mg$

$$-F_{sp} + f = m\ddot{x}$$

$$-kx + f = m\ddot{x} \longrightarrow \boxed{m\ddot{x} + kx = f}$$

← This is the equation of motion representing motion in the $\bar{I}$ direction

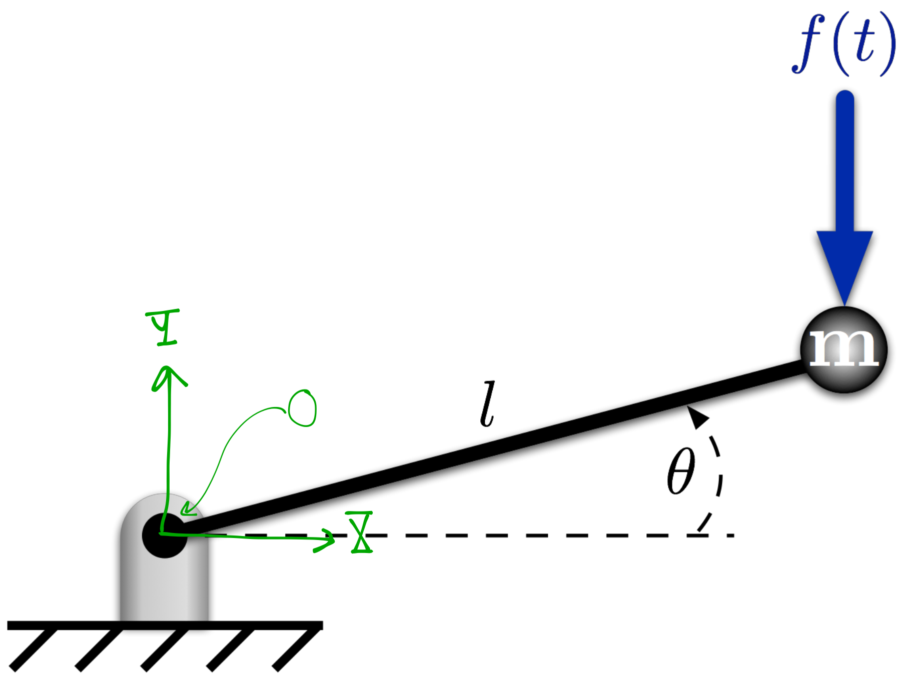
Q: Now what?

Solve the ODE to get $x(t)$ ← This is the response of the mass in the $\bar{I}$ direction

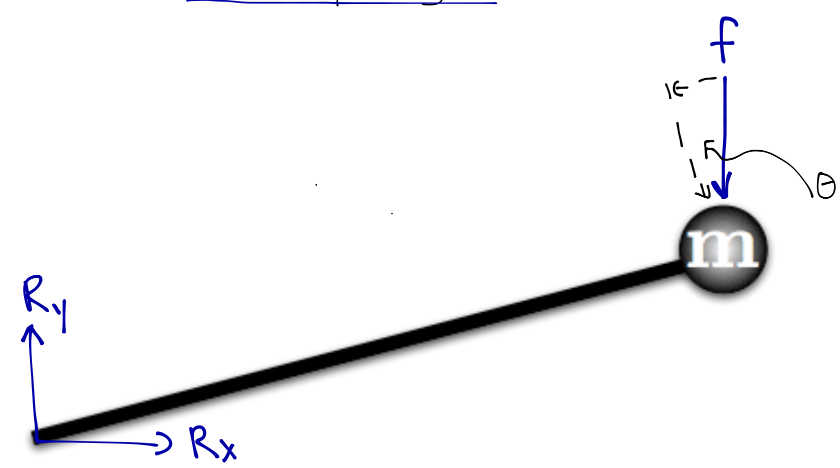
Rotational Motion

Q: $\sum \bar{M}_A$?

- pure torques acting on a body
- forces acting at a distance from a rotation point



Free Body Diagram



We have a choice of where to sum moments about. So let's pick a point that will allow us to use the "simple" form of the rotation equations.

Q: Which point should we choose?

The system is in pure rotation about point O $(\bar{a}_o = 0)$, so let's use it.

$\sum \bar{M}_o = ?$ $\longrightarrow$ Q: What is the moment generated by force F?

$$\sum \bar{M}_o = \bar{r}_{F/o} \times \bar{F} = (l\cos\theta\,\bar{I} + l\sin\theta\,\bar{J}) \times (-F\bar{J})$$

$$= (-Fl\cos\theta)\bar{K}$$ $\longleftarrow$ This is the only component that can create motion in θ

$$\sum \bar{M}_o = \frac{d}{dt}(I_o\bar{\omega}) = I_o\bar{\alpha}$$

$\bar{\omega}$ = angular velocity
$\bar{\alpha}$ = angular acceleration

Q: What are $\bar{\omega}$ and $\bar{\alpha}$ in this problem? $\longrightarrow$ $\bar{\omega} = \dot{\theta}\bar{K}$ $\quad \bar{\alpha} = \ddot{\theta}\bar{K}$

$$\sum \bar{M}_o = I_o\ddot{\theta}\bar{K}$$

$$(-Fl\cos\theta)\cancel{\bar{K}} = (I_o\ddot{\theta})\cancel{\bar{K}}$$

$$\boxed{I_o\ddot{\theta} = -Fl\cos\theta}$$