MCHE 485: Mechanical Vibrations
Spring 2019 – Homework 1

Assigned:	Friday, January 25th
Due:	Friday, February 1st, 5pm

Assignment: From "Principles of Vibration" by Benson Tongue, problems:
1.1, 1.3, 1.21, 1.23, 1.24, 1.31, 1.54

Submission: Emailed *single* pdf document:

- to `joshua.vaughan@louisiana.edu`
- with subject line and filename `ULID-MCHE485-HW1,` where `ULID` is your ULID
- *Note:* Submissions with incorrect filenames or submitted as multiple images/pdfs will be rejected.

Problem 1.1

1.1. If $x(t) = a_1 e^{i\omega t} + a_2 e^{-i\omega t}$, $x(0) = 4$ and $\dot{x}(0) = 2$, what are a_1 and a_2 equal to?

$$\dot{x}(t) = i a_1 \omega e^{i\omega t} - i a_2 e^{-i\omega t}$$

$$x(0) = 4 = a_1 + a_2 \longrightarrow a_2 = 4 - a_1$$

$$\dot{x}(0) = i a_1 \omega - i a_2 \omega = 2$$

sub into the 2nd equation

$$i a_1 \omega - i\omega(4 - a_1) = 2$$

$$2 i a_1 \omega - 4 i\omega = 2$$

$$a_1 = \frac{2 + 4i\omega}{2i\omega}$$

$$\boxed{a_1 = \frac{1 + 2i\omega}{i\omega}}$$

so $a_2 = 4 - \left(\frac{1+2i\omega}{i\omega}\right) = \frac{4i\omega - (1 + 2i\omega)}{i\omega}$

$$\boxed{a_2 = \frac{2i\omega - 1}{i\omega}}$$

Note: We generally don't like to leave i in the denominator, so:

$$a_1 = \frac{1 + 2i\omega}{i\omega}\left(\frac{i}{i}\right) = \frac{i - 2\omega}{-\omega} = \frac{2\omega - i}{\omega}$$

$$a_2 = \frac{2i\omega - 1}{i\omega}\left(\frac{i}{i}\right) = \frac{-2\omega - i}{-\omega} = \frac{2\omega + i}{\omega}$$

Problem 1.3

1.1. If $x(t) = a_1 e^{i\omega t} + a_2 e^{-i\omega t}$, $x(0) = 4$ and $\dot{x}(0) = 2$, what are a_1 and a_2 equal to?

1.2. Express $(1 + 2i)e^{i\omega t} + (1 - 2i)e^{-i\omega t}$ in terms of $\sin(\omega t)$ and $\cos(\omega t)$.

1.3. If $x(t) = b_1 \cos(\omega t) + b_2 \sin(\omega t)$, with the same initial conditions as in Problem 1.1, what are b_1 and b_2 equal to?

If $x(t) = b_1 \cos(\omega t) + b_2 \sin(\omega t)$ then $\dot{x}(t) = -b_1\omega \sin \omega t + b_2 \omega \cos \omega t$

Use these to solve the initial condition problem:

$$x(0) = 4 = b_1 \cos(0) + \cancel{b_2 \sin(0)}^{\,0} \rightarrow \boxed{b_1 = 4}$$

$$\dot{x}(0) = 2 = \cancel{-b_1 \omega \sin(0)}^{\,0} + b_2 \omega \cos \omega \rightarrow b_2 \omega = 2 \rightarrow \boxed{b_2 = \frac{2}{\omega}}$$

Problem 1.21

1.21. What is ω_n for the system illustrated in Figure P1.21 in terms of m, k_1, k_2, and k_3?

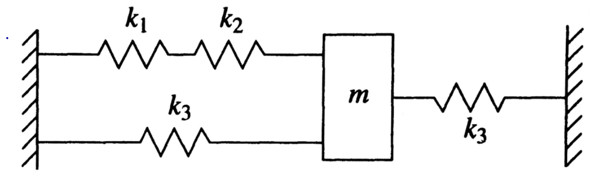
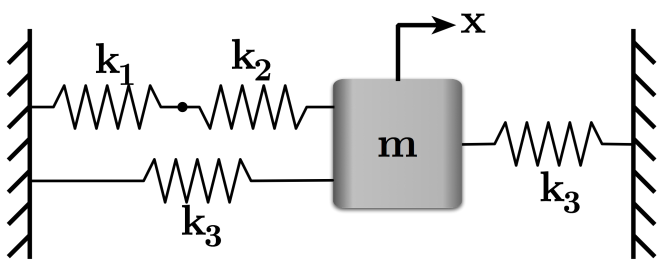
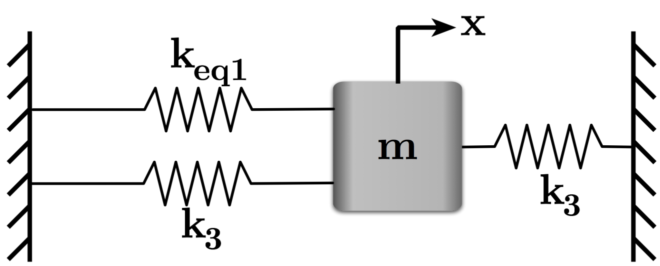


Figure P1.21

Use equivalent springs to simplify this problem.

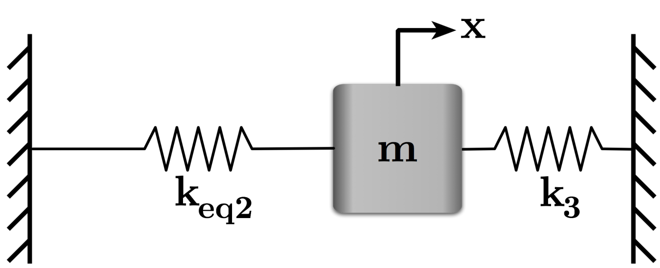


k₁ and k₂ are in series, so



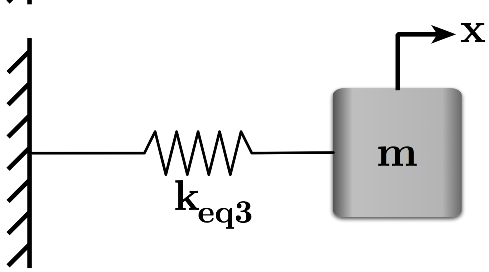
$$\frac{1}{K_{eq1}} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$K_{eq1} = \frac{k_1 k_2}{k_1 + k_2}$$



K_{eq1} and k_3 are in parallel

$$K_{eq2} = K_{eq1} + k_3$$



K_{eq2} and k_3 are in parallel

$$K_{eq3} = K_{eq2} + k_3$$

So $\boxed{K_{eq3} = \left(\frac{k_1 k_2}{k_1 + k_2}\right) + 2k_3}$

We know the natural frequency of the equivalent system is $\sqrt{\frac{K_{eq3}}{M}}$

Problem 1.23

1.23. Find the equivalent spring constant for the two springs with widely differing spring constants shown in Figure P1.23. Comment on your result.

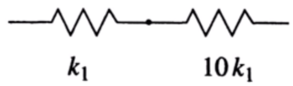


Figure P1.23

The springs are in series, so:

$$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{10K_1}$$

$$= \frac{10}{10K_1} + \frac{1}{10k_1} = \frac{11}{10K_1}$$

$$K_{eq} = \frac{10K_1}{11}$$

Here, the lower spring constant, k1, clearly dominates the effective spring constant of the two in series.

This should make intuitive sense; imagine a scenario just like the one in the problem. There are two springs in series, one much stiffer than the other. The softer one (lower spring constant) will compress much more easily. As such, it dominates the characteristics of their combination.

Another way to picture this is to look at the characteristics as we approach physical limits. As the stiff spring becomes infinitely stiff, the soft spring is the only remaining flexible part, so it completely determines the spring constant.

Problem 1.24

1.24. Find the equivalent spring constant for the two springs with widely differing spring constants shown in Figure P1.24. Comment on your result.

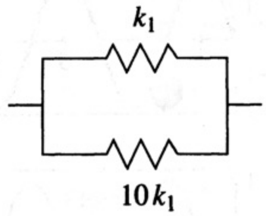
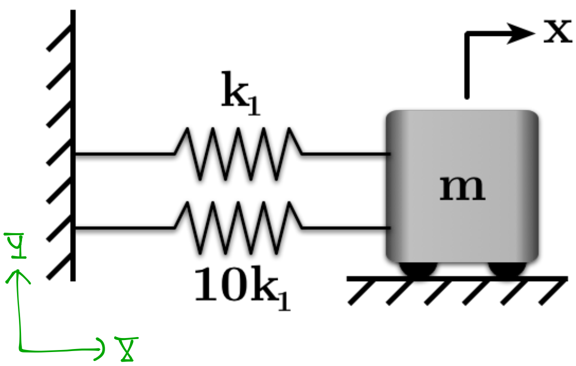
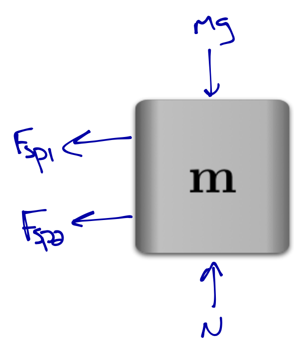


Figure P1.24

$$K_{eq} = k_1 + 10k_1 = 11k_1$$

In this case, the stiffer spring dominates the resulting equivalent spring constant. The reason is easy to see if we look at the forces these springs would cause. For example:





$$\sum \bar{F} = -F_{sp1} - F_{sp2} \quad \leftarrow \text{all in } \bar{I}\text{-direction}$$

$$= -k_1\delta - 10k_1\delta \quad \leftarrow \text{here } \delta = x$$

$$= -k_1 x - 10k_1 x$$

$$= \underbrace{(-11k_1)}_{K_{eq} = 11k_1} x$$

The stiffer spring contributes more force for the same deflection.

Problem 1.31

1.31. Consider the system illustrated in Figure P1.31, which has two linear springs. The unstretched length of k_1 is .5 m and the unstretched length of k_2 is .25 m. $k_1 = 1000$ N/m, $k_2 = 2000$ N/m, $m = 2$ kg, $l = .5$ m. Find the equilibrium position of the mass and determine the natural frequency of the system. Compare this natural frequency to that associated with $l = .75$ m (i.e., no precompression in the spring).

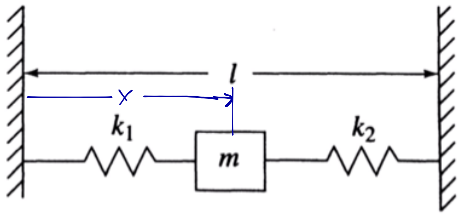
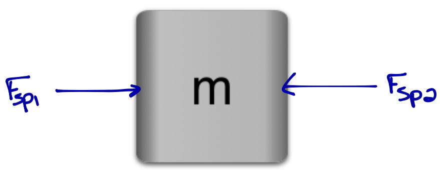


Figure P1.31

FBD



$$m\ddot{x} = -F_{sp1} - F_{sp2}$$

$$F_{sp1} = k_1\delta_1 = k_1(0.5 - x)$$

$$F_{sp2} = k_2\delta_2 = k_2(0.25 - (l - x))$$ ← order of terms inside (·) chosen to match the sign on the FBD

Let's start by looking at the equil position. Here, the two spring forces balance, so:

$$k_1(0.5 - x) = k_2(0.25 - (l - x))$$

$$0.5k_1 - k_1 x = 0.25k_2 - k_2 l + k_2 x$$

$$-(k_1 + k_2)x = 0.25k_2 - k_2 l - 0.5k_1$$

$$x = -\frac{0.25k_2 - k_2 l - 0.5k_1}{k_1 + k_2} = 0.33\text{m}$$

Define $x(t) = x_{eq} + y(t)$

$$m\ddot{y} = k_1(0.5 - (x_{eq} + y)) - k_2(0.25 - (l - (x_{eq} + y)))$$

$$m\ddot{y} + (k_1 + k_2)y = \underbrace{\cancel{k_1(0.5 - x_{eq}) - k_2(0.25 - (l - x_{eq}))}}_{=0 \text{ at equil.}}$$

$$\ddot{y} + \frac{k_1 + k_2}{m}y = 0 \quad \text{so} \quad \boxed{\omega_n = \sqrt{\frac{k_1 + k_2}{m}} = 38.7 \frac{\text{rad}}{\text{s}}}$$

If the springs are not precompressed then the system is already operating about its equil, so:

$$m\ddot{y} = -k_1 y - k_2 y$$

$$m\ddot{y} + (k_1 + k_2)y = 0$$

$$\boxed{\omega_n = \sqrt{\frac{k_1 + k_2}{m}} = 38.7 \frac{\text{rad}}{\text{s}} \quad \leftarrow \text{Same as above}}$$

Problem 1.54

1.54. As a practical joke, you've put some fast-acting glue in your friend's slingshot as in Figure P1.54. Your friend puts a mass of .06 kg in the slingshot, pulls it back 1 m, and then releases it. If the mass travels out 1 m beyond the slingshot and then returns to strike his hand .5 second after the release, what must the spring constant be for the massless elastic band? Assume that the elastic band has an unstretched length equal to 0 m.

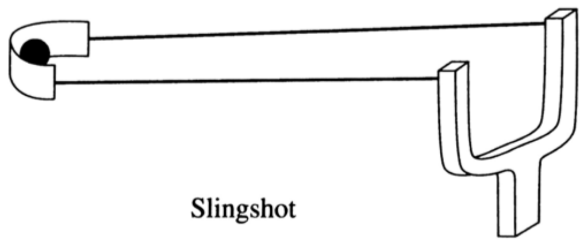


Figure P1.54

The key to this problem is realizing that traveling from release to 1m beyond the slingshot to strike the hand is represents one period of oscillation.

So, that tells us the 0.5s = 1 period for this system.

This means the frequency $\omega_n = \frac{2\pi}{\tau} \frac{rad}{s} = \frac{2\pi}{0.5s} \frac{rad}{s} = 4\pi \frac{rad}{s}$

We know the natural frequency is $\omega_n = \sqrt{\frac{k}{m}}$ and $m = 0.06kg$. Just solve for k.

$$4\pi = \sqrt{\frac{k}{0.06}} \rightarrow \boxed{k = 9.47 \, N/m}$$