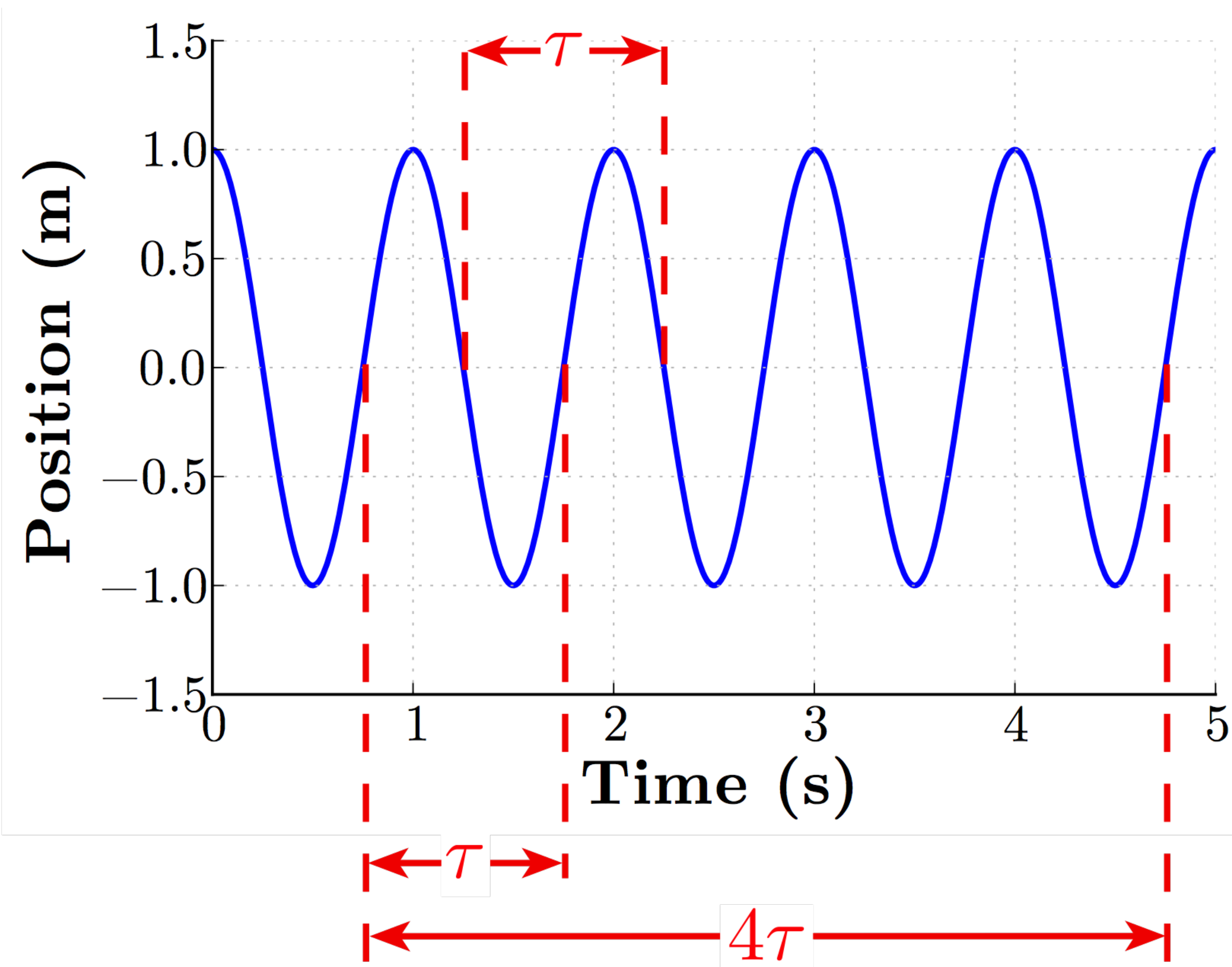
System Identification

Given some vibrating system, how can we accurately estimate natural frequency and damping ratio?

Let's start with systems with 1 dominant mode (1 DOF is important, like all the systems we've studied so far).

Q: For undamped systems, how could we estimate natural frequency?

- Time between "zero" crossings
- It's better to count N periods and divide
- We can also get an idea about the linearity of a system:
  - Is the time between zeros always (approx) equal?
    - Yes -> System is linear
    - No --> System may be nonlinear



Some key points:

- We're really counting about equil. If the system has a nonzero equil., it's often best to remove this offset.
- Be sure to count entire periods
- You can do this by looking at the slope (i.e. count from one positive-slope zero-intersection to the next positive-slope intersection
- It's usually best to get multiple estimates, the compare and average (or otherwise decide which is the best to use)

System Identification (cont.)

Q: What about damping?

Changes the freq of oscillation to $\omega_d = \omega_n\sqrt{1-\zeta^2}$ ← if $\zeta \ll 1$ $\omega_d \approx \omega_n$

But, remember that strictly speaking you will be calculating the damped natural frequency, not the "pure" natural frequency

Q: How could we estimate the damping itself?

Remember that the response is $x(t) = \underbrace{e^{-\zeta\omega_n t}}\left(a\cos\omega_d t + b\sin\omega_d t\right)$

We can get an estimate of damping from this term

It sets the decay rate of $x(t)$, so look at that envelope to estimate ζ

Log Decrement (To estimate damping ratio from free responses)

$x(t) = e^{-\zeta\omega_n t}\left(a\cos\omega_d t + b\sin\omega_d t\right)$ define $x(0) = x_0$ and $\dot{x}(0) = 0 \longrightarrow a = x_0$ and $b = 0$

$x(t) = e^{-\zeta\omega_n t}\left(x_0\cos\omega_d t\right)$ ← Now, let's look at the response N-periods later

Each period $\tau = \frac{2\pi}{\omega_d}$ so N-periods $= N\tau = \frac{2N\pi}{\omega_d}$

$$x\left(\frac{2N\pi}{\omega_d}\right) = \exp\left[-\zeta\omega_n\frac{2N\pi}{\omega_d}\right]\left[x_0\underbrace{\cos\left(\omega_d\frac{2N\pi}{\omega_d}\right)}_{=1\ \forall N}\right] = x_0\exp\left[\frac{-2N\pi\zeta}{\sqrt{1-\zeta^2}}\right]$$

Now, let's look at the ratio between the 1^{st} peak $(x(0)=x_0)$ and the N^{th} peak:

$$\frac{x(0)}{x(N\tau)} = \frac{x(0)}{x\left(\frac{2N\pi}{\omega_d}\right)} = \frac{\cancel{x_0}}{\cancel{x_0}\exp\left[\frac{-2N\pi\zeta}{\sqrt{1-\zeta^2}}\right]} = \exp\left[\frac{2N\pi\zeta}{\sqrt{1-\zeta^2}}\right]$$

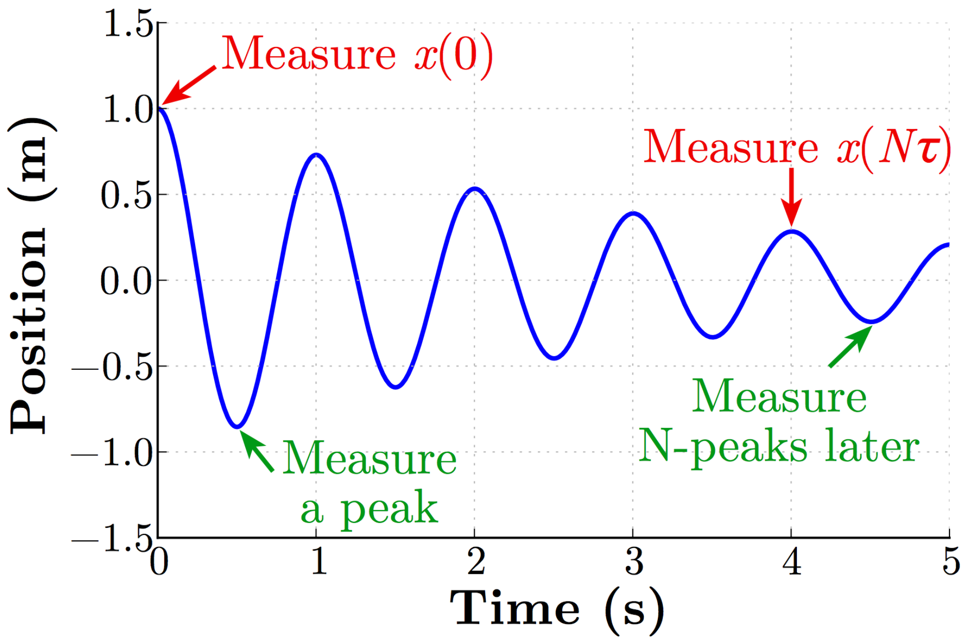
We can measure x_0 and $x\left(\frac{2N\pi}{\omega_d}\right)$ from our experimental response. To get ζ, we just need to solve this equation for it.

$$\ln\left(\frac{x(0)}{x(N\tau)}\right) = \frac{2N\pi\zeta}{\sqrt{1-\zeta^2}}$$

define $\sigma = \frac{1}{N}\ln\left(\frac{x(0)}{x(N\tau)}\right)$ then $\zeta = \frac{\sigma}{\sqrt{4\pi^2+\sigma^2}}$ $\left[\text{for } \zeta \ll 1,\ \zeta \approx \frac{\sigma}{2\pi}\right]$

Log Dec procedure summary

1. Measure x(0) and x($N\tau$) from the response.



2. Plug these into $\sigma = \frac{1}{N}\ln\left[\frac{x(0)}{x(N\tau)}\right]$ to get σ .

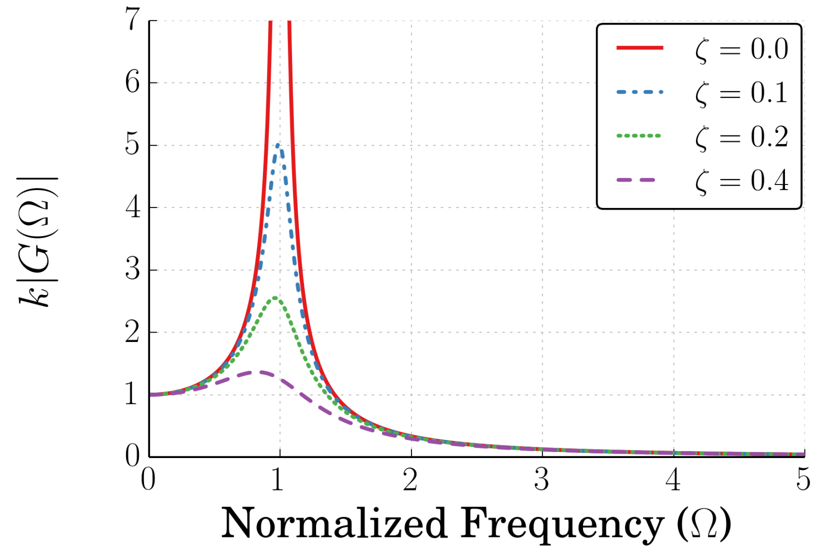
3. Use $\zeta = \frac{\sigma}{\sqrt{4\pi^2+\sigma^2}}$ to estimate ζ .

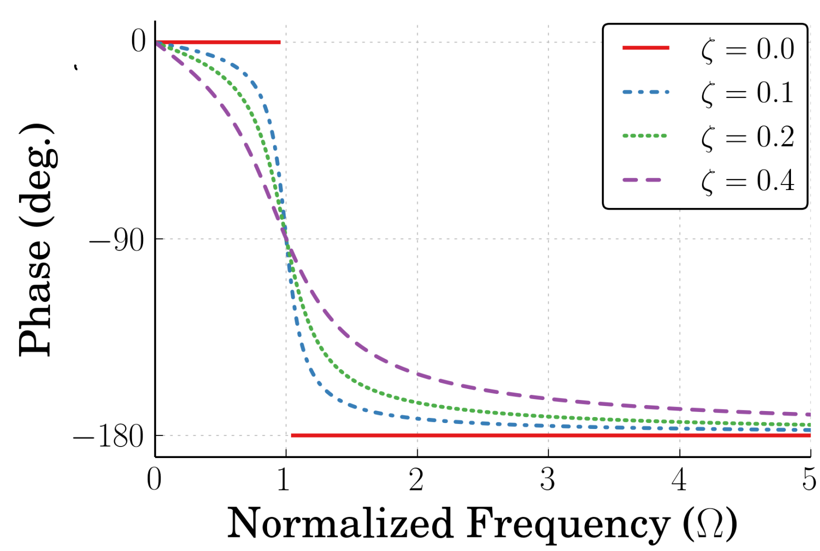
We can also use this to help determine linearity:

- Estimate damping for various ranges (different "Ns")
- If linear, the damping should be (approx.) equal for all

System Identification from Forced Responses

Q: How can we estimate system properties from forced responses?





Q: How can we (experimentally) generate plots like those above?

1. Select the frequency of our harmonic input, measure the amplitude of response.

2. Repeat over a range of frequencies.

Q: How can we estimate frequency?

Directly from frequency location of the peak
(Okay for lightly damped systems, $\zeta \ll 1$)

Q: What about damping?

1. Look at the ratio of the peak amplitude to the static reponse ($\omega = 0$)

Remember that for a direct-forced response:

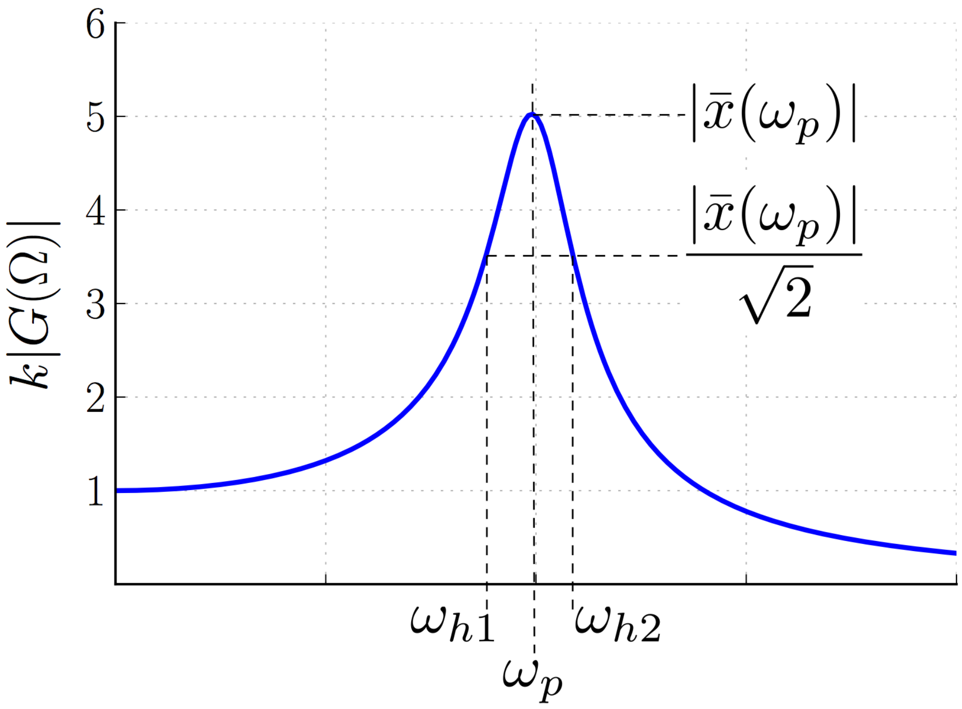
$$|X| = \frac{\bar{F}}{m} \frac{1}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega\omega_n)^2}}$$ so this ratio is:

$$\frac{|G(\omega_p)|}{|G(0)|} = \frac{\frac{\bar{F}}{m} \frac{1}{\sqrt{(\omega_n^2 - \omega_p^2)^2 + (2\zeta\omega_p\omega_n)^2}}}{\frac{\bar{F}}{m\omega_n^2}} \Longrightarrow \frac{1}{2\zeta\sqrt{1-\zeta^2}} \approx \frac{1}{2\zeta} \text{ when } \zeta \ll 1$$

2. Look at half-power points

2. Look at half-power points (cont.)

See the book for derivation



Define $\delta_h = \omega_{h2} - \omega_{h1}$

We can estimate:

$$\zeta = \frac{\delta_h}{2\omega_n}$$

Q: See any problems with this?

- for low damping, peak is narrow. It can be difficult to find $1/\sqrt{2}$ points
- for high damping, not much difference between the peak and half-power amps

System Identification Summary

- There are many methods to estimate natural freq. and damping ratio.
- The ones we looked at so far are for 1DOF systems
  (or systems with 1 dominant mode)
- We often need to filter the data before these calculations
  (Real data is noisy.)
- Often preferable to combine methods.