Input Shaping (cont.)

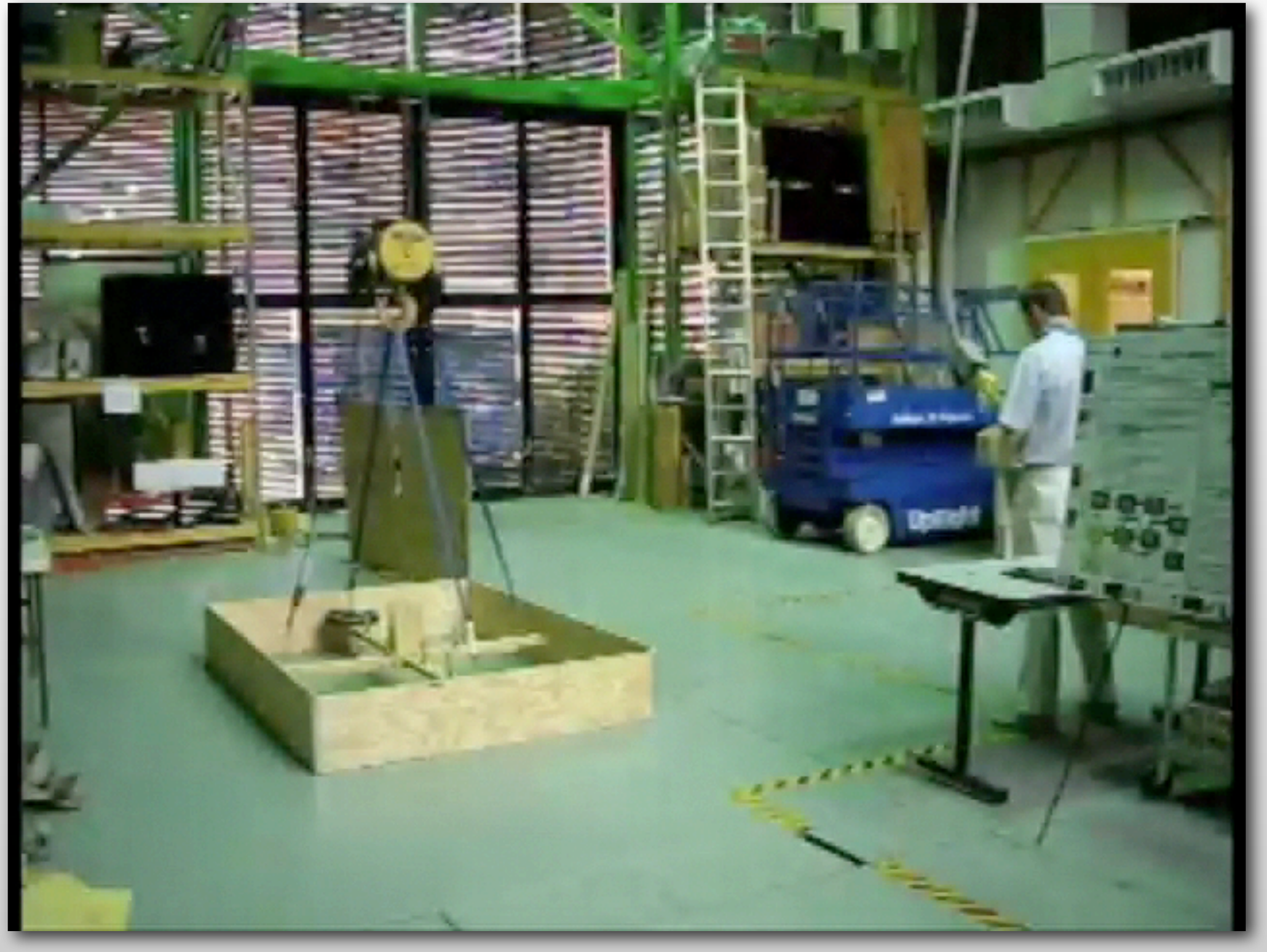
Dr. Joshua Vaughan

Rougeou 225

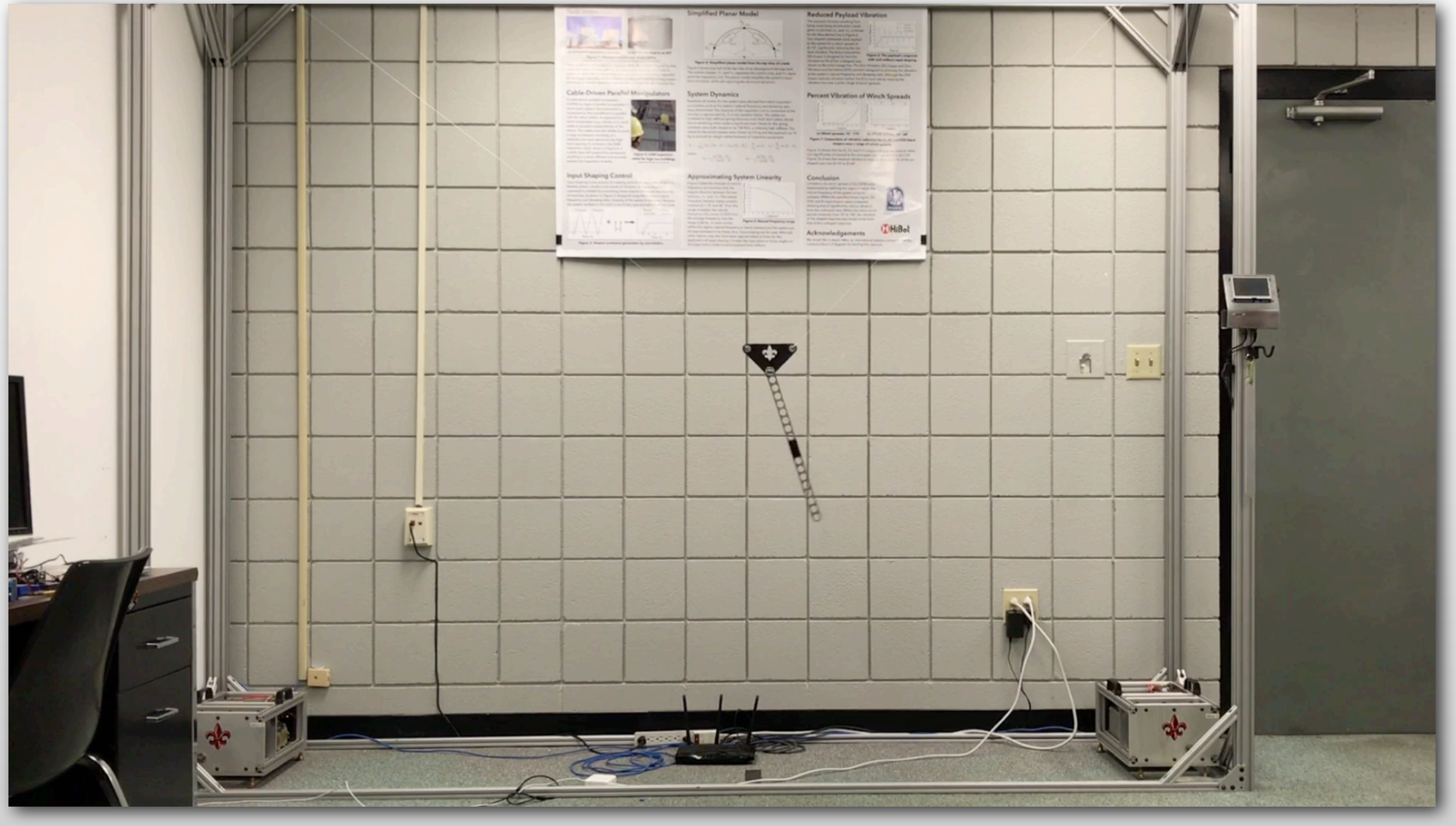
joshua.vaughan@louisiana.edu

@Doc_Vaughan

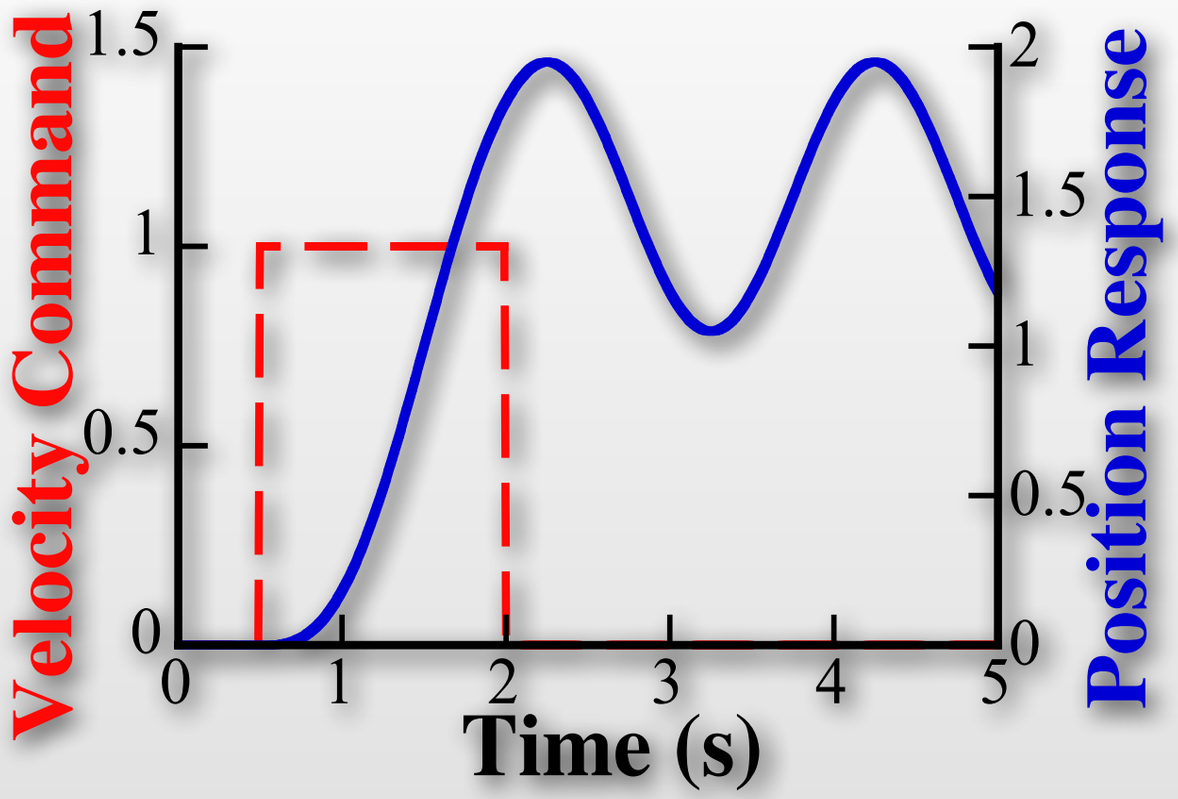
10-ton Bridge Crane

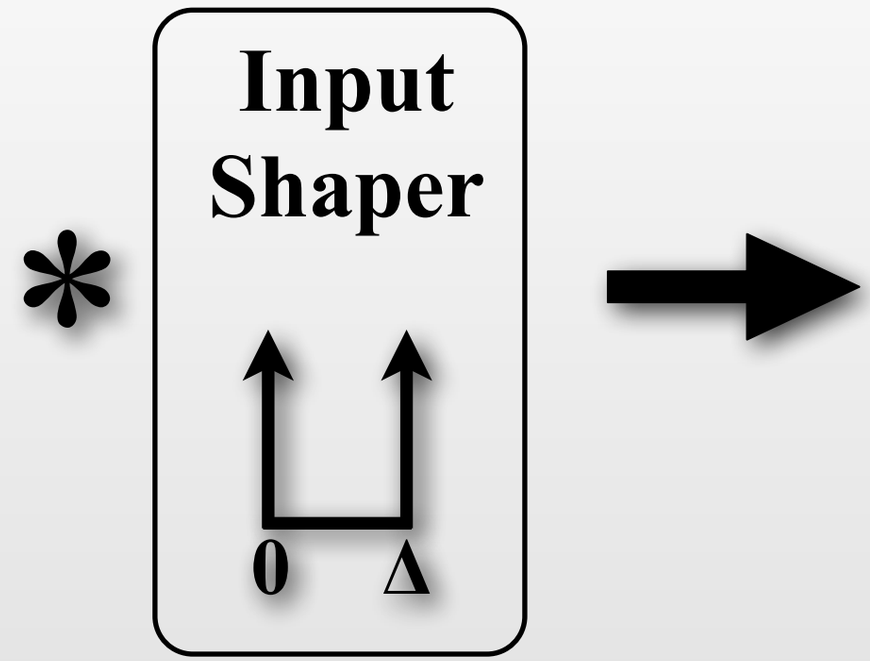


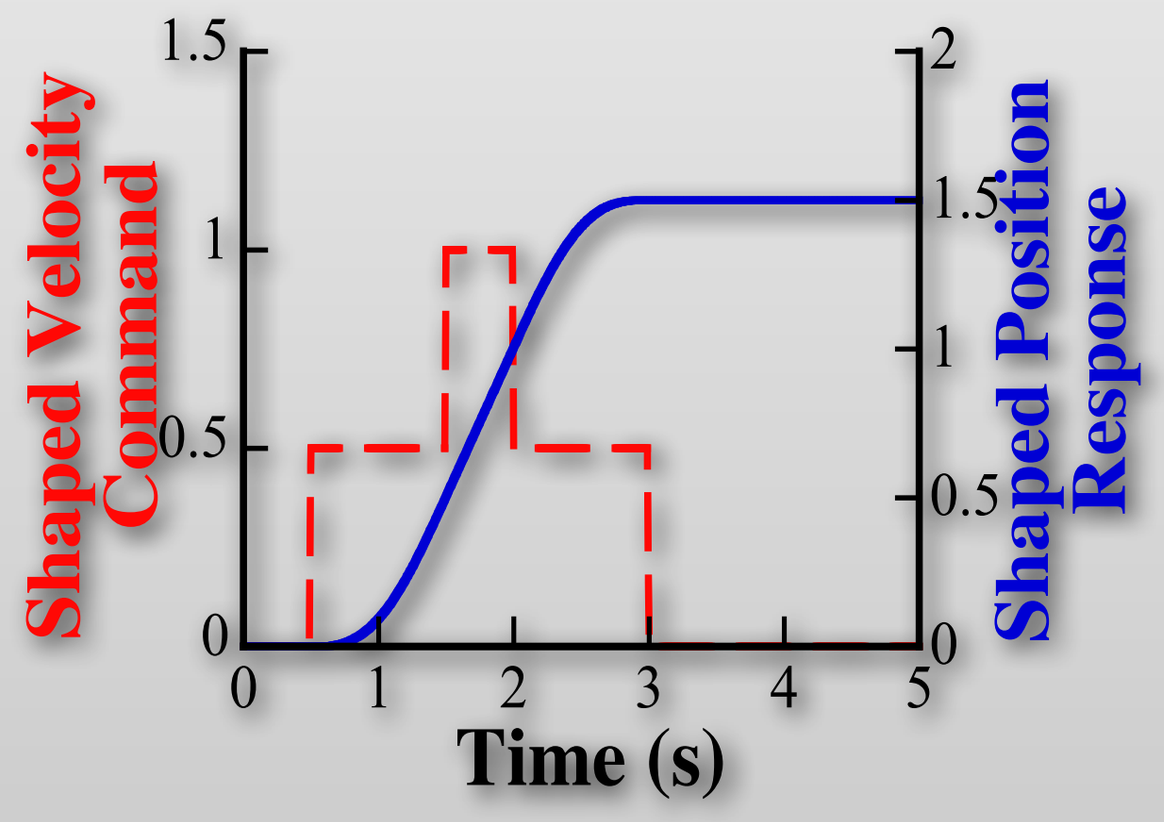
Lab-scale CDPM/CSPM



Input Shaping Process

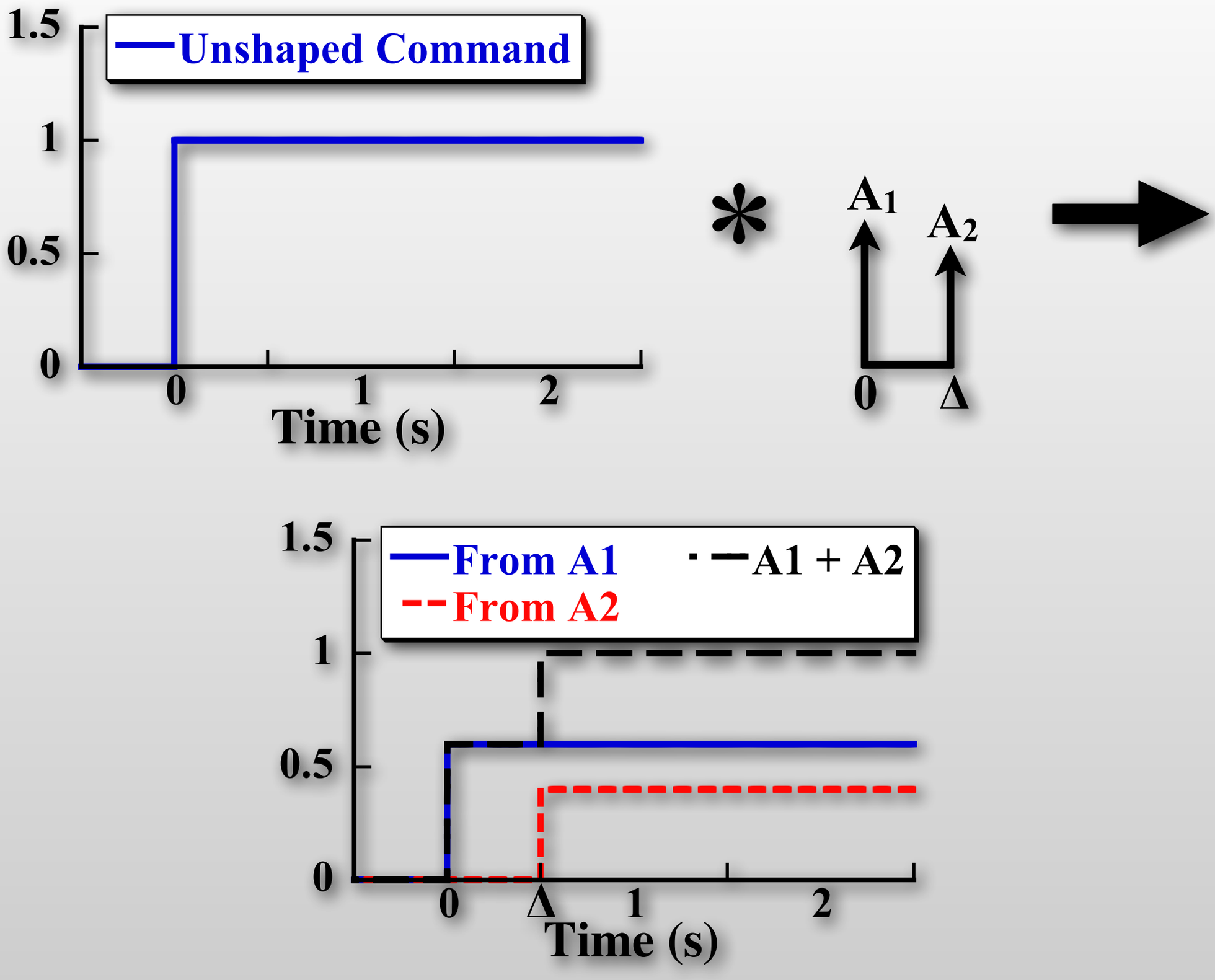




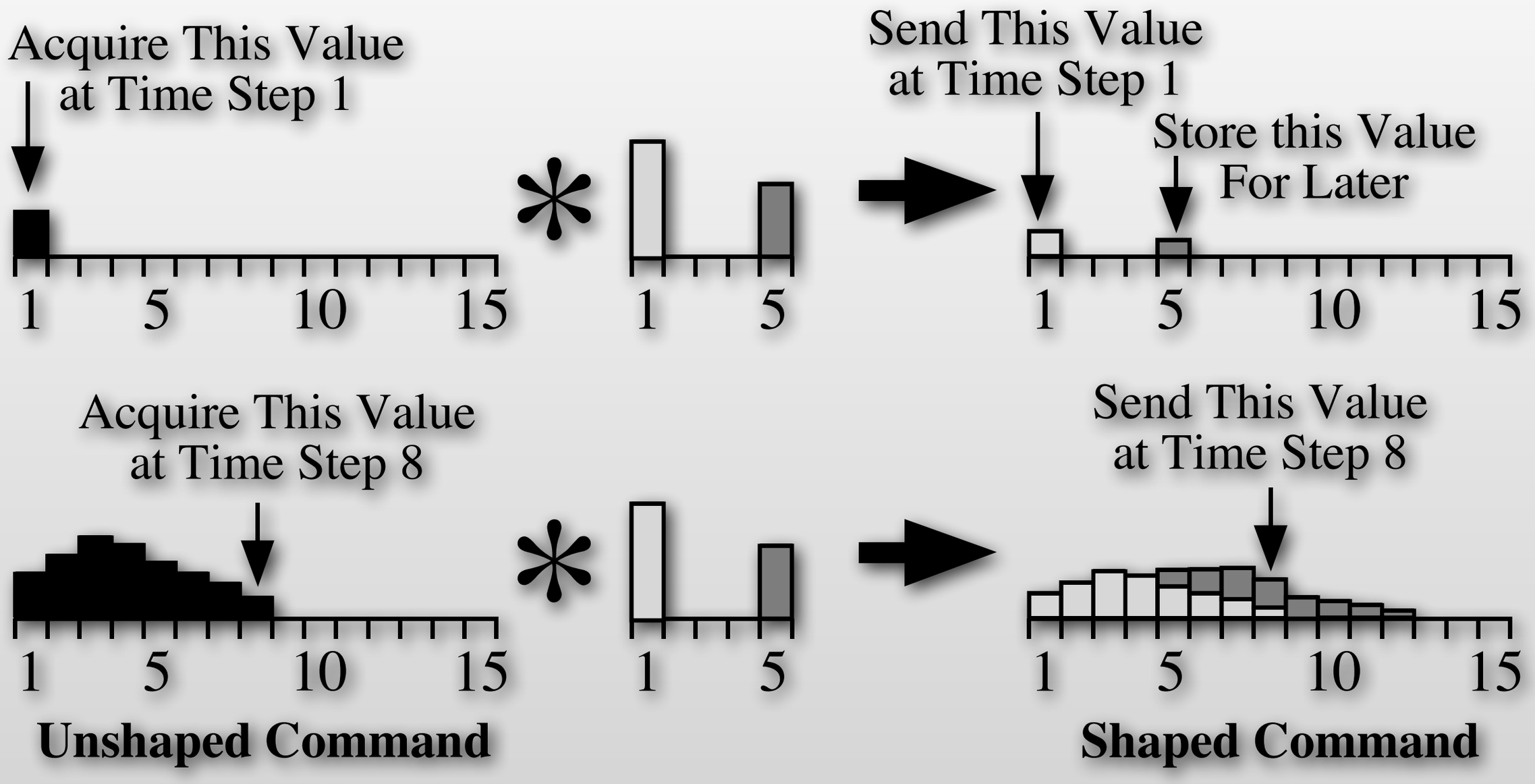


- Shaper design:
  - Natural Freq.
  - Damping Ratio
- Slight increase in command duration

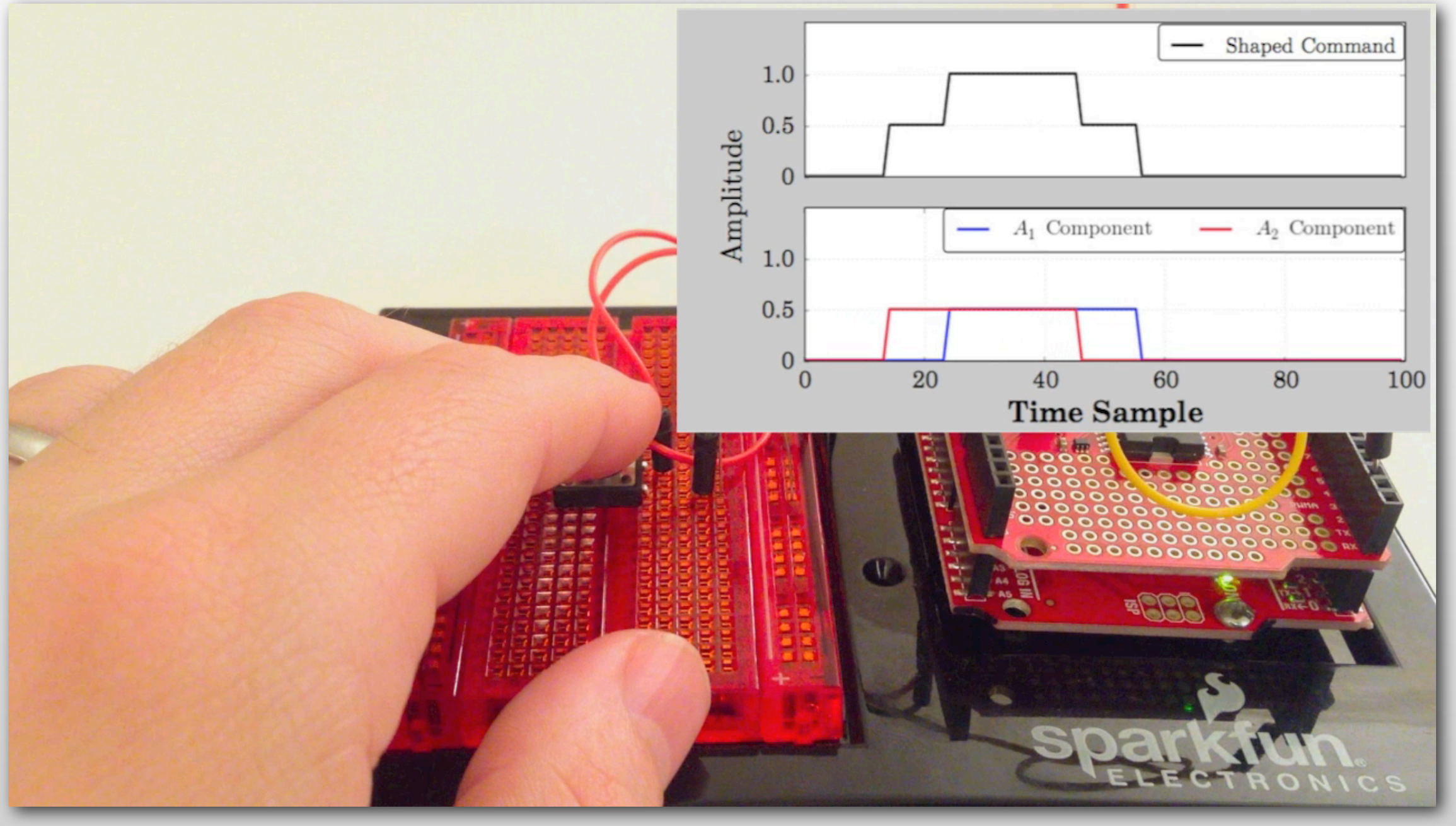
Convolution with Impulses



Implementation

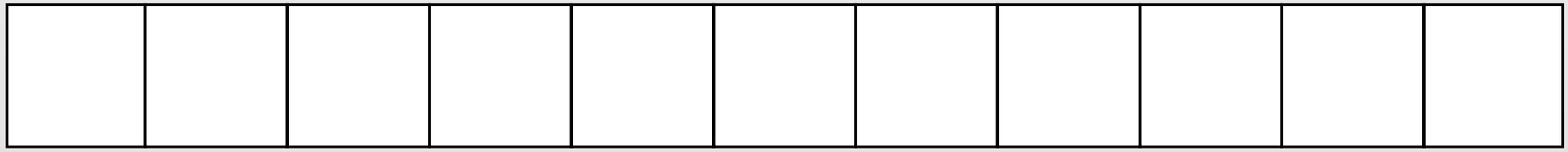


Implementation



“Real” Implementation

- We don’t have infinite memory → Use a ring buffer



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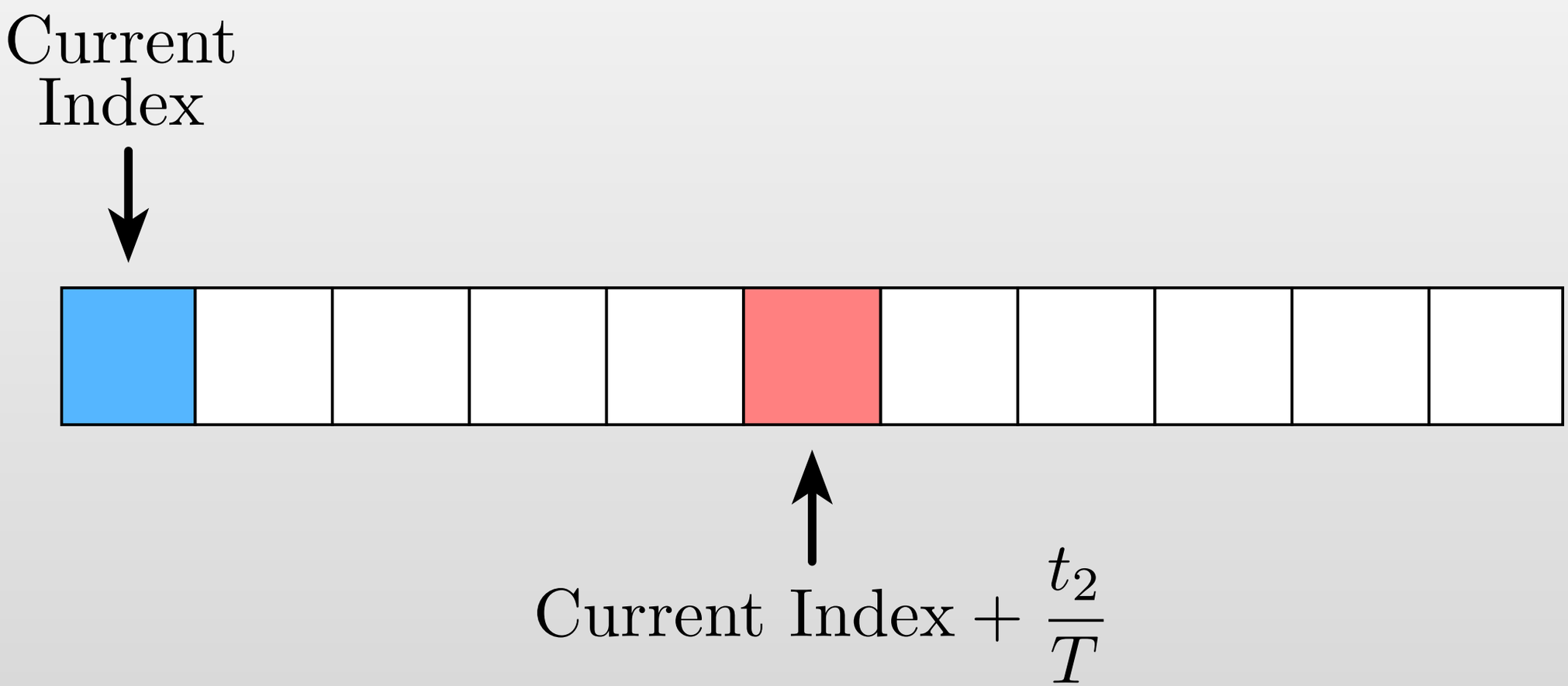

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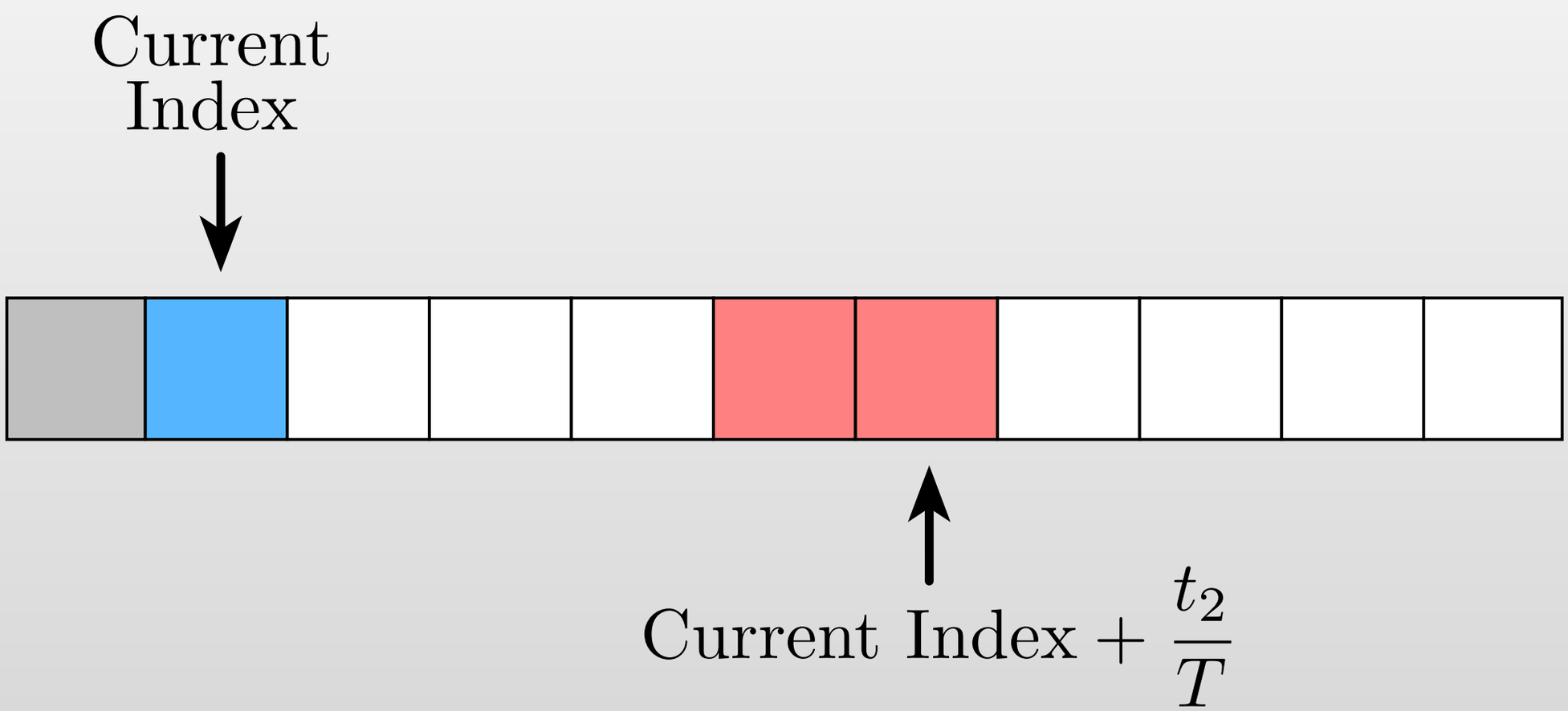

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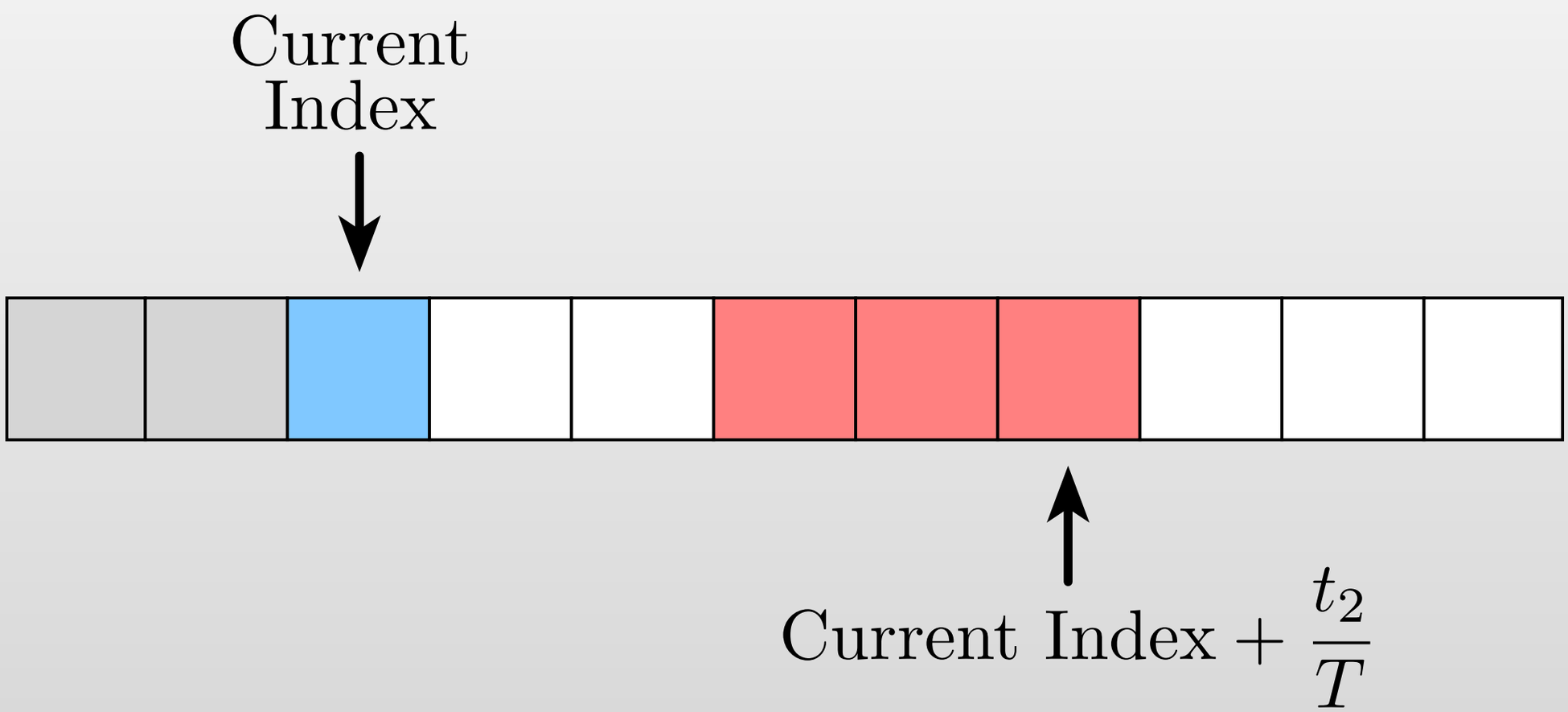

“Real” Implementation

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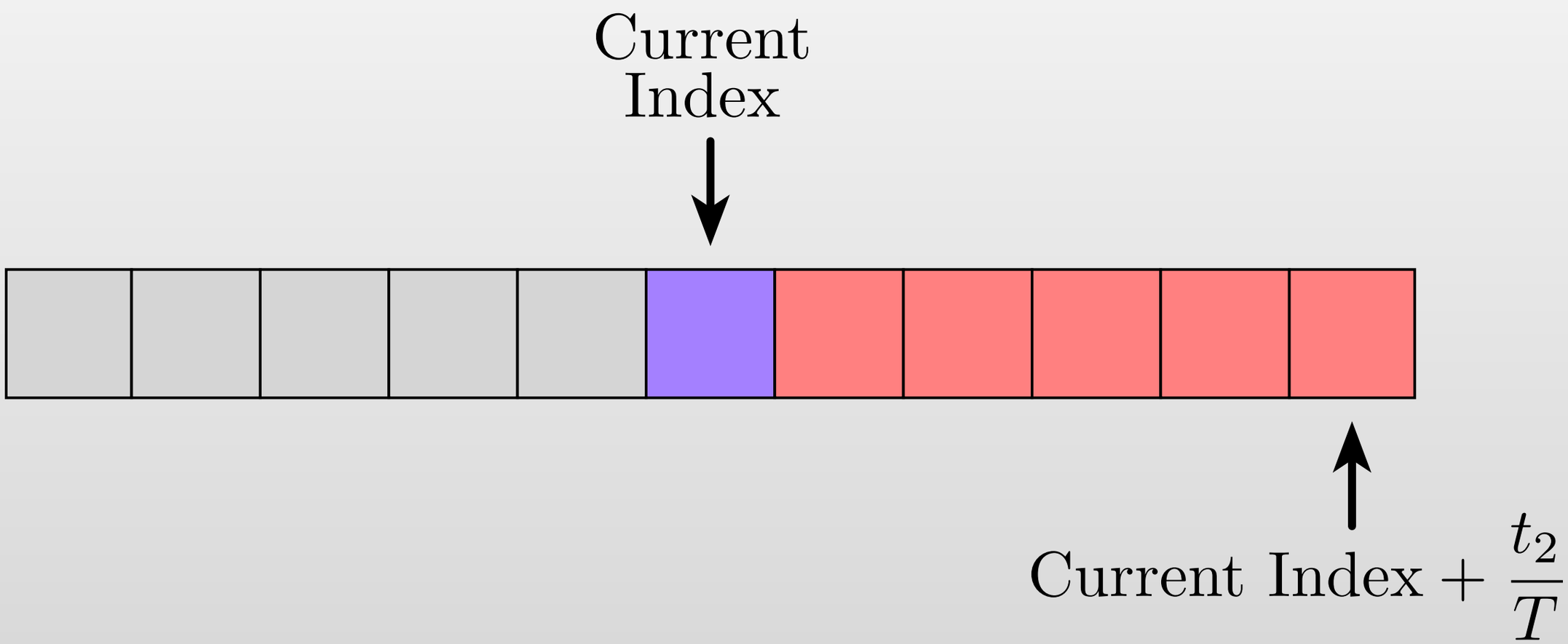

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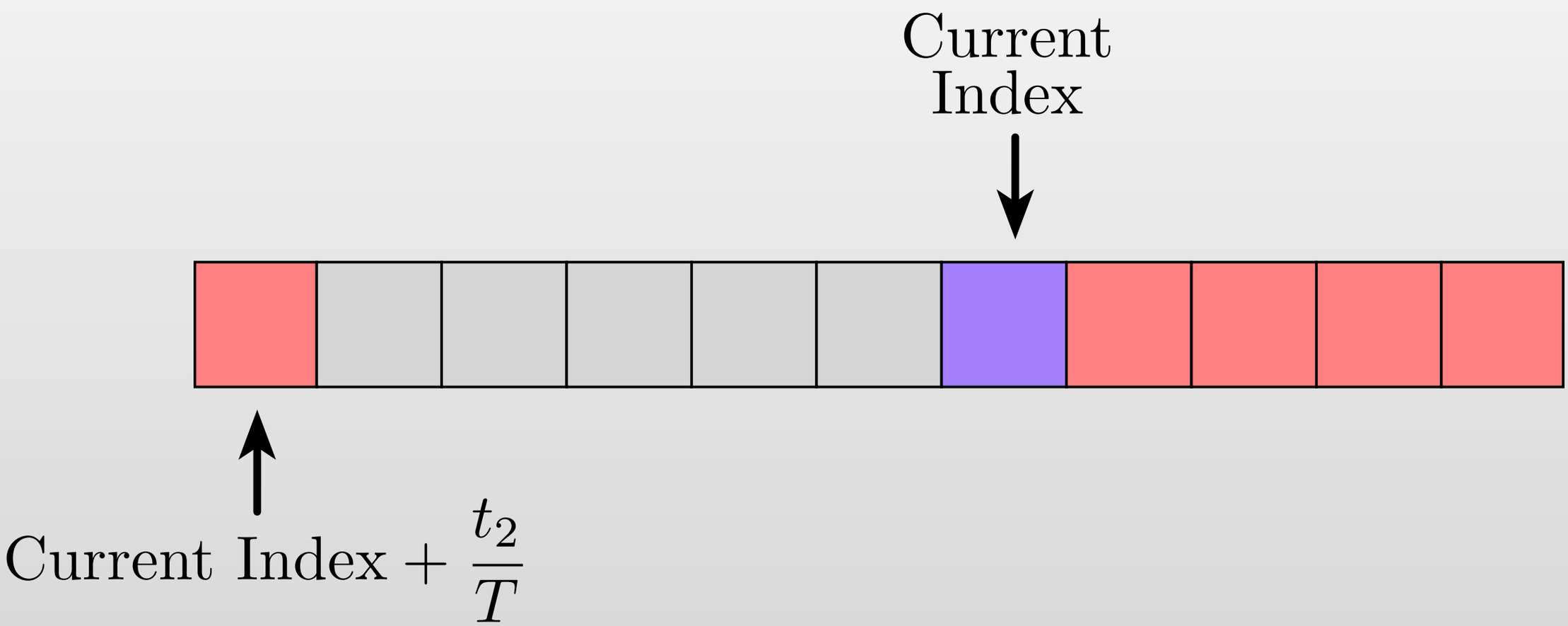


Vibration Equation

$$V(\omega,\zeta) = e^{-\zeta\omega t_n}\sqrt{[C(\omega,\zeta)]^2 + [S(\omega,\zeta)]^2}$$

$$C(\omega,\zeta) = \sum_{i=1}^{n} A_i e^{\zeta\omega t_i}\cos(\omega t_i\sqrt{1-\zeta^2})$$

$$S(\omega,\zeta) = \sum_{i=1}^{n} A_i e^{\zeta\omega t_i}\sin(\omega t_i\sqrt{1-\zeta^2})$$

$V(\omega,\zeta)$ is the vibration excited by *n*-impulses.

$$V(\omega,\zeta) \leq V_{tol}$$

Constraint is that vibration less than V_{tol}

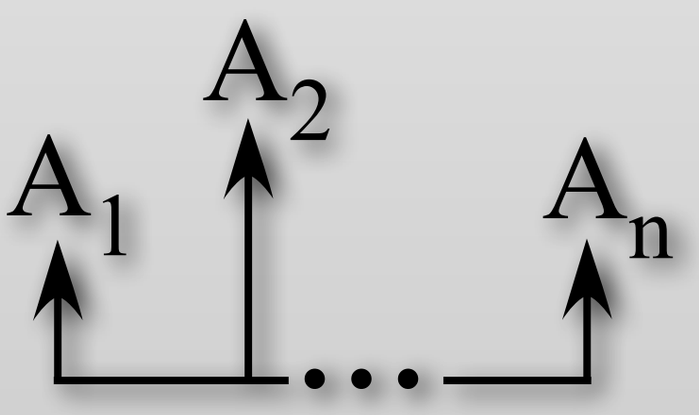
Impulse Amplitude Constraints

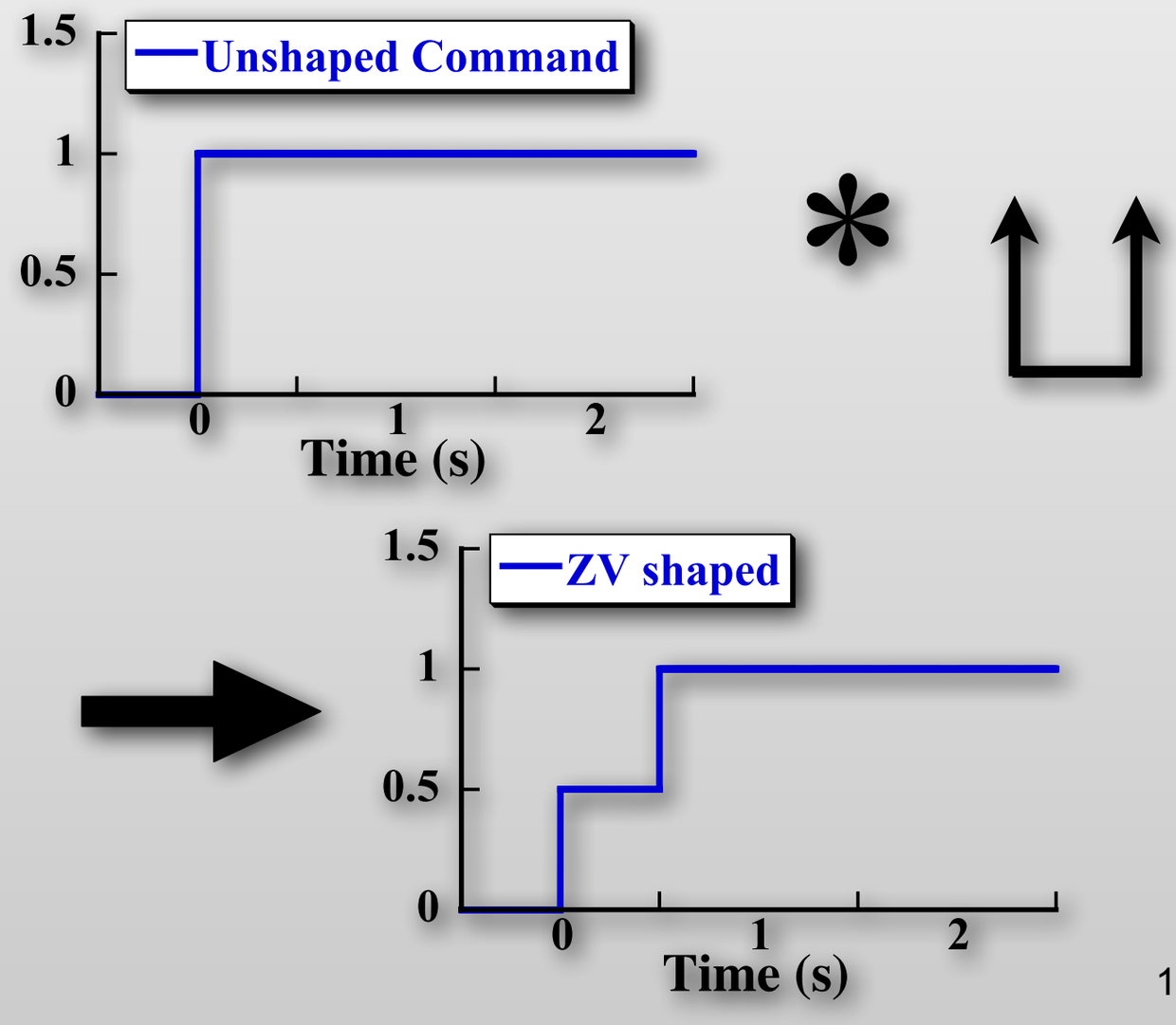
<u>All impulses sum to one</u>

$$\sum A_i = 1, \qquad i = 1, \ldots, n$$

<u>Positive Shapers</u>

$$\begin{aligned} A_i &\geq 0 \\ i &= 1, \ldots, n \end{aligned}$$





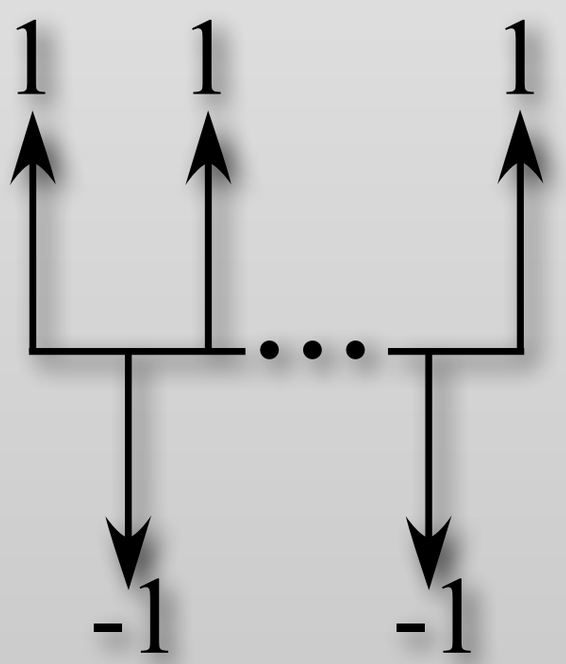
Impulse Amplitude Constraints

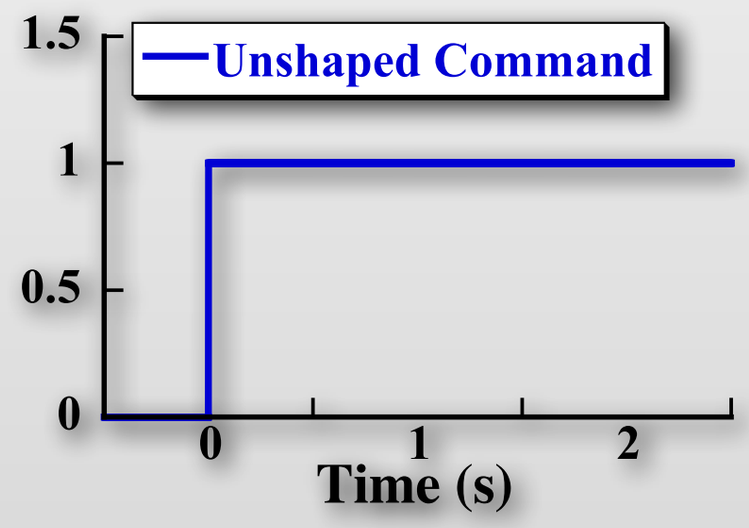
<u>All impulses sum to one</u>

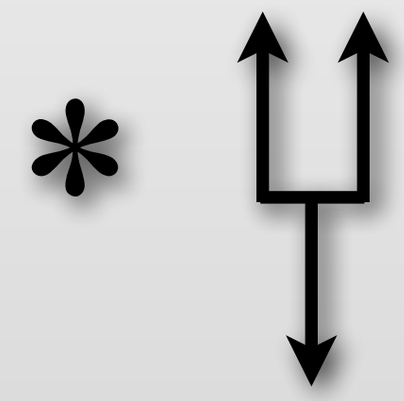
$$\sum A_i = 1, \qquad i = 1, \ldots, n$$

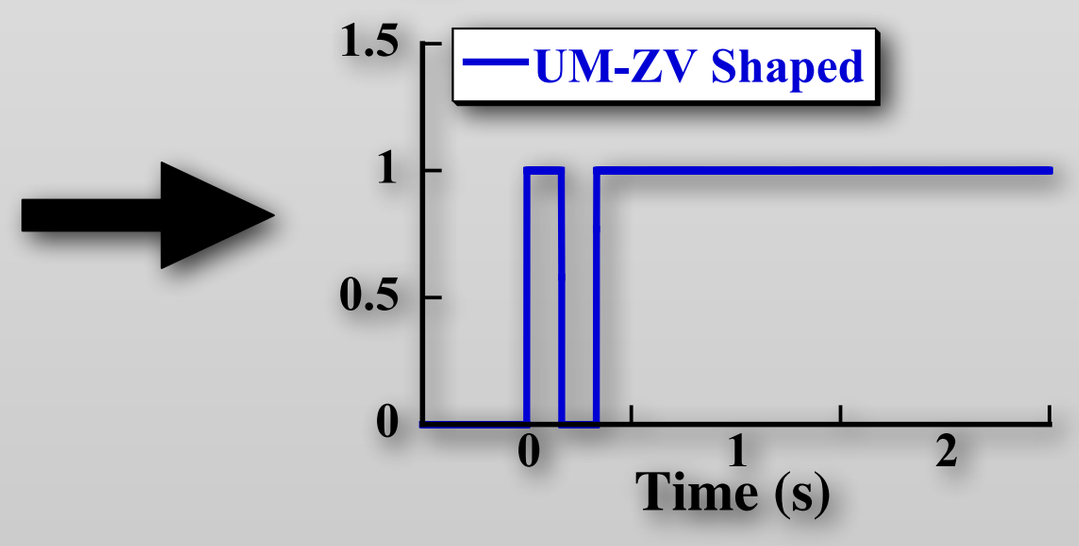
<u>Unity Magnitude (UM)</u>

$$\begin{aligned} A_i &= (-1)^{i+1} \\ i &= 1, \ldots, n \end{aligned}$$









Impulse Amplitude Constraints

<u>All impulses sum to one</u>

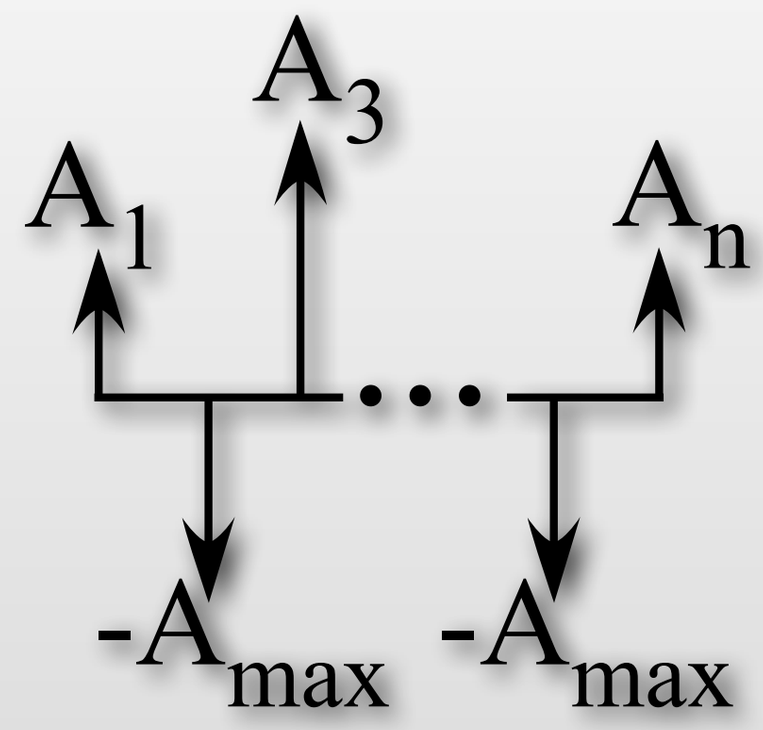
$$\sum A_i = 1, \qquad i = 1, \ldots, n$$

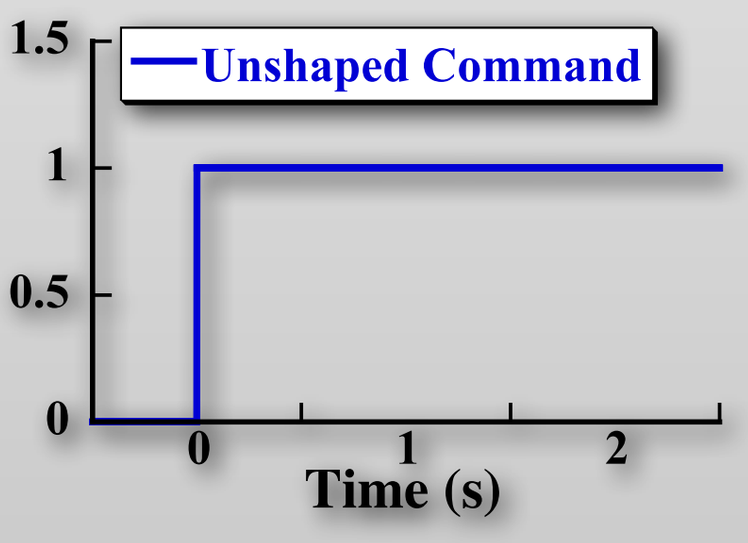
<u>Specified Negative Amplitude (SNA)</u>

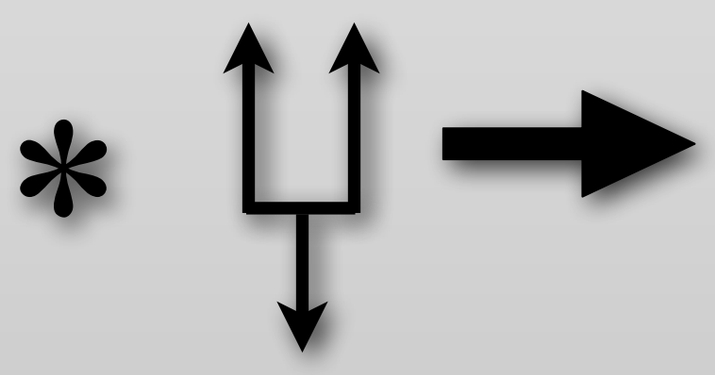
$$0 < A_i \leq 1 \qquad \text{when } i \text{ is odd}$$

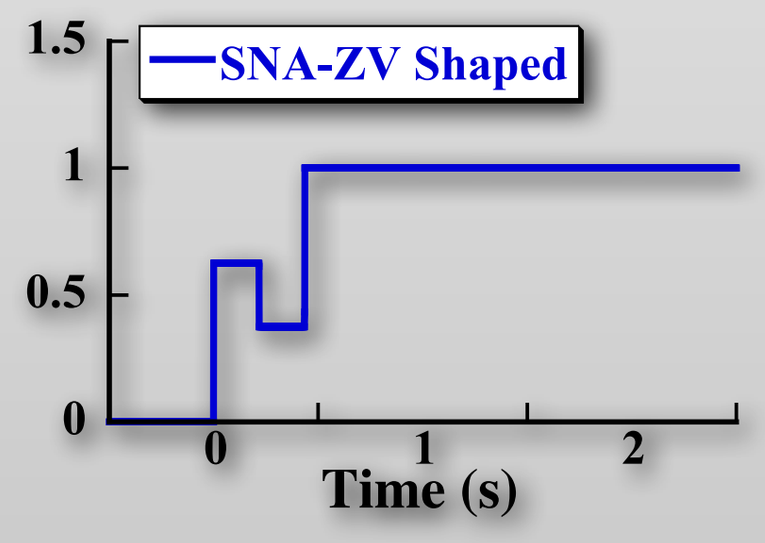
$$A_i = -A_{max} \qquad \text{when } i \text{ is even}$$

$$0 \leq \sum_{i=1}^{k} A_i \leq 1 \quad k = 1, \ldots, n$$









Example Closed-Form Shapers

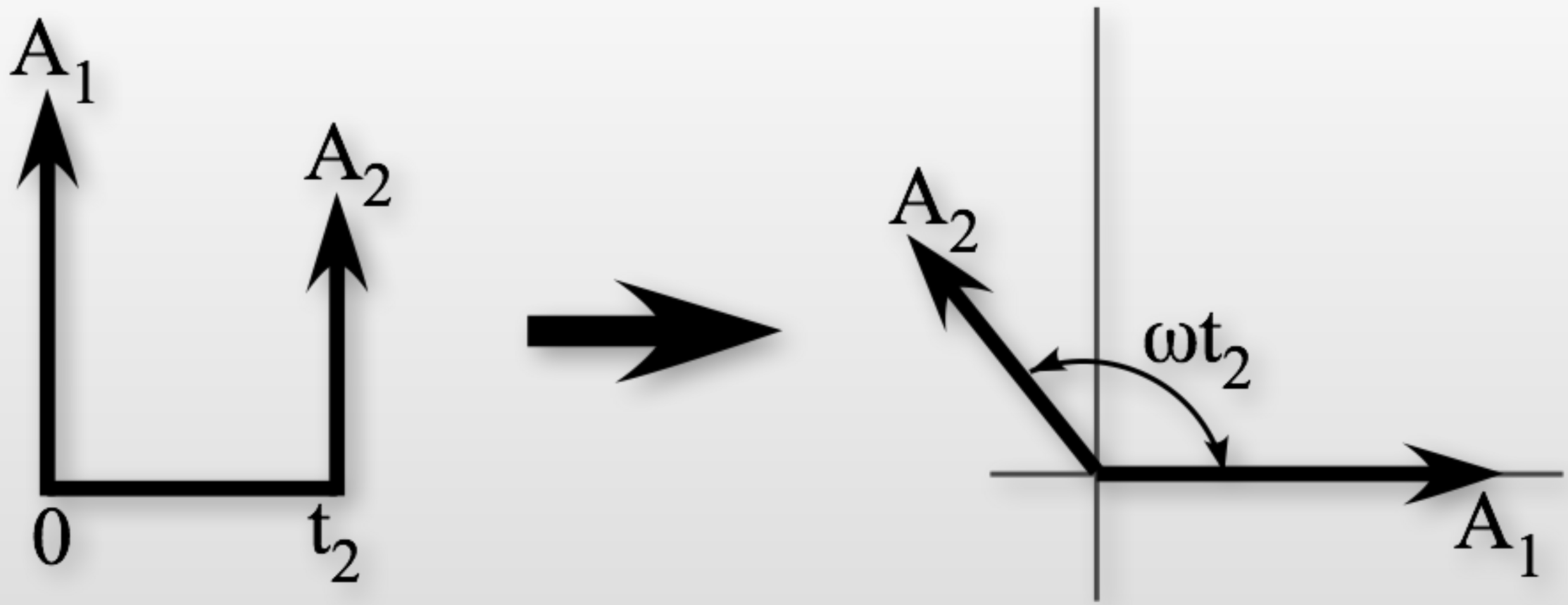
$$\mathrm{ZV} \equiv \begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \frac{1}{1+K} & \frac{K}{1+K} \\ 0 & \frac{\tau_d}{2} \end{bmatrix}$$

$$\mathrm{ZVD} \equiv \begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \frac{1}{1+2K+K^2} & \frac{2K}{1+2K+K^2} & \frac{K^2}{1+2K+K^2} \\ 0 & \frac{\tau_d}{2} & \tau_d \end{bmatrix}$$

where

- τ_d is the damped vibration period
- $K = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$

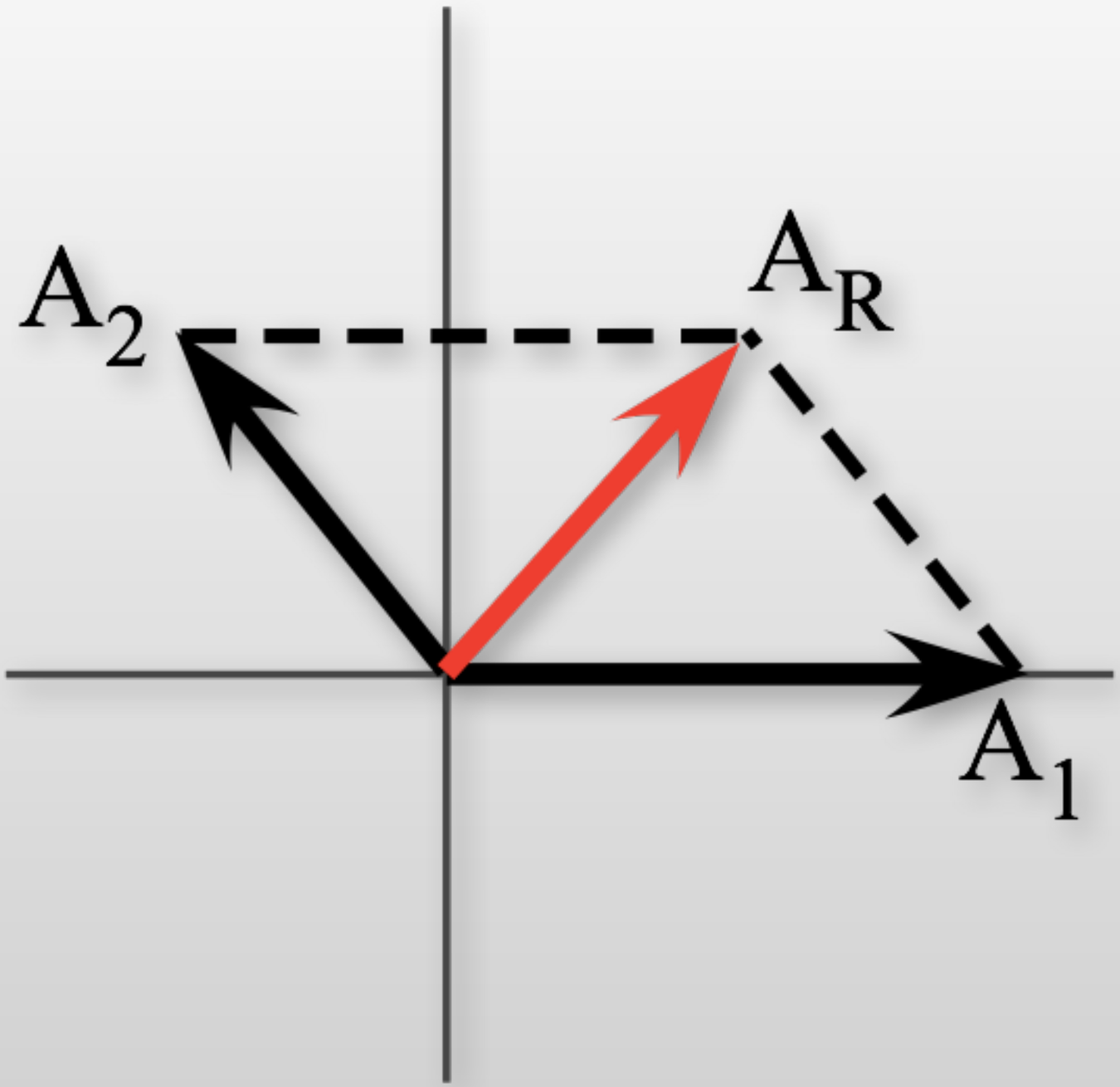
Vector Diagrams



- Vector magnitude = $\left|A_i\right|$
- Vector angle, $\theta_i = \omega t_i$

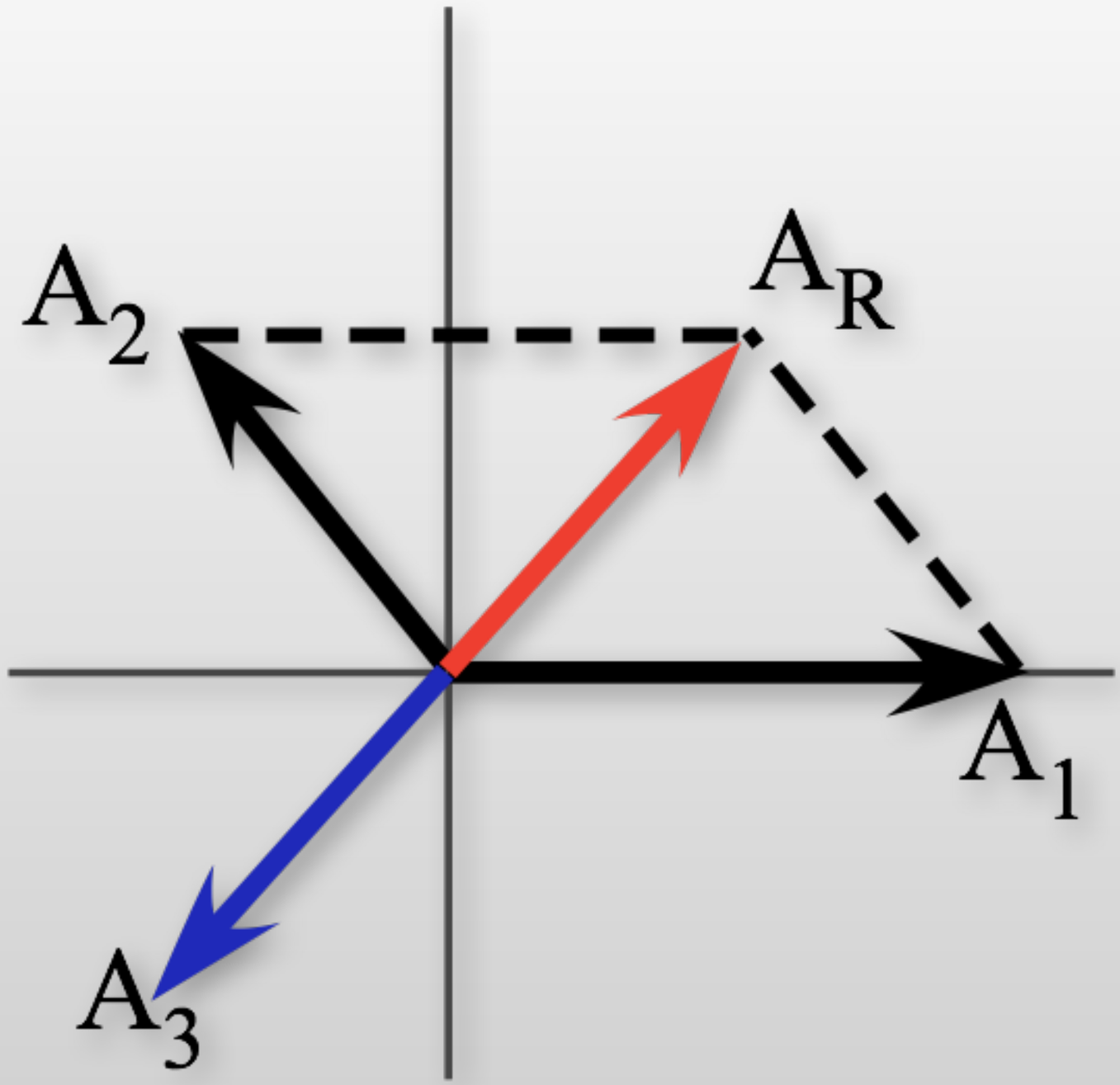
Vector Diagram Use

- Resultant vector, A_R, is proportional to vibration



Vector Diagram Use

- Resultant vector, A_R, is proportional to vibration
- Can add a 3rd impulse to eliminate vibration



Vector Diagram Use

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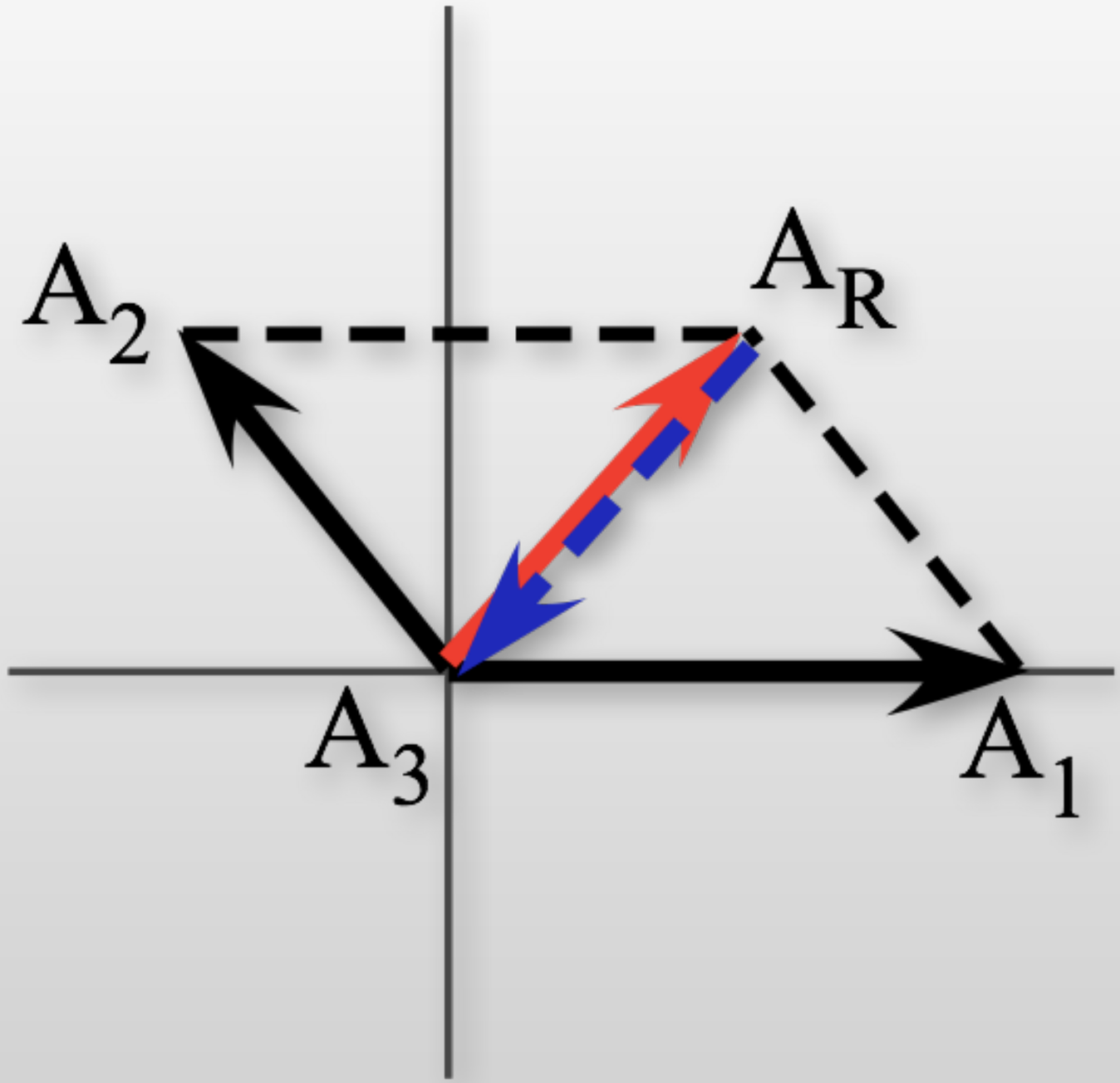
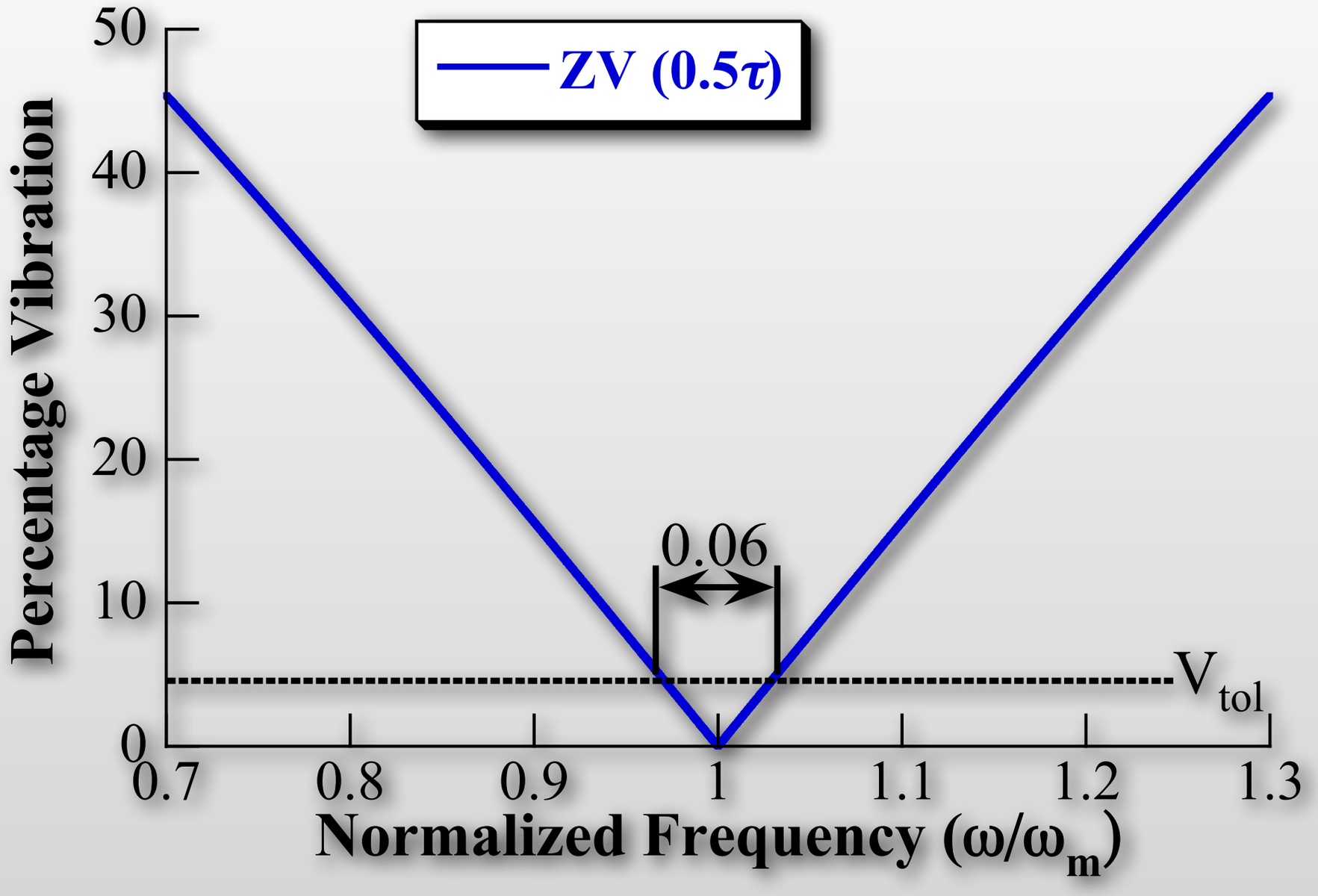
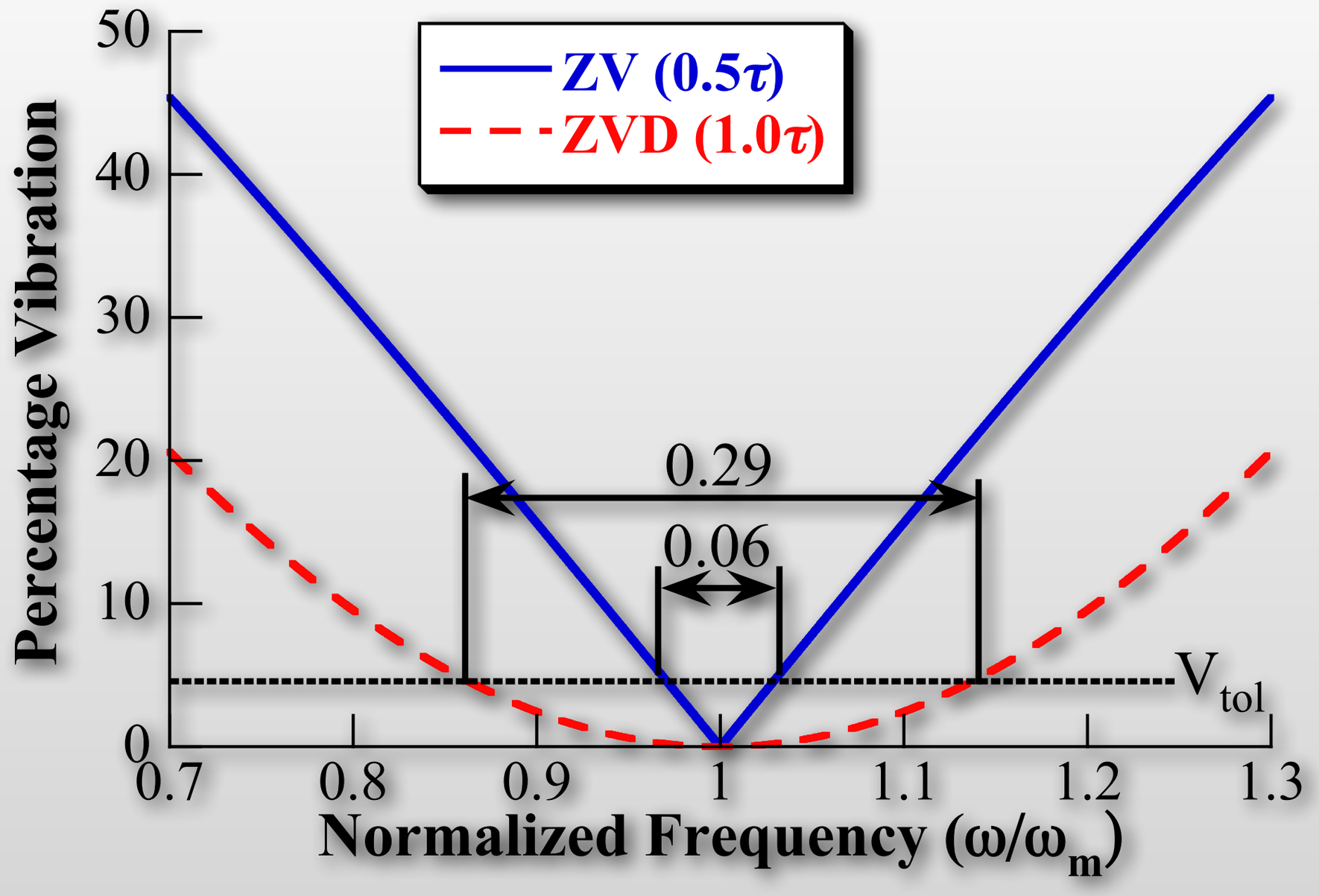


Measuring Robustness



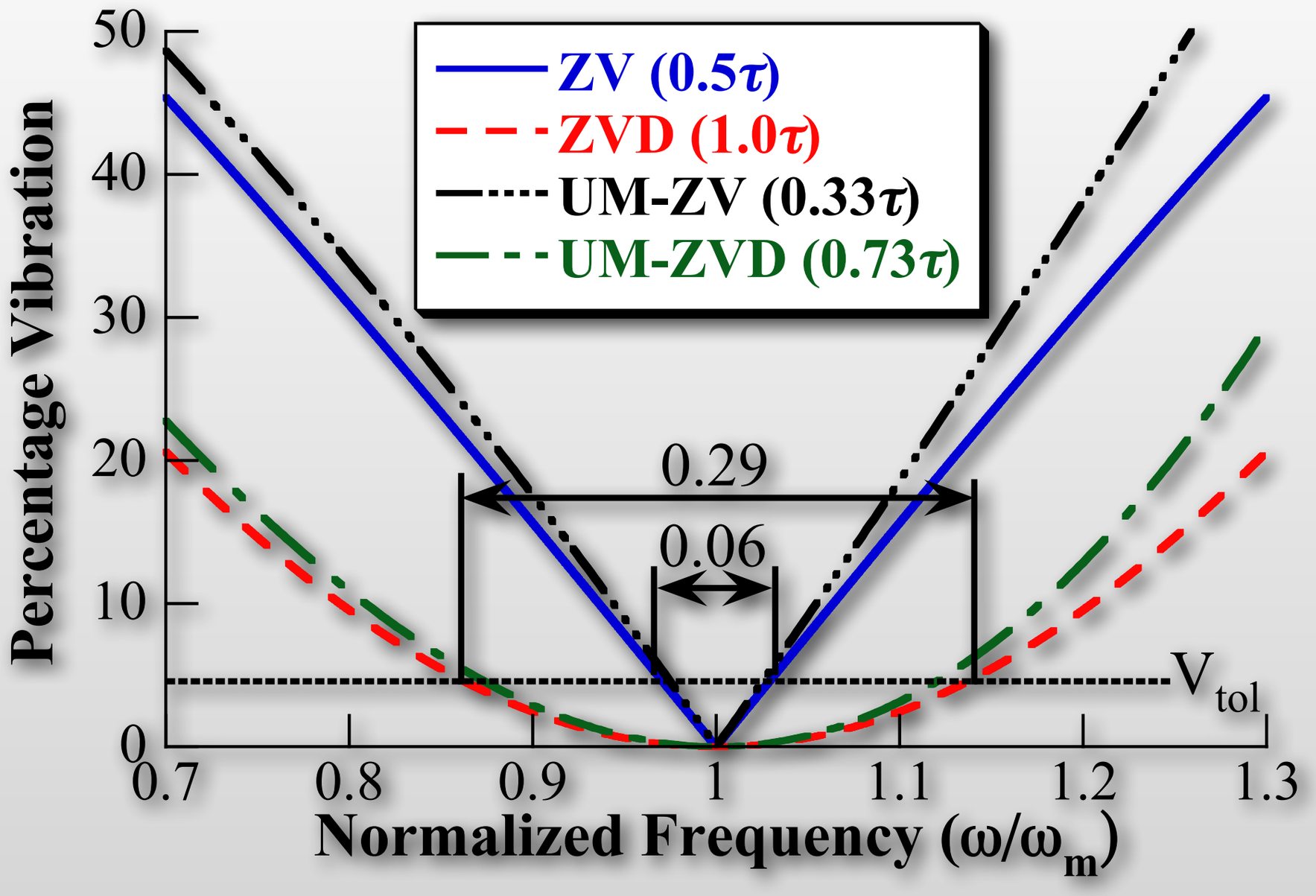
Insensitivity - the width of a Sensitivity curve where vibration remains under V_{tol}

Measuring Robustness



Insensitivity - the width of a Sensitivity curve where vibration remains under V_{tol}

Measuring Robustness

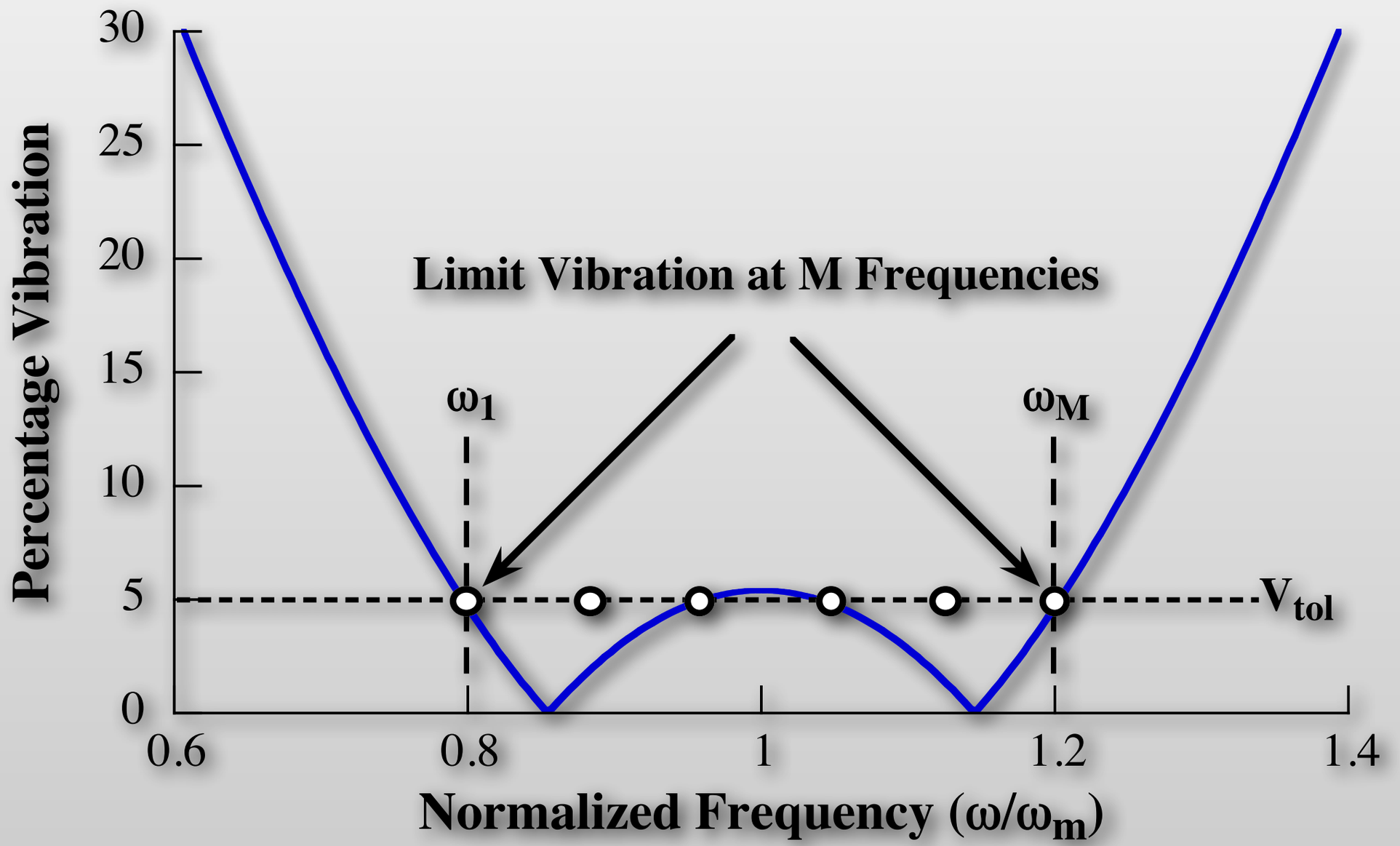


Insensitivity - the width of a Sensitivity curve where vibration remains under V_{tol}

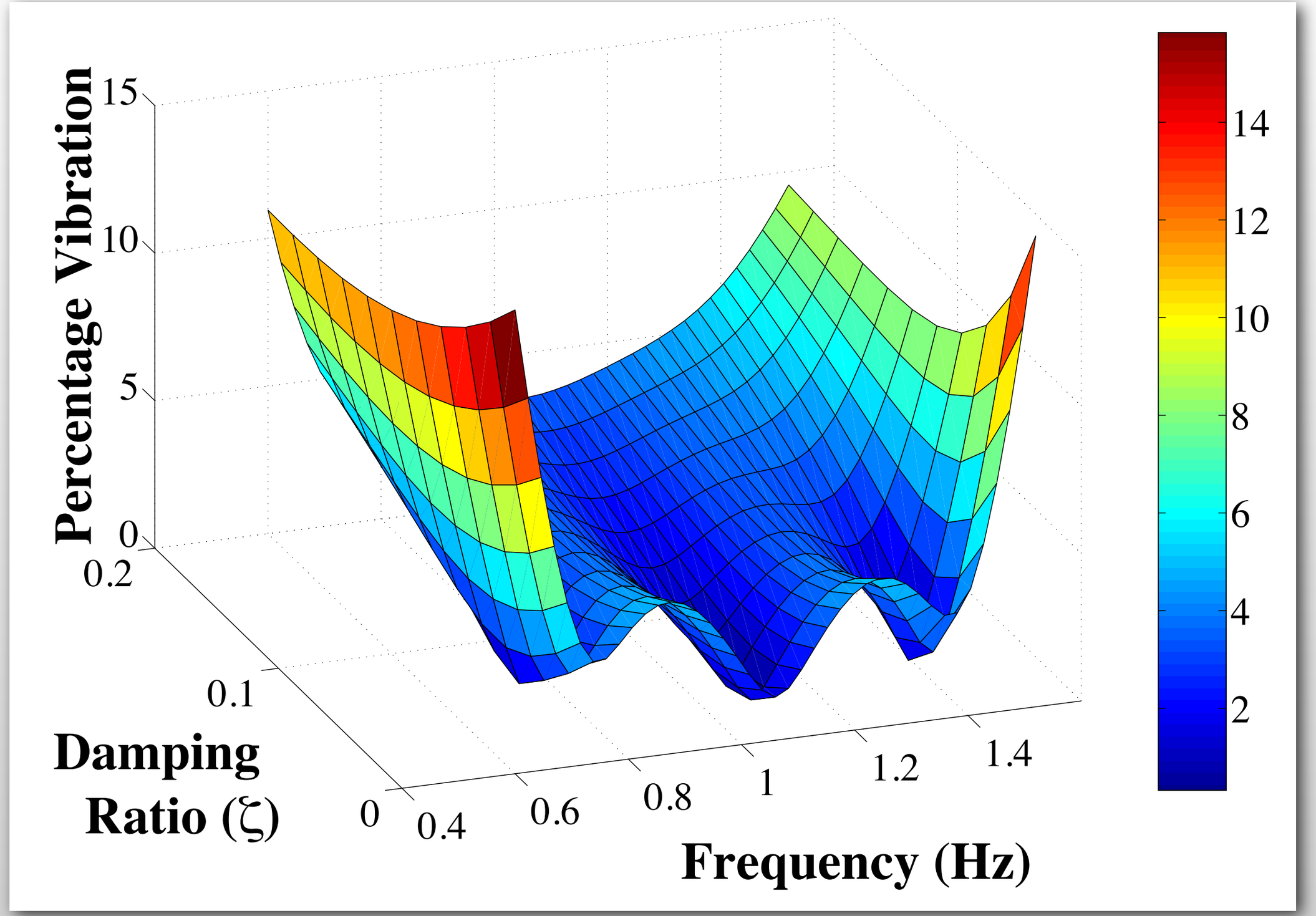
Specified Insensitivity (SI) Constraints

$$V(\omega_k, \zeta) = e^{-\zeta \omega_k t_n} \sqrt{[C(\omega_k, \zeta)]^2 + [S(\omega_k, \zeta)]^2} \leq V_{tol}$$

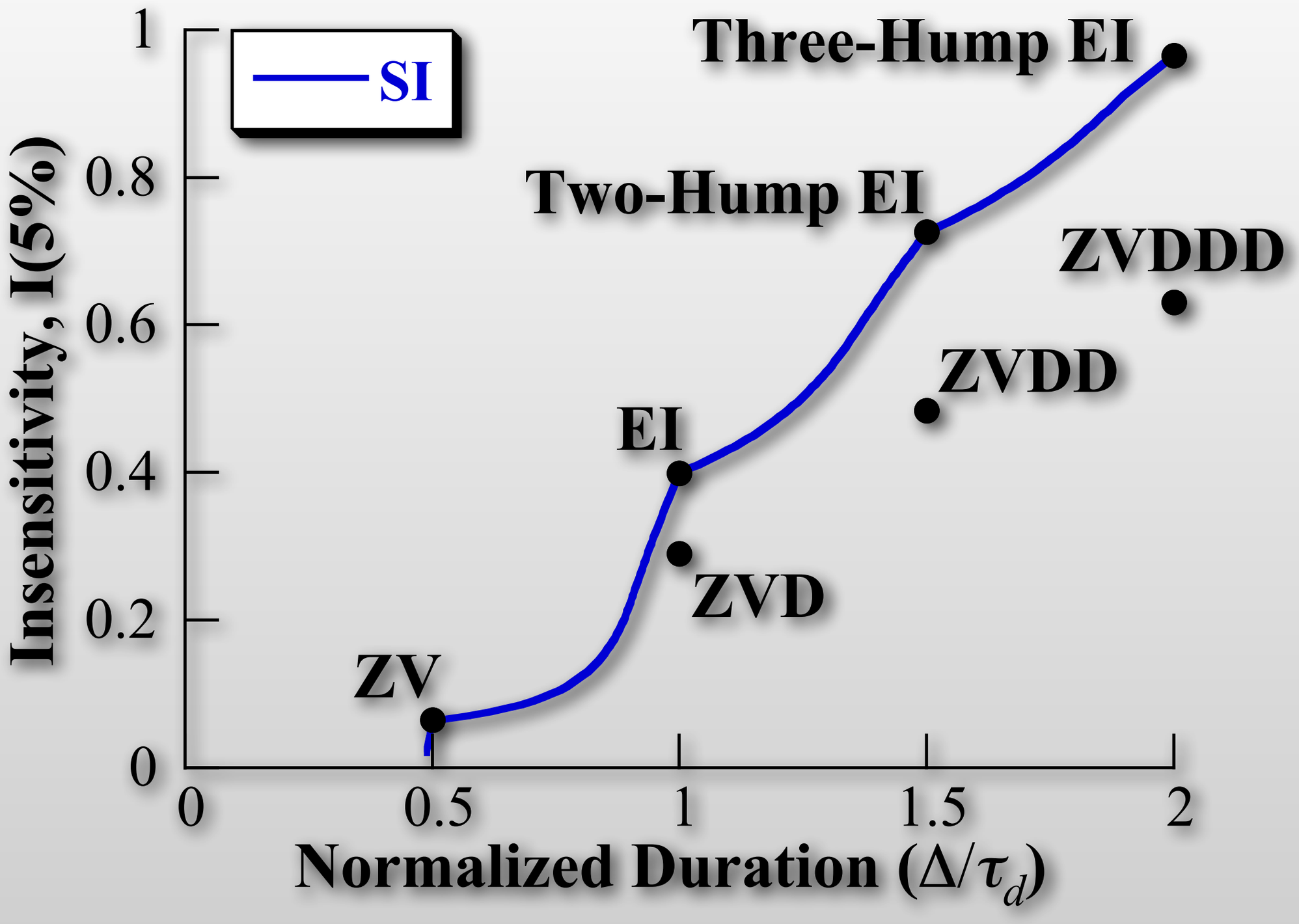
$$k = 1, \ldots, M$$



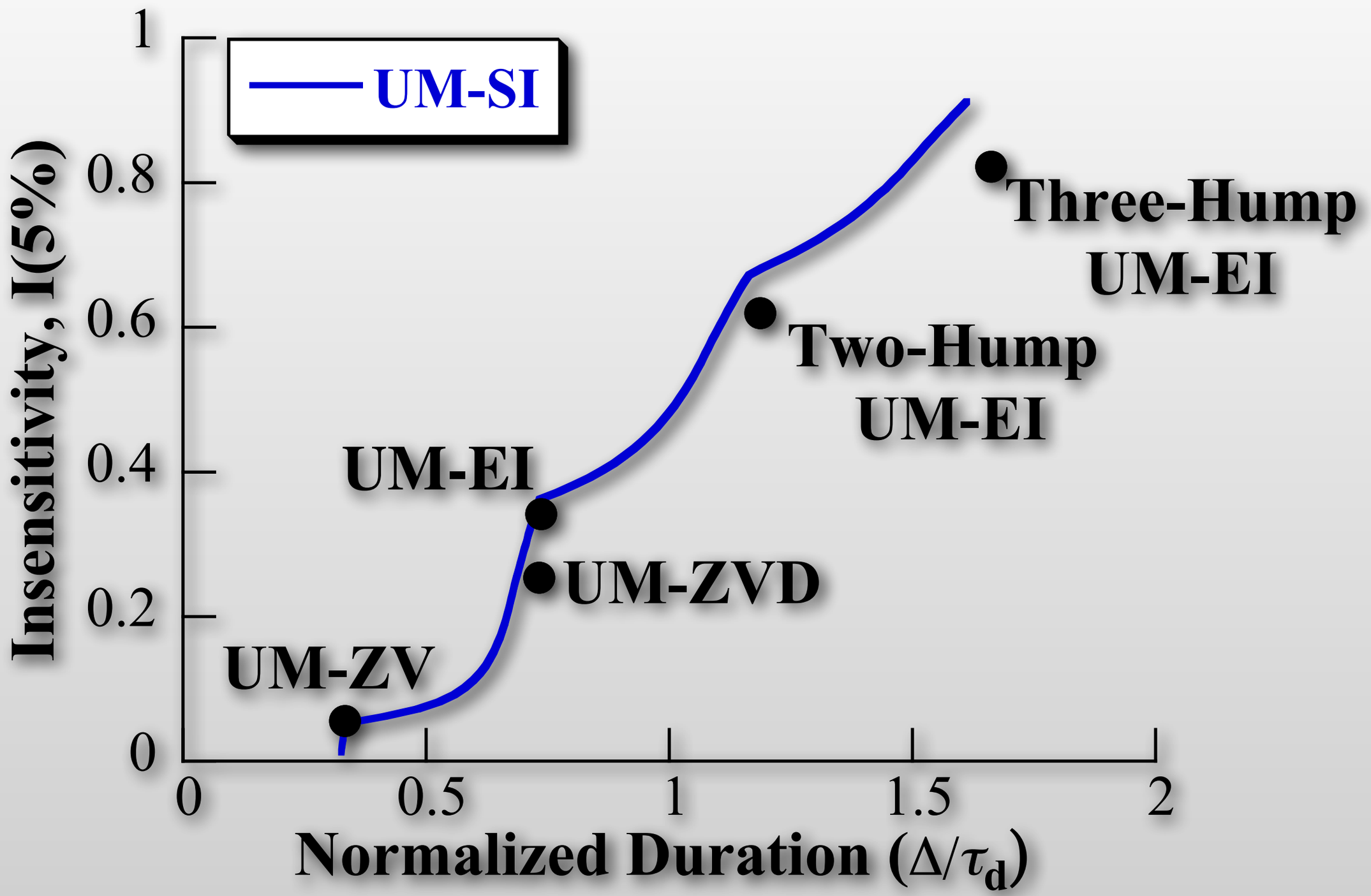
Damping Matters, but less than Freq.



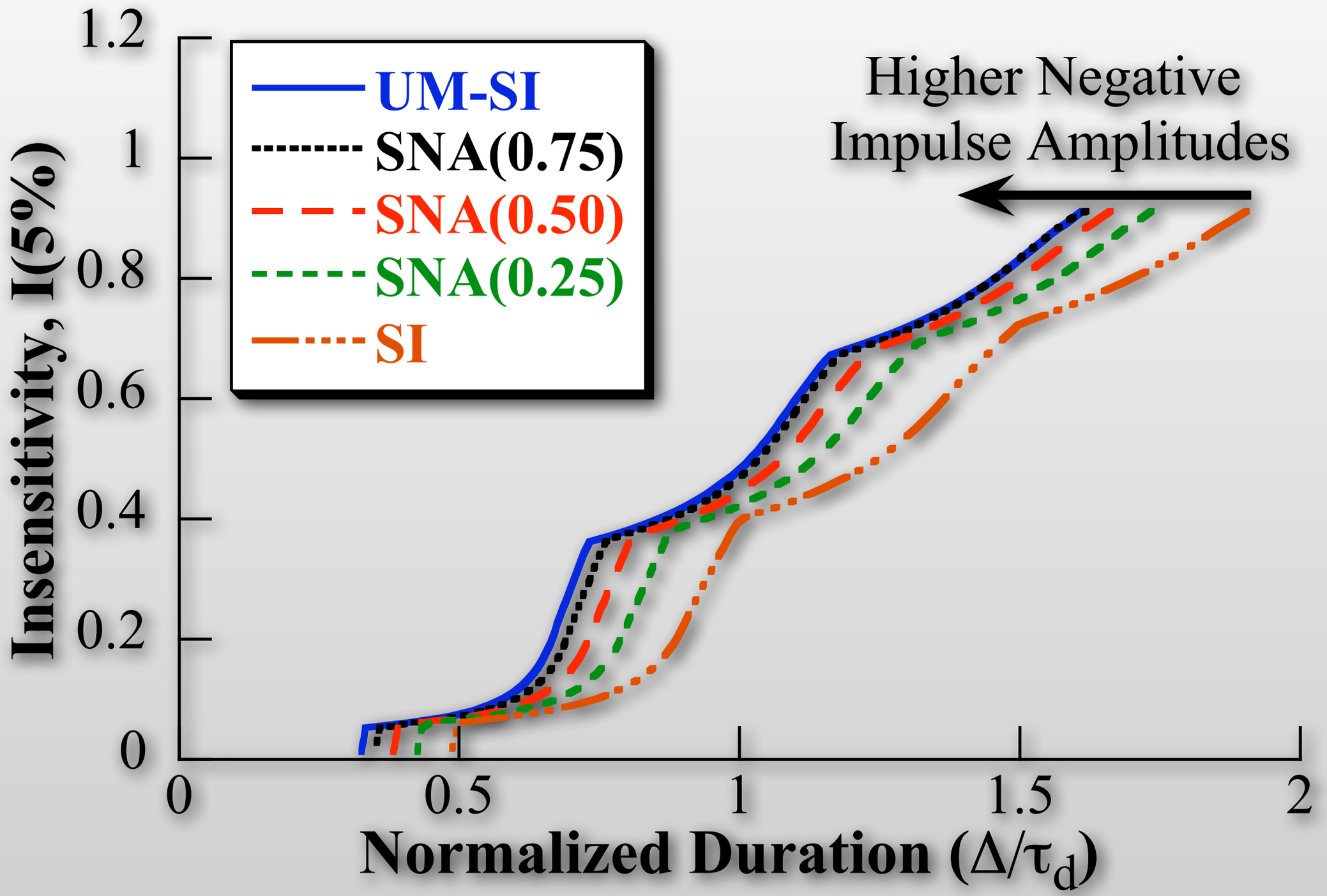
Shaper Duration v. Insensitivity



Shaper Duration v. Insensitivity



Shaper Duration v. Insensitivity



General Design Procedure

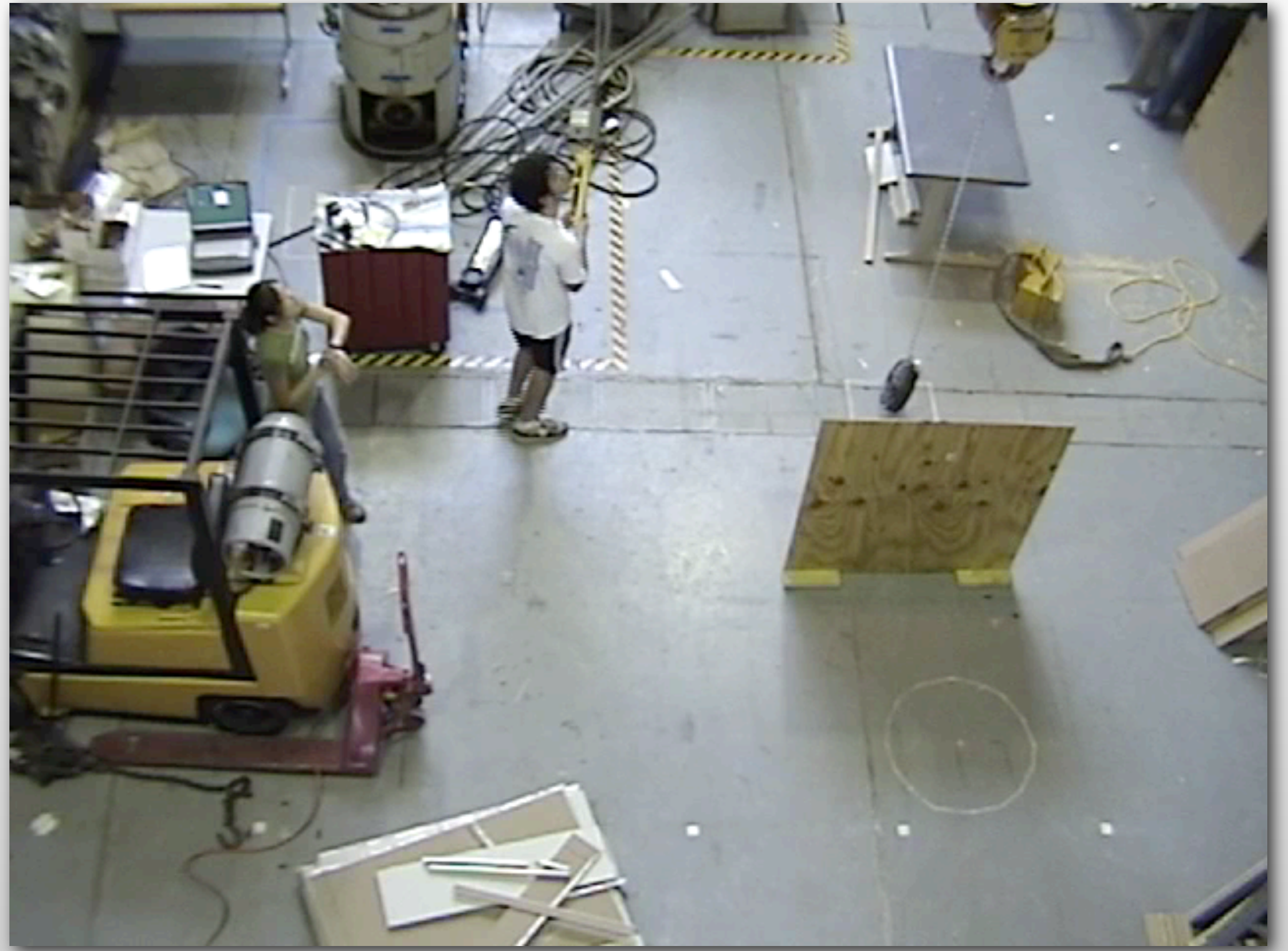
- Determine nominal system parameters
  - Dominant vibration frequencies (often only 1)
  - Associated damping ratios

- Determine variation of parameters
  - How must robustness is needed?

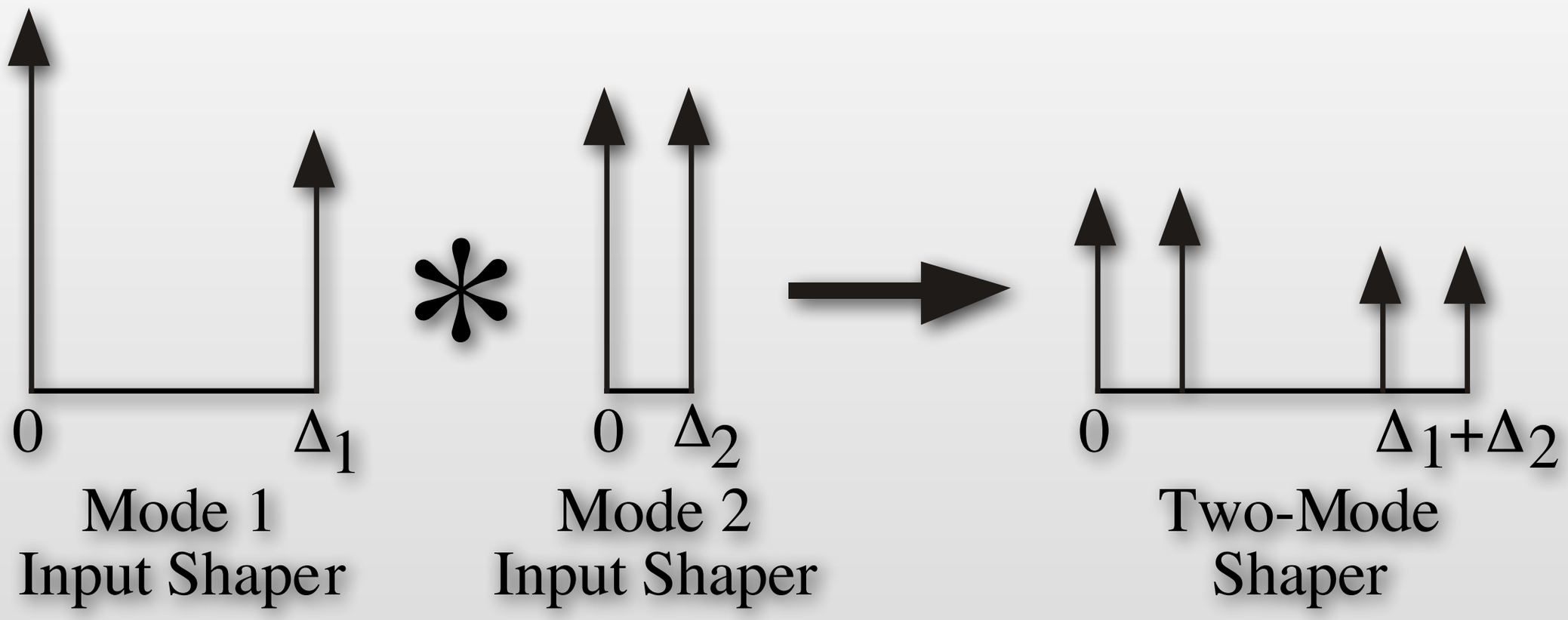
- Pick shaper or shapers to use

- Plug in frequency and damping to get:
  - Impulse amplitudes
  - Impulse times

Example Multi-mode Crane Oscillation

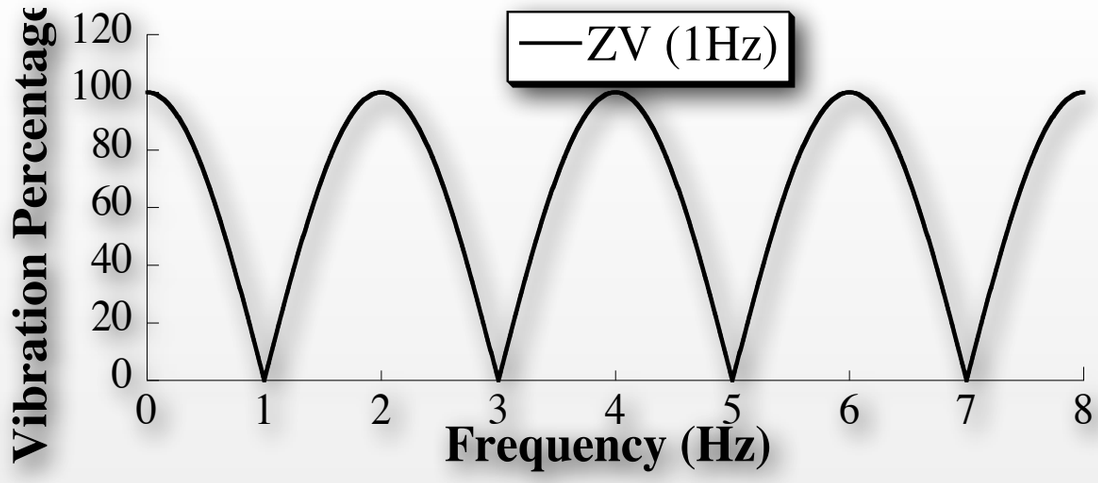


Convolved Two-Mode Shaper



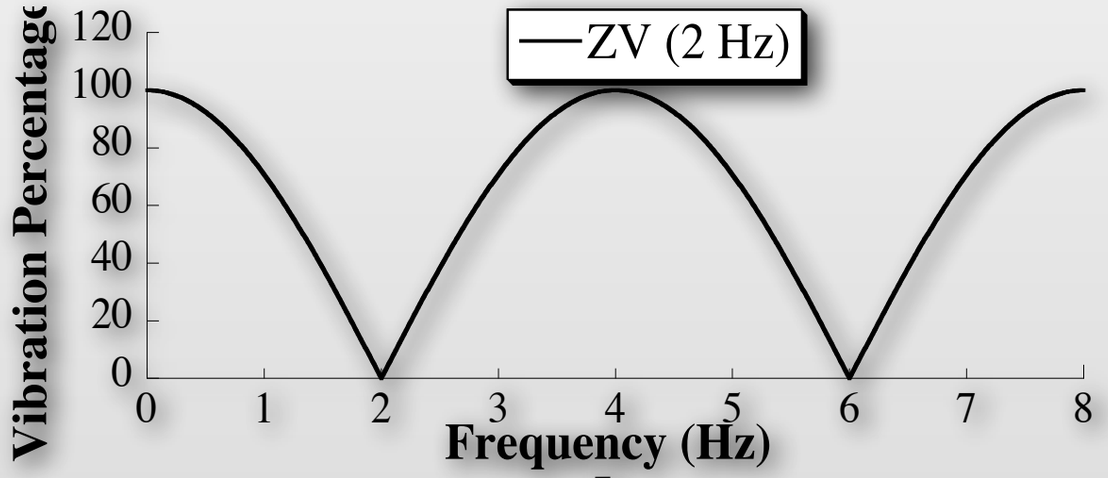
- Design shaper for each mode, then convolve to get a shaper that eliminates both modes

ZV Shaper for 1 Hz

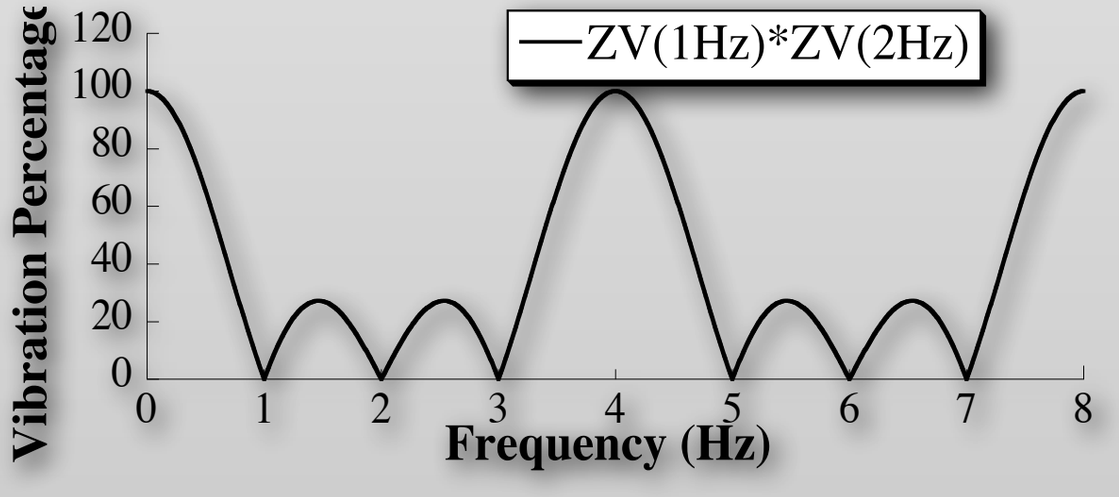


*

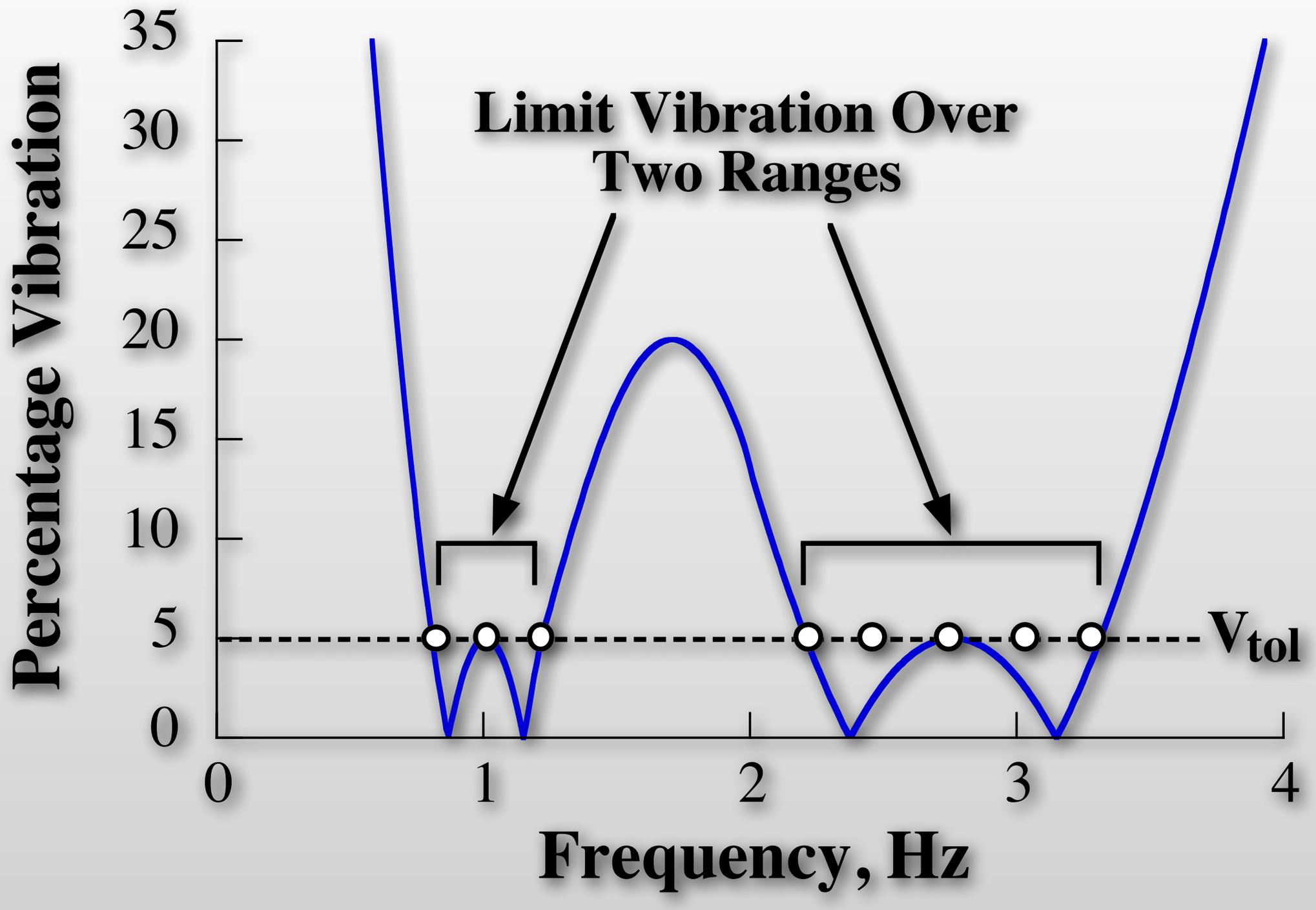
ZV Shaper for 2 Hz



ZV Shaper for 1 Hz and 2 Hz



Multi-mode SI Shapers



- Solve for all modes simultaneously → faster shapers

Example Multi-mode Crane Oscillation