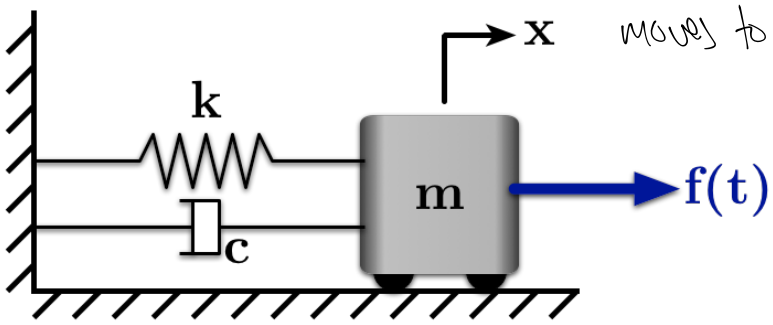
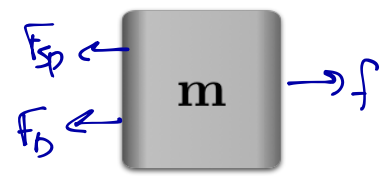
Example

Goal: Design controller to define $f(t)$ such that mass moves to desired position x_d



FBD



$$m\ddot{x} = f - F_{sp} - F_D$$
$$= f - kx - c\dot{x}$$

$$m\ddot{x} + c\dot{x} + kx = f$$

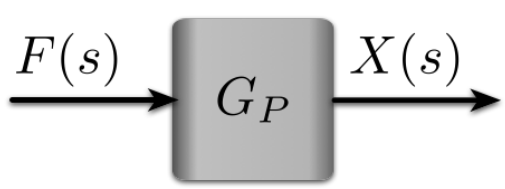
$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \frac{f}{m} \rightarrow \ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \frac{f}{m} \rightarrow (s^2 + 2\zeta\omega_n s + \omega_n^2)\overline{X} = \frac{1}{m}F$$

$$\frac{\overline{X}}{F} = \frac{1/m}{s^2 + 2\zeta\omega_n s + \omega_n^2} = G_P$$

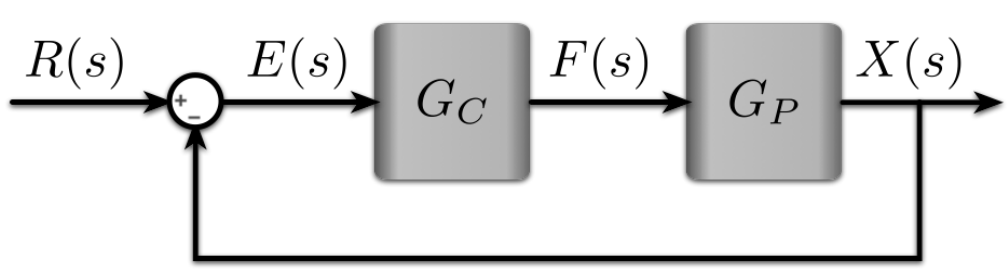
Assume "light" damping ...

Q: what are the poles of this system? $\rightarrow -\zeta\omega_n \pm i\omega_d$

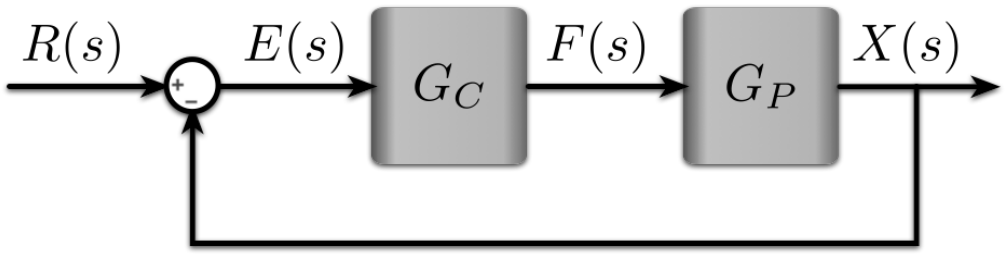
Q: What's its current block diagram?



Q: If we want to create a feedback controller for this system, what does it become?



Example (cont.)



$$\frac{X}{F} = \frac{1/m}{s^2 + 2\zeta\omega_n s + \omega_n^2} = G_P$$

Let's start by trying just a proportional controller.

$$G_c = k_p$$

So,

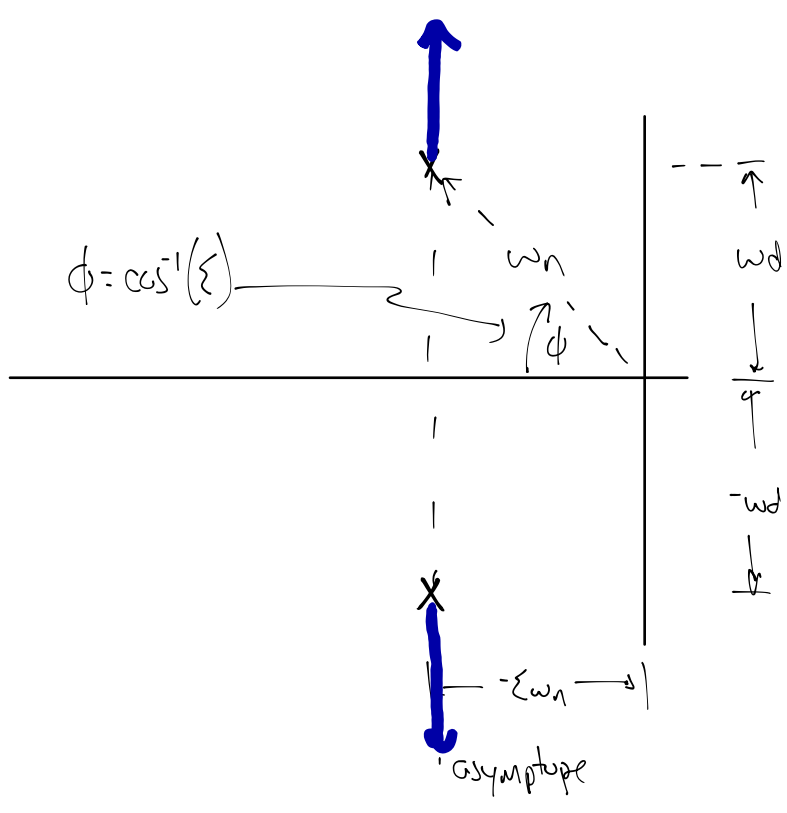
$$\frac{X}{R} = \frac{G_c G_p}{1 + G_c G_p} = \frac{k_p/m}{s^2 + 2\zeta\omega_n s + \left(\omega_n^2 + k_p/m\right)}$$

Q: What is the prop. controller going to do?

Let's quickly plot a Root Locus to confirm/inspect further

$$G_c G_p = \frac{k_p/m}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

poles at $-\zeta\omega_n \pm i\omega_d$



- Number of separate loci? = 2
- Asymptotes:

$$\sigma_A = \frac{-2\zeta\omega_n}{2} = -\zeta\omega_n$$

$$\phi_A = \left(\frac{2k+1}{n-m}\right)180^\circ \quad k = 0, 1, \ldots, n-m-1 = 0, 1$$

$$= \left(\frac{2k+1}{2}\right)180^\circ = 90^\circ, 270^\circ = \pm 90^\circ$$

- Breakaway angle

$$= 180^\circ - (\Sigma \text{ angle to all other poles}) + (\Sigma \text{ angle to all other zeros})$$

$$= 180^\circ - 90^\circ = 90^\circ$$

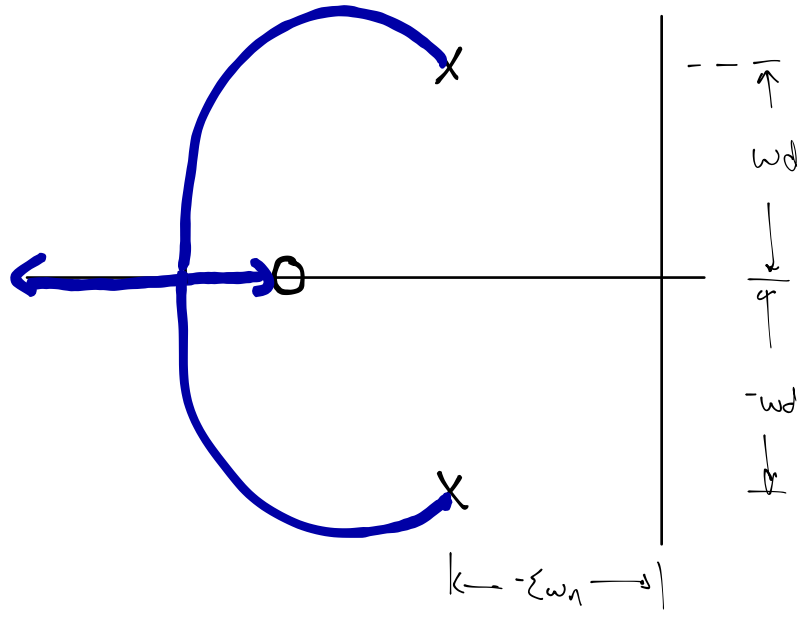
So, response is always oscillatory ... and
- becomes <u>less</u> damped with increasing k_p
- Settling time is approx constant

Example (cont.)

Let's try PD control, so $G_c = k_p + k_d s$ and $G_c G_p = \dfrac{(k_p + k_d s)\,1/m}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$$\frac{X}{R} = \frac{(k_p + k_d s)\,1/m}{s^2 + \left(2\zeta\omega_n + \frac{k_d}{m}\right)s + \left(\omega_n^2 + \frac{k_p}{m}\right)}$$

← Now, we can affect damping in closed-loop response by our choice of k_d



OL poles at $-\zeta\omega_n \pm j\omega_d$

OL zero at $-\dfrac{k_p}{k_d}$

$SL = 2$

$$\sigma_A = \frac{-2\zeta\omega_n + \frac{k_p}{k_d}}{1} = -2\zeta\omega_n + \frac{k_p}{k_d}$$

$$\phi_A = \left(\frac{2k+1}{1}\right)180^\circ \quad k: 0,1 \Rightarrow \phi_A = 180^\circ$$

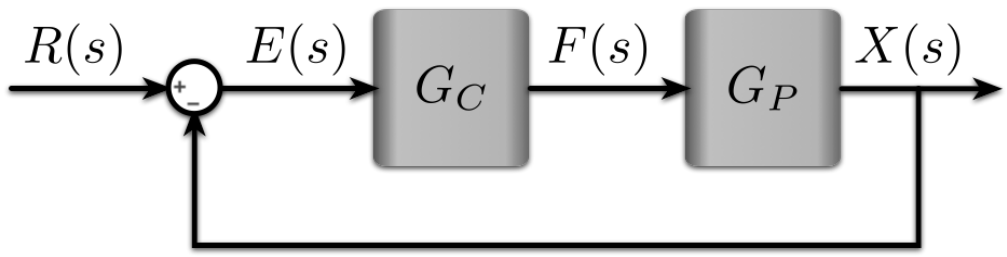
breakaway points

Now we can place our poles based on perf. measures on t_s, t_p, M_p, etc.

Q: Are there any alternatives to this??

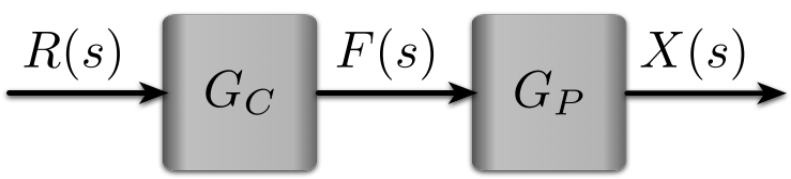
of course... Let's look at the block diagram again

Plant/Model Inversion

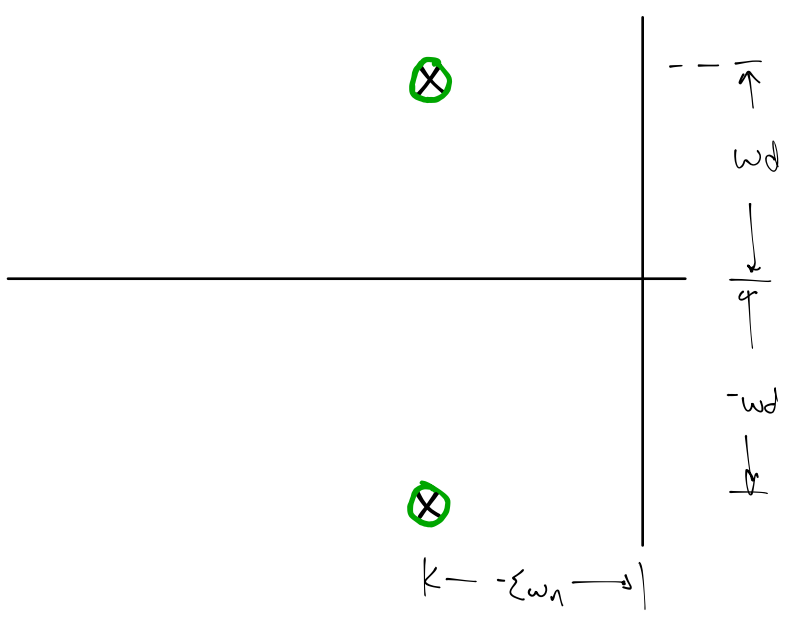


Q: What if $G_c = G_p^{-1}$?

$G_c G_p$ is then $G_p^{-1} G_p = 1$ ← We could make the system do exactly what we want!!!



$$\frac{X}{R} = G_c G_p = G_p^{-1} G_p = 1$$



X - poles from G_p

O - zeros from $G_c = G_p^{-1}$

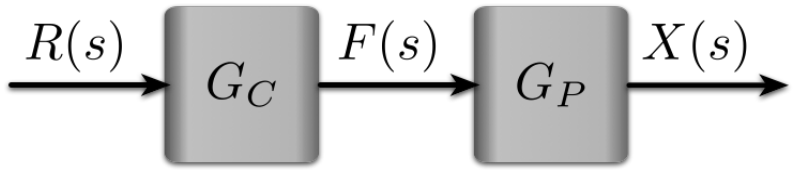
Q: Any problems with this?

- To be 1, need to know the model/system <u>exactly</u> ← which we can never do!
- Assumes F can grow as large as needed ← which is never true!

So, we can't (blindly) do model inversion... However...

Pole/zero Cancellation

Let's look back at:



Pick $G_c = A_1 + A_2 e^{-t_2 s}$ where $A_2 = A_1 e^{-\pi\zeta\omega_n/\omega_d}$ and $t_2 = \frac{\pi}{\omega_d}$

$$G_c = A_1 + A_1 \exp\left[\frac{-\pi}{\omega_d}(\zeta\omega_n + \sigma + i\omega)\right]$$ ← evaluating a $s = \sigma + i\omega$

We'd like the output of G_c to be 0 at the flexible modes of the system
(Vibration will not be excited ... like tuning an EQ)

For this to happen, we need

$$\exp\left[\frac{-\pi}{\omega_d}(\zeta\omega_n + \sigma + i\omega)\right] = -1$$

Let's break into real and imaginary parts

$$\underbrace{\exp\left[\frac{-\pi}{\omega_d}(\zeta\omega_n + \sigma)\right]}_{\text{want: } 1} \cdot \underbrace{\exp\left[\frac{-\pi i\omega}{\omega_d}\right]}_{\text{want: } -1} = -1$$

$$\exp\left[\frac{-\pi}{\omega_d}(\zeta\omega_n + \sigma)\right] = 1 \longrightarrow \text{True when } \sigma = -\zeta\omega_n$$

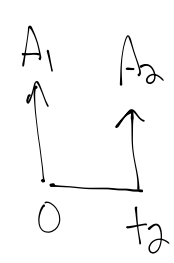
$$\exp\left[\frac{-\pi i\omega}{\omega_d}\right] = -1 \longrightarrow \cos\left(\frac{\pi\omega}{\omega_d}\right) - i\sin\left(\frac{\pi\omega}{\omega_d}\right) = 1 \longrightarrow \text{True when } \omega = \omega_d$$

So, we want $s = -\zeta\omega_n + i\omega_d$ ⟵ exactly matching flexible pole locations

Q: We assumed some form: $G_c = A_1 + A_2 e^{-t_2 s}$?

What does this represent?

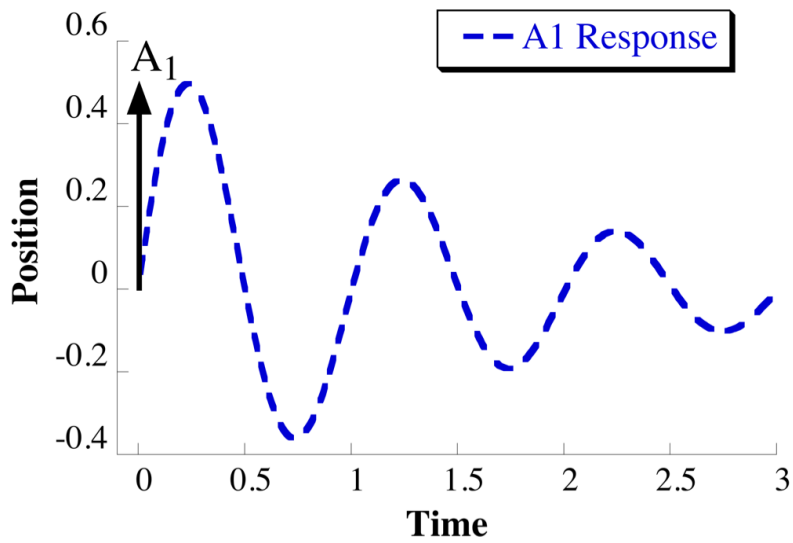
impulse of amp A_1 at $t=0$ — impulse of amp. A_2 — delay of t_2



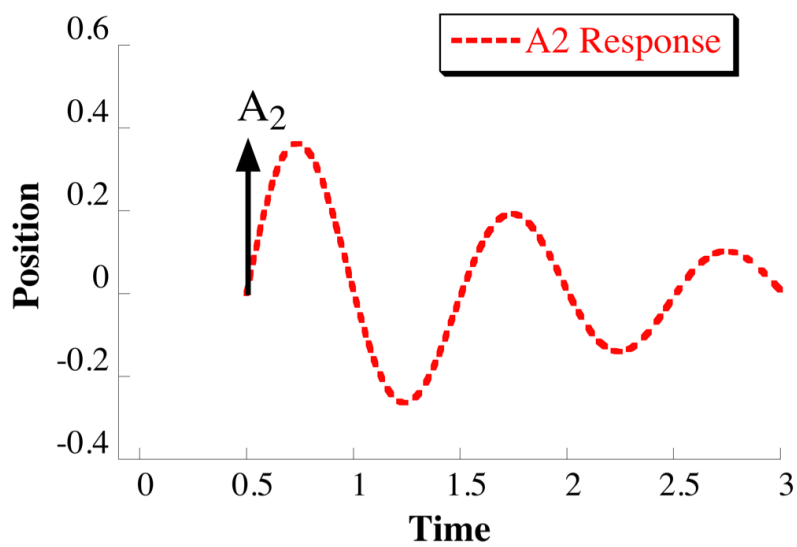
Input Shaping

Q: So, why would this impulse sequence work to create 0 vibration?

Look at response to A_1 at $t=0$

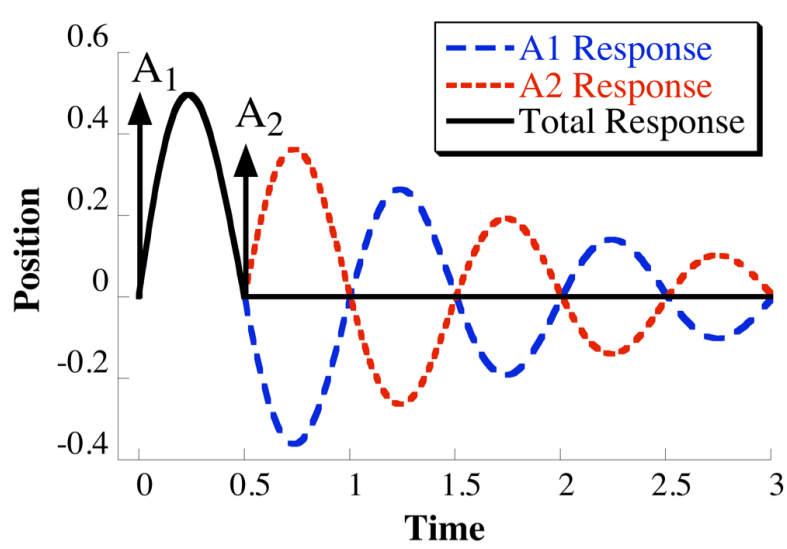


A linear system would respond similarly to impulse A_2 at t_2



If both are issued to the same system sequentially, their responses add due to

Superposition



The total response for all $t > t_2$ is 0!!!

Q: What's the problem with this?

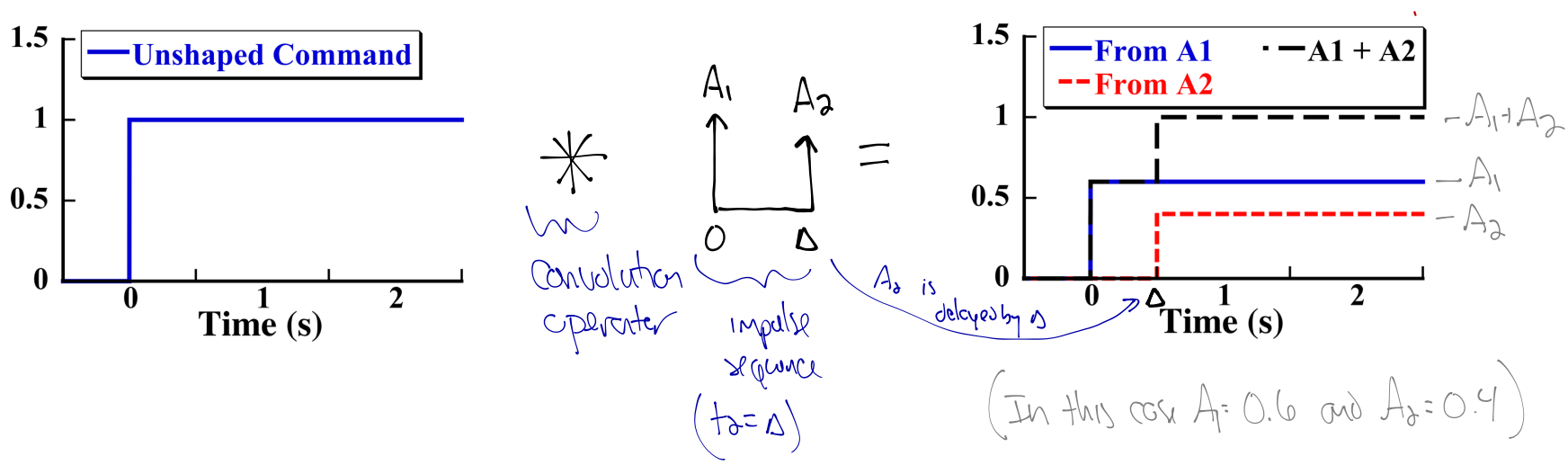
We don't drive systems with impulses

Input Shaping (cont.)

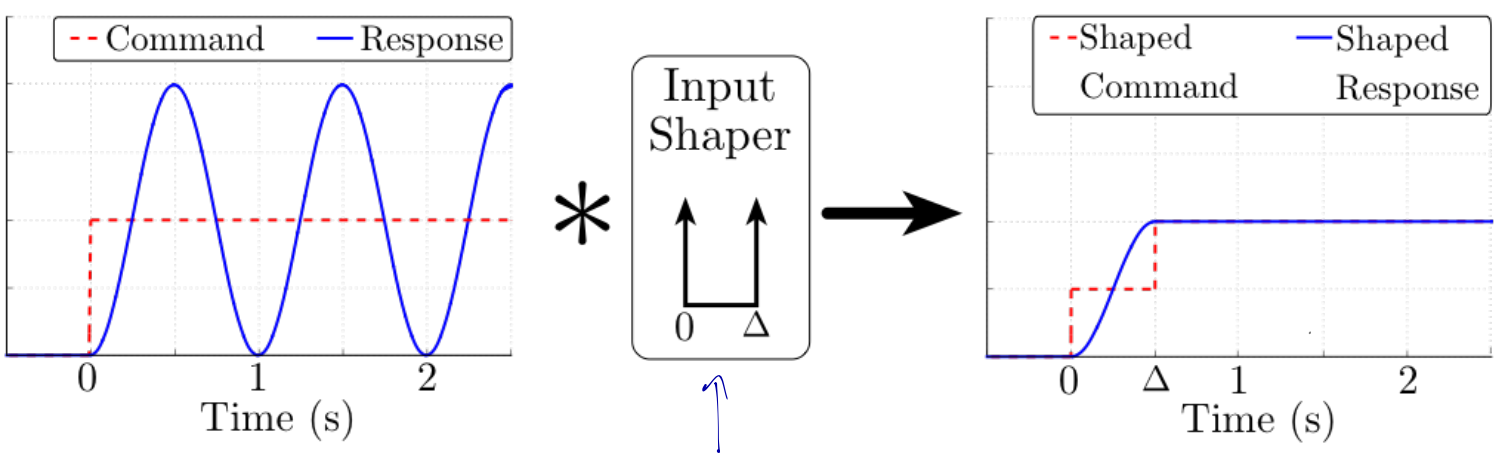
We use convolution. → passes freq. domain (s-plane, included) properties to the result

With impulses, it's easy. Will convolve some original "unshaped" reference command with the sequence of impulses.

Let's look at a step command:



(In this case $A_1 = 0.6$ and $A_2 = 0.4$)



The impulse sequence is called an Input Shaper

This process/control method is called Input Shaping