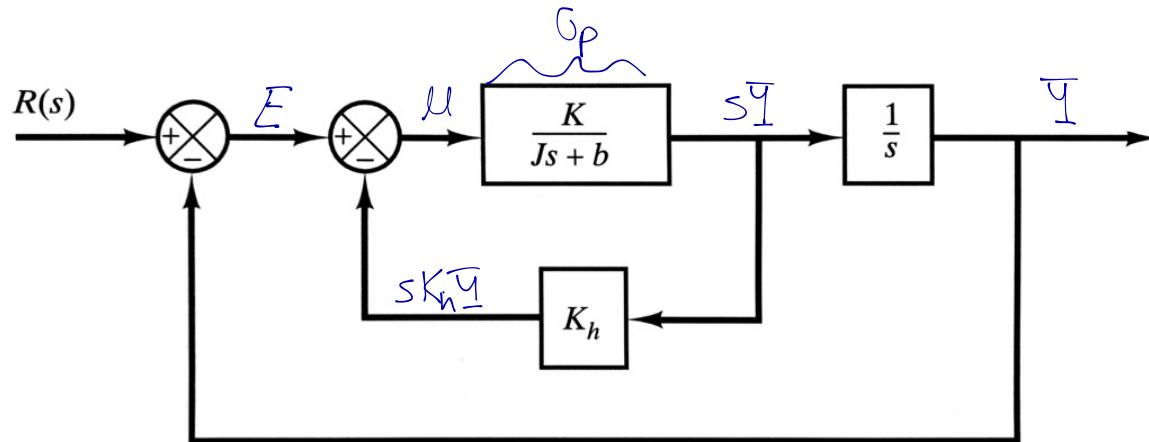


Position Control Systems with Velocity Feedback

So far, we've only fed back the output. We often have the option of using other states for feedback too. One common is the velocity in a position control system.



$$E = R - Y$$

$$U = E - sK_h Y$$

$$sY = G_p U$$

$$sY = G_p (E - sK_h Y)$$

$$sY + sK_h G_p Y = G_p E$$

$$sY + sK_h G_p Y = G_p [R - Y]$$

$$sY + sK_h G_p Y + G_p Y = G_p R$$

$$\frac{Y}{R} = \frac{G_p}{G_p + s(1 + K_h G_p)} = \frac{\frac{K}{Js+b}}{\frac{K}{Js+b} + s\left(\frac{Js+b + K_h K}{Js+b}\right)}$$

$$\text{CLTF} \rightarrow \frac{Y}{R} = \frac{K}{Js^2 + (b + K_k h)s + K}$$

$$\frac{Y}{R} = \frac{\frac{K/J}{s^2 + (\frac{b+KK_h}{J})s + \frac{K}{J}}}{s^2 + (\frac{b+KK_h}{J})s + \frac{K}{J}}$$

$$2\zeta\omega_n \quad \omega_n^2 \rightarrow \omega_n = \sqrt{\frac{K}{J}}$$

$$2\zeta\omega_n = \frac{b + K_k h}{J}$$

$$\zeta = \frac{b + K_k h}{2J\omega_n}$$

Feedback velocity increases damping,

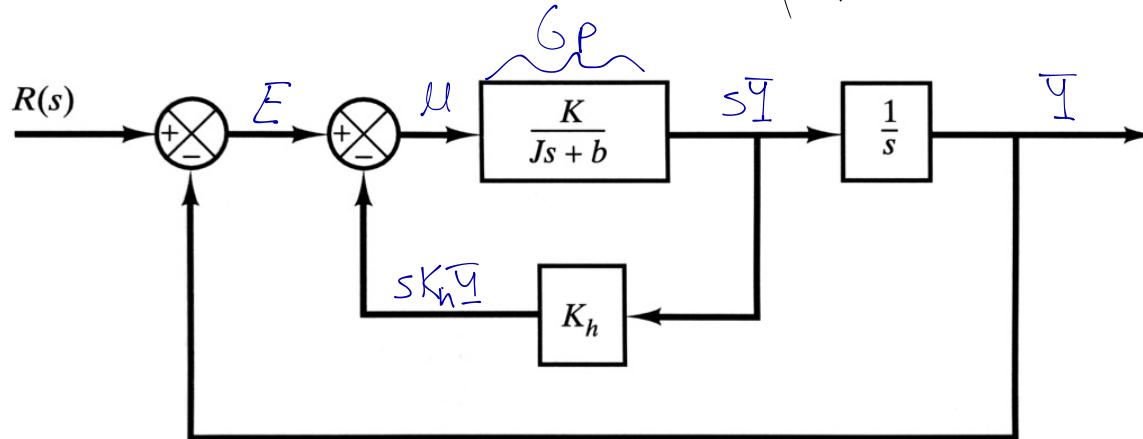
while not increasing natural freq

Position Control Systems with Velocity Feedback Example

For the system below, $J = 2 \text{ kg-m}^2$ and $b = 1 \text{ Nms}$

Determine K and K_h such that percent overshoot = 0.2

peak time = 1s



$$\% \text{ overshoot} = \exp\left[-\xi\pi/\sqrt{1-\xi^2}\right]$$

$$\text{Peak Time} = \pi/\omega_d$$

Q: What damping ratio is necessary to meet overshoot spec?

$$0.2 = \exp\left[-\xi\pi/\sqrt{1-\xi^2}\right] \rightarrow \xi = 0.456$$

Q: What damped natural freq is required to meet the peak time spec?

$$1s = \frac{\pi}{\omega_d} \rightarrow \omega_d = \pi = 3.14 \text{ rad/s}$$

Q: So, what ω_n is required?

$$\omega_d = \omega_n \sqrt{1-\xi^2} \rightarrow 3.14 = \omega_n \sqrt{1-(0.456)^2} \rightarrow \omega_n = 3.53 \text{ rad/s}$$

Q: Can we find any gains yet?

$$\frac{Y}{R} = \frac{\frac{K}{J}}{s^2 + \left(\frac{b+KK_h}{J}\right)s + \frac{K}{J}}$$

$$\underbrace{2\xi\omega_n}_{2\xi\omega_n} \quad \underbrace{\omega_n^2}_{\omega_n^2} \rightarrow \omega_n = \sqrt{\frac{K}{J}} \leftarrow 3.53 = \sqrt{\frac{K}{J}} \rightarrow K = 24.92$$

Q: K_h ?

$$2\xi\omega_n = \frac{b+KK_h}{J}$$

$$\xi = \frac{b+KK_h}{2J\omega_n} \rightarrow$$

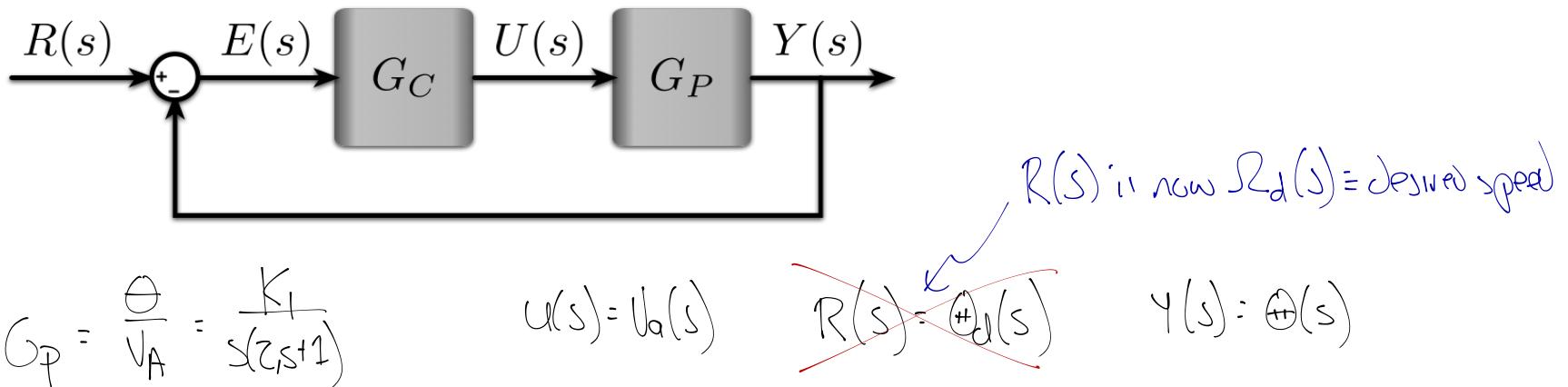
$$0.456 = \frac{1 + 24.92 K_h}{2J\omega_n} \rightarrow$$

$$K_h = 0.218$$

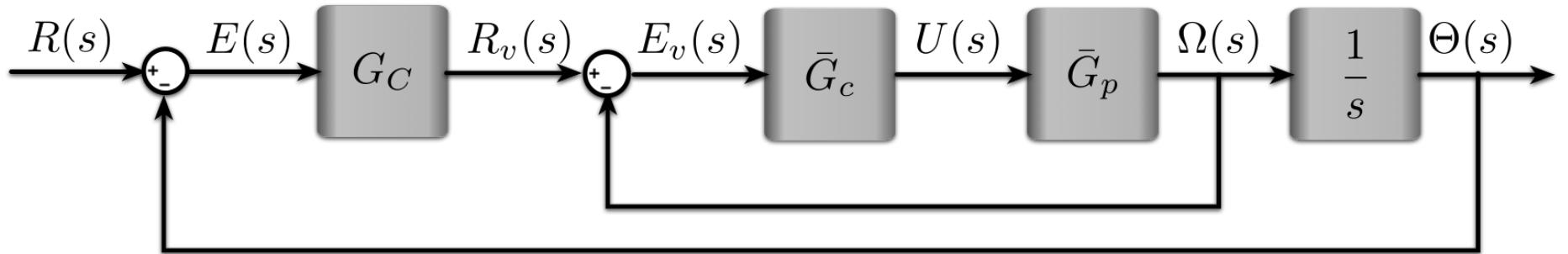
Example

We worked this example last week. Position control of a DC motor.

How can we control its speed?

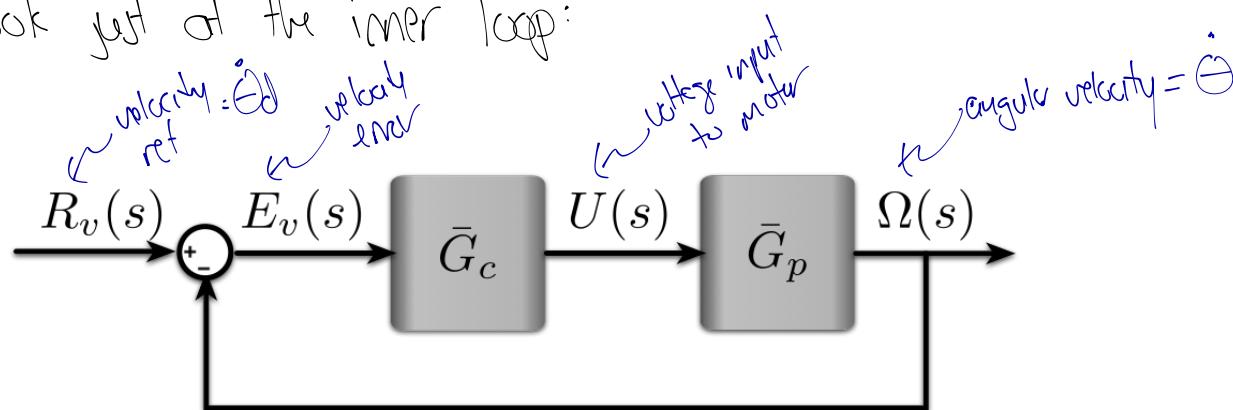


Let's try feedback on the velocity



$$\text{where } \bar{G}_p = sG_p = \frac{sK_1}{s(z_1 s + 1)}$$

Let's look just at the inner loop:



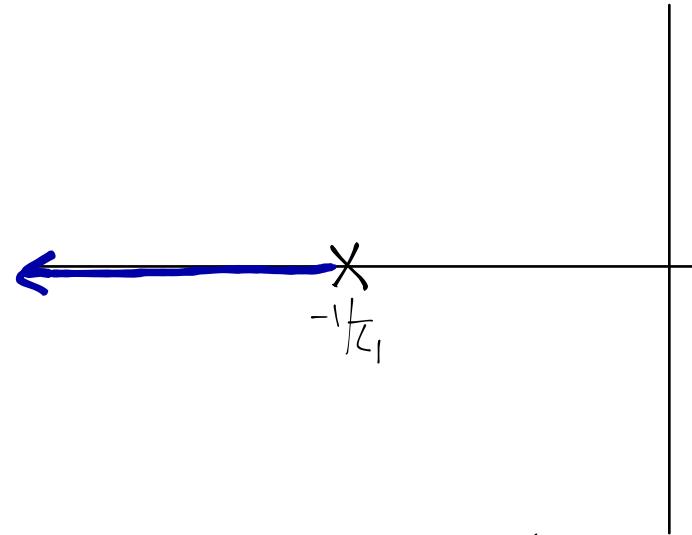
Let \bar{G}_c = velocity controller = k_p ← just proportional control for now

$$\frac{r(s)}{R_v(s)} = \frac{\bar{G}_c \bar{G}_p}{1 + \bar{G}_c \bar{G}_p} = \frac{k_p \bar{G}_p}{1 + k_p \bar{G}_p}$$

Example (cont)

$$\frac{R(s)}{R_U(s)} = \frac{\bar{G}_c \bar{G}_p}{1 + \bar{G}_c \bar{G}_p} = \frac{k_p \bar{G}_p}{1 + k_p \bar{G}_p}$$

$$\bar{k_p \bar{G}_p} = \frac{K k_p}{T_1 s + 1}$$



It seems that we can make this system as fast as we want... perfectly track vel. command

Let's look at a couple examples in the time domain

$$\text{Desired speed} = \dot{\theta}_d \quad \text{actual speed} = \dot{\theta}$$

$$u(t) = K_p (\dot{\theta}_d - \dot{\theta})$$

If $\dot{\theta}_d > \dot{\theta} \rightarrow \text{positive voltage}$

If $\dot{\theta}_d < \dot{\theta} \rightarrow \text{negative voltage}$

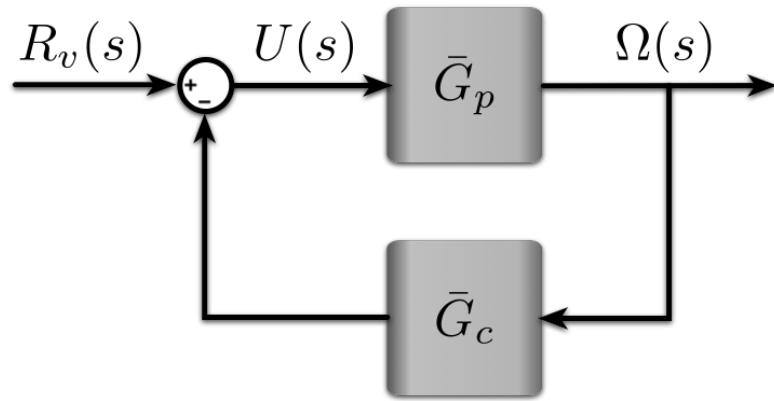
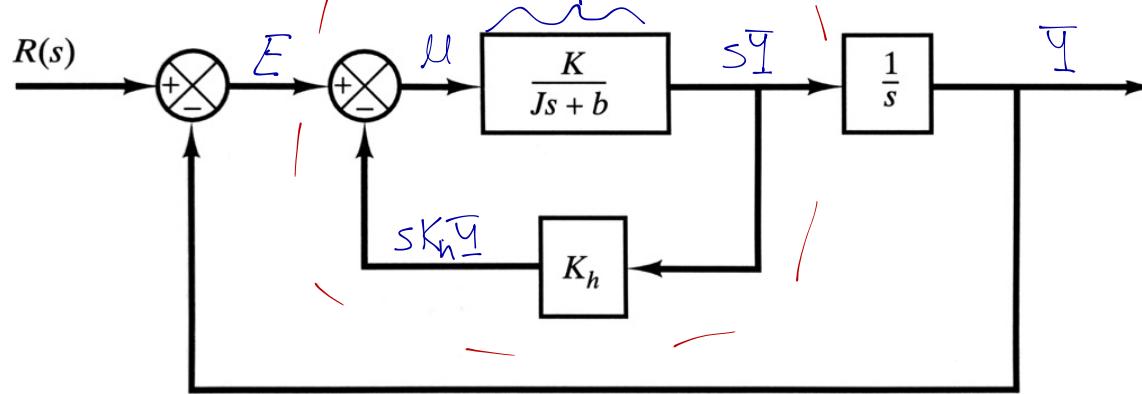
Q: What does this mean for the motor???

If it's spinning faster than desired... apply negative voltage? \rightarrow reverse?

not good!!

Example (cont)

Look back to the 1st example today
Let's try a loop like this one



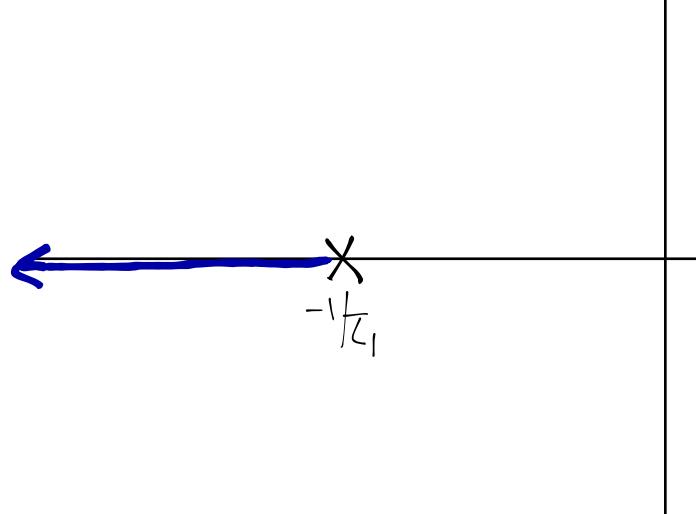
Let $\bar{G}_c \equiv$ velocity controller $\equiv k_p$ ← just proportional control for now

$$\Omega = \bar{G}_p U = \bar{G}_p (R_v - \bar{G}_c \Omega)$$

$$(1 + \bar{G}_c \bar{G}_p) \Omega = \bar{G}_p R_v$$

$$\frac{\Omega}{R_v} = \frac{\bar{G}_p}{1 + \bar{G}_c \bar{G}_p} = \frac{\bar{G}_p}{1 + k_p \bar{G}_p}$$

$$\bar{k}_p \bar{G}_p = \frac{K k_p}{\zeta_1 s + 1}$$



Let's look at the time domain again

$$U(s) = R_v - K \Omega : \dot{\theta}_d - K \dot{\theta} \quad \text{still no good!!} \\ (\text{same problem as before})$$

Example (cont)

Q: What can we do... what do we want to happen?

$\dot{\theta}_d > \dot{\theta} \rightarrow$ increase voltage input from current level $\leftarrow \dot{\theta}_d - \dot{\theta}$ is pos

$\dot{\theta}_d < \dot{\theta} \rightarrow$ reduce voltage input from current level $\leftarrow \dot{\theta}_d - \dot{\theta}$ is neg

Signs do correlate to direction of change

Q: How about? ...

$$u(t) = u(t_{\text{prev}}) + k_p(\dot{\theta}_d - \dot{\theta}) = u(t_{\text{prev}}) + du(t)$$

input immediately
prior to now



PID Controller contributes
this term

We don't have a great way to represent this term yet
but it is easily done in the digital/discrete domain.