

PID Controller Design Example

Think back to the armature-controlled DC Motor example (Ex. 2.5)

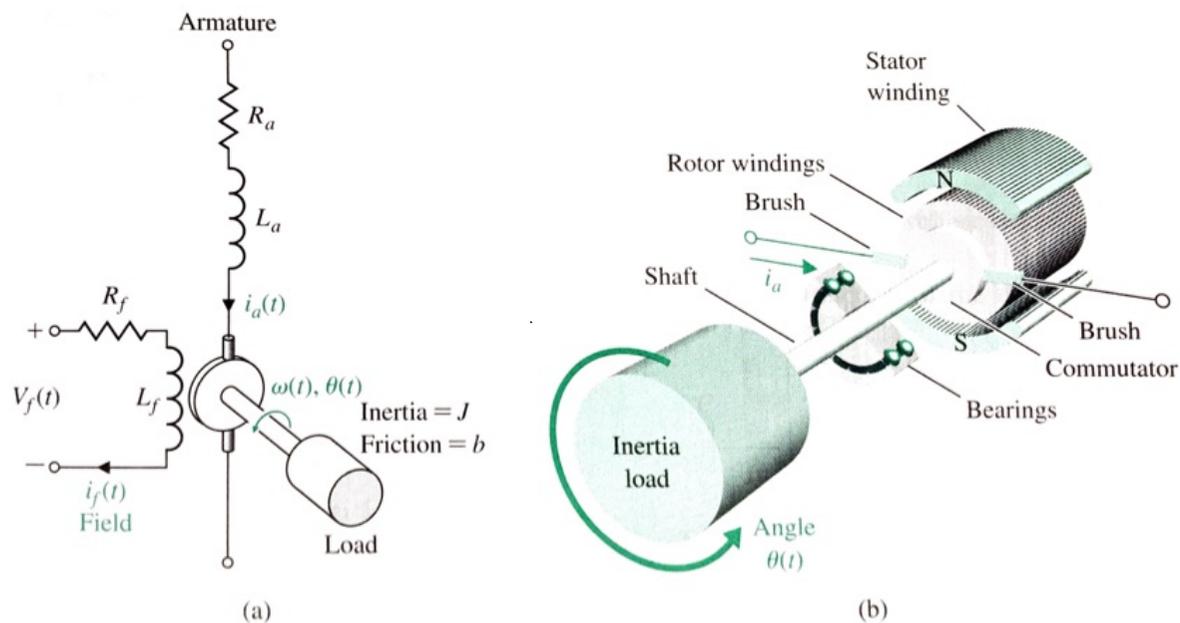


FIGURE 2.18
A DC motor
(a) electrical
diagram and
(b) sketch.

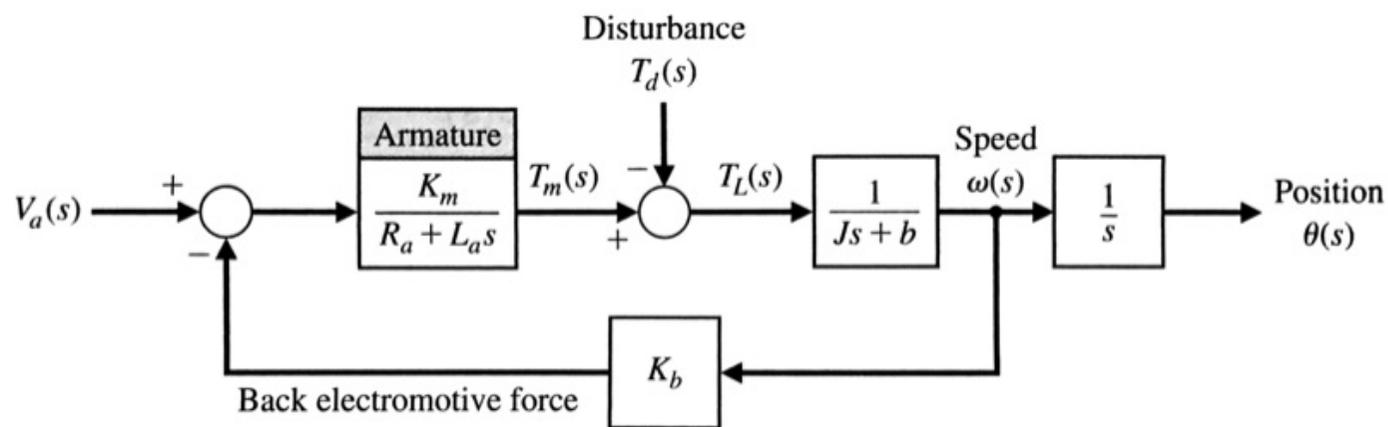


FIGURE 2.20
Armature-controlled
DC motor.

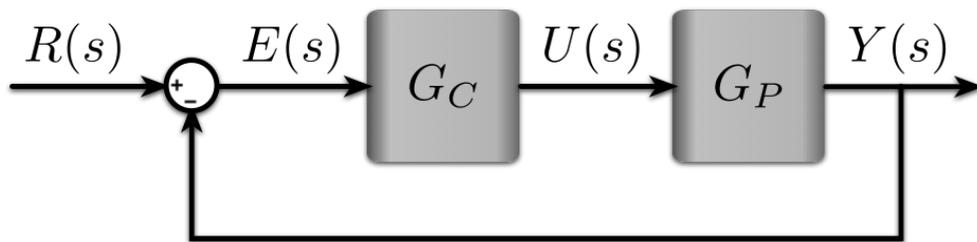
We found the TF from V_a to θ to be

$$\frac{\Theta}{V_a} = \frac{K_1}{s(\tau_1 s + 1)} \quad \text{where } K_1 = \frac{K_m}{R_a b + K_b K_m} \quad \text{and} \quad \tau_1 \equiv \frac{R_a J}{K_b K_m + R_a b}$$

PID Controller Design Example (cont.)

Goal: Make Θ track a desired angle Θ_d

Start by rewriting the block diagram using $\frac{\Theta}{V_A}$ as the plant:



$$G_P = \frac{\Theta}{V_A} = \frac{K_I}{s(z, s+1)} \quad U(s) = V_A(s) \quad R(s) = \Theta_d(s) \quad Y(s) = \Theta(s)$$

Q: What should G_C be?

Let's look at a PD controller first

$$G_C = k_p + k_d s$$

Q: What is the closed-loop TF?

$$\frac{\Theta}{\Theta_d} = \frac{G_C G_P}{1 + G_C G_P} = \frac{\left(\frac{K_I}{s(z, s+1)}\right)(k_p + k_d s)}{1 + \left(\frac{K_I}{s(z, s+1)}\right)(k_p + k_d s)} = \frac{K_I(k_p + k_d s)}{z_1 s^2 + (1 + K_I k_d)s + K_I k_p}$$

$$\frac{\Theta}{\Theta_d} = \frac{K_I k_d s + K_I k_p}{z_1 s^2 + (1 + K_I k_d)s + K_I k_p} = \frac{\frac{K_I k_d}{z_1} s + \frac{K_I k_p}{z_1}}{s^2 + \left(\frac{1 + K_I k_d}{z_1}\right)s + \frac{K_I k_p}{z_1}}$$

Damping ratio
dictated by this
term ($= 2\zeta\omega_n$)

natural freq determined
by this term ($= \omega_n^2$)

Q: Would we have steady-state error to a step input?

Hint: What's the system type? \rightarrow Type 1 system (1 pure integrator)

Remember that
this is determined
by $G_C G_P$ not by
CL TF

e_{ss} to step is 0

One reason we wouldn't use integral
control here

PID Controller Design Example (cont.)

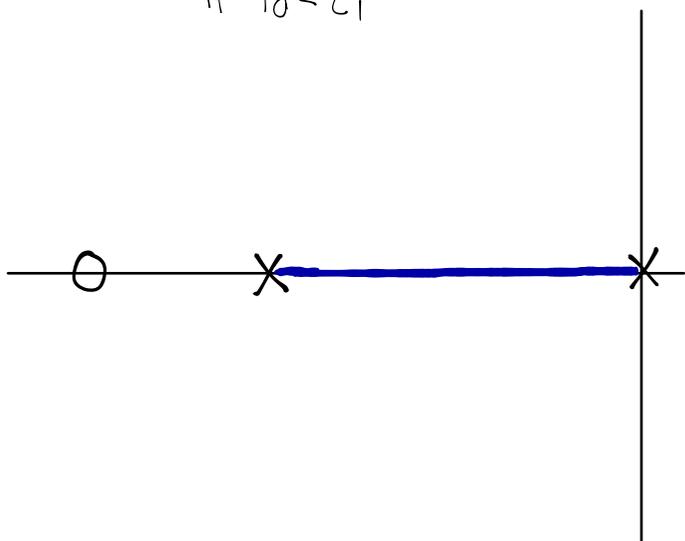
Q: How could we draw the root locus for this system?

need to put it in a form where only 1 parameter is varying

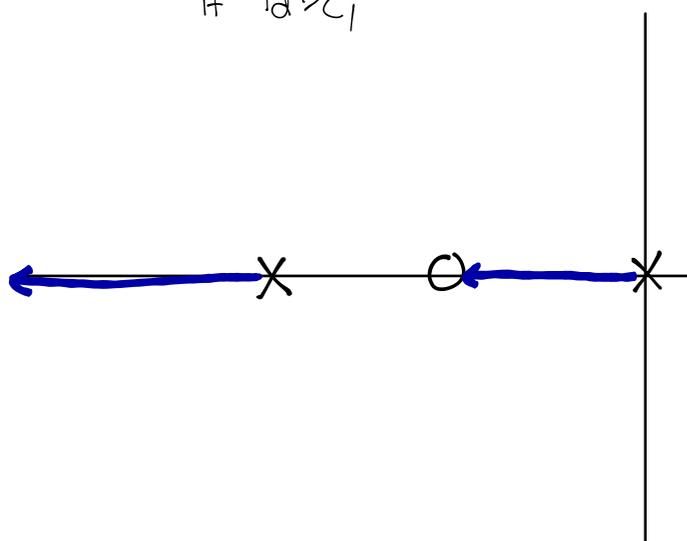
Rewrite G_c as $G_c = k_p(1 + T_d s)$ where $T_d = \frac{k_d}{k_p}$

So $G_c G_p = \frac{k_p(k_1(1 + T_d s))}{s(\tau_1 s + 1)} \leftarrow$ zero at $s = -\frac{1}{T_d} = -\frac{k_p}{k_d}$
 \leftarrow poles at 0 and $-\frac{1}{\tau_1}$

if $T_d < \tau_1$



if $T_d > \tau_1$



Q: How many separate loci? \rightarrow more poles than zeros and 2 poles so 2 loci?

\leftarrow This case is done then

Q: Asymptotes?

$$\sigma_A = \frac{\sum(-p_j) - \sum(z_i)}{n-m} = \frac{(-1/\tau_1 - 1/T_d)}{2-1} = -\frac{1}{\tau_1} - \frac{1}{T_d}$$

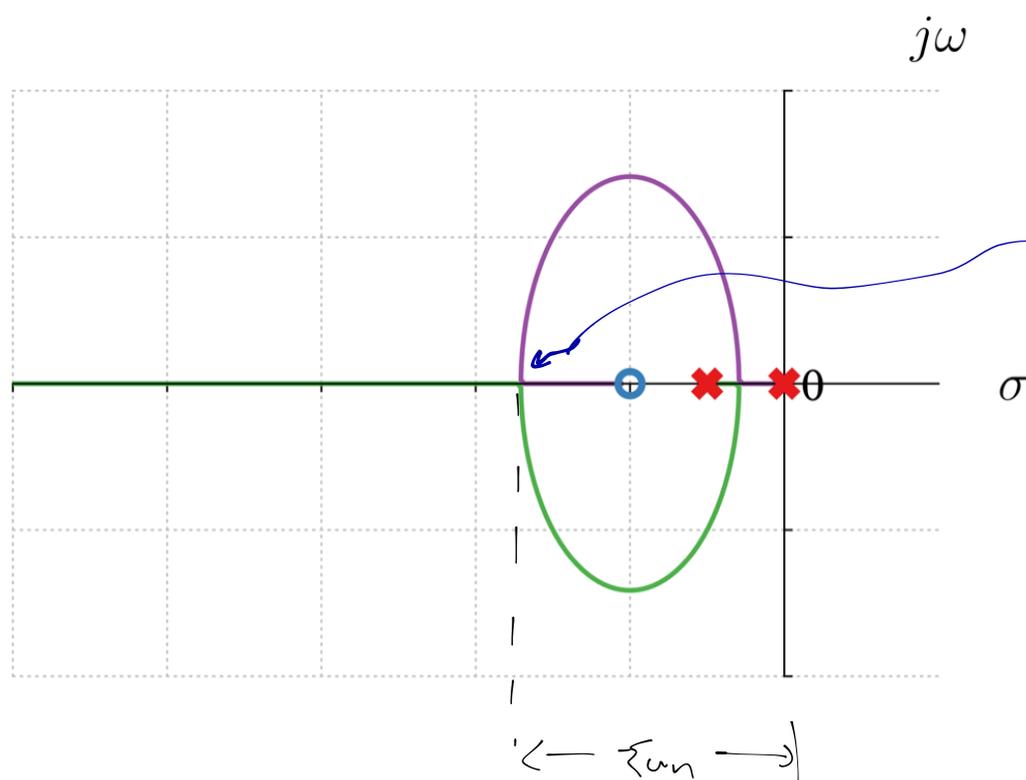
$$\phi_A = \left(\frac{2k+1}{n-m}\right) 180^\circ = (2k+1) 180^\circ = \pm 180^\circ$$

Q: Breakaway Point(s)?

$$1 + k_p \left(\frac{k_1(1 + T_d s)}{s(\tau_1 s + 1)} \right) \rightarrow k_p = \frac{-s(\tau_1 s + 1)}{k_1(1 + T_d s)}$$

Solve $\frac{dk_p}{ds} = 0 \leftarrow$ obviously tedious to do

PID Controller Design Example (cont.)



Q: For this case, where would we want to place the ζ poles?

Here it probably a good place.

↑ damping
but fast response

Remember $t_s \hat{=} \frac{4}{\zeta \omega_n}$

so placing the poles here would give a "local min" settling time

"Manual Tuning" for this example

1) Set $k_d=0$ so the TF becomes:

$$\frac{\Theta}{\Theta_d} = \frac{k_1 k_p}{z_1 s^2 + s + k_1 k_p} = \frac{\frac{K_1 k_p}{z_1}}{s^2 + \frac{1}{z_1} s + \frac{K_1 k_p}{z_1}}$$

$$\omega_n^2 = \frac{K_1 k_p}{z_1} \quad \leftarrow \text{increasing } k_p \text{ increases } \omega_n$$

$$2\zeta \omega_n = \frac{1}{z_1}$$

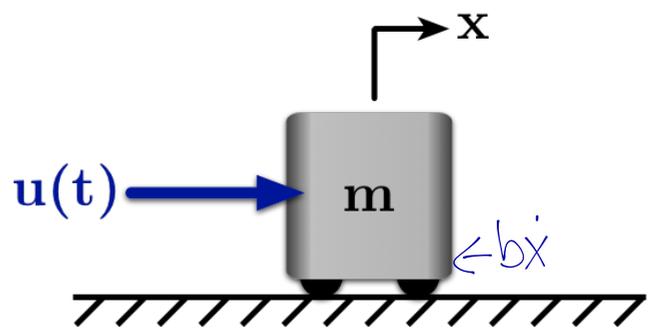
$$\zeta = \frac{1}{2z_1 \omega_n} \quad \leftarrow \text{and decreases } \zeta$$

Find k_p such that $\zeta \rightarrow 0$

2) Find k_p such that amp is $1/4$ of peak after 1 cycle
(start with $k_p = 1/2$ of value from step 1)

3) Adjust k_d to lower overshoot + improve settling time as desired

PI Control of a Mass with Viscous Friction



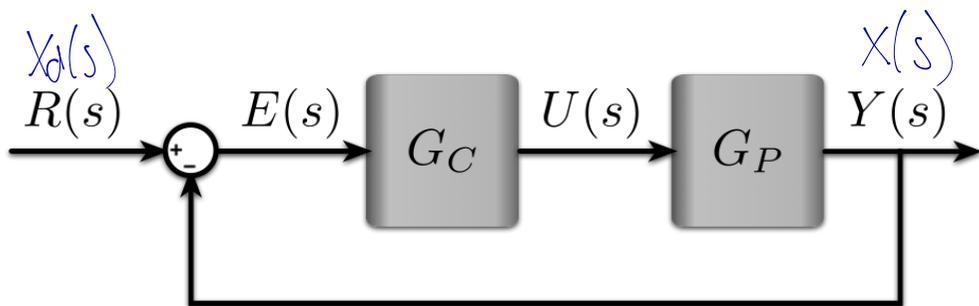
Goal: choose $u(t)$ to drive x to x_d

$$m\ddot{x} = u - b\dot{x} \quad \leftarrow \text{equation of motion}$$

$$\frac{X}{U} = G_p = \frac{1}{ms^2 + bs} = \frac{1}{s(ms+b)}$$

Let's try a PI controller for u :

$$u(t) = k_p e + k_I \int e dt \quad \rightarrow \quad G_c = k_p + \frac{k_I}{s} = k_p \left(1 + \frac{1}{T_I s} \right) = \frac{k_p (T_I s + 1)}{T_I s}$$

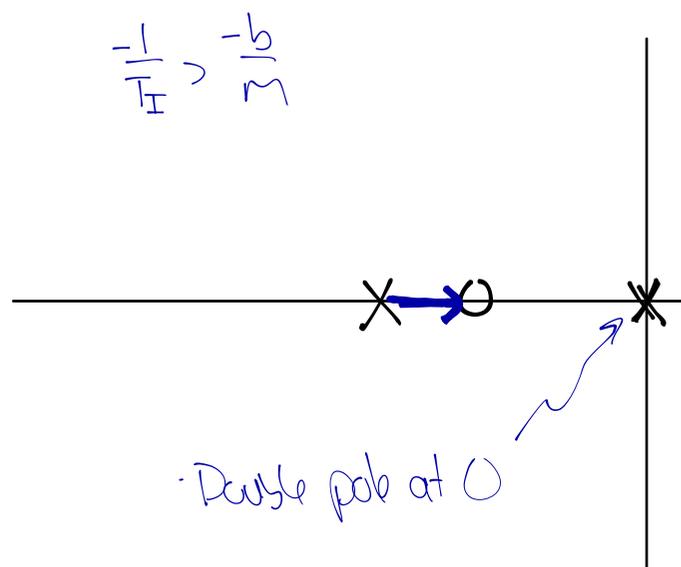
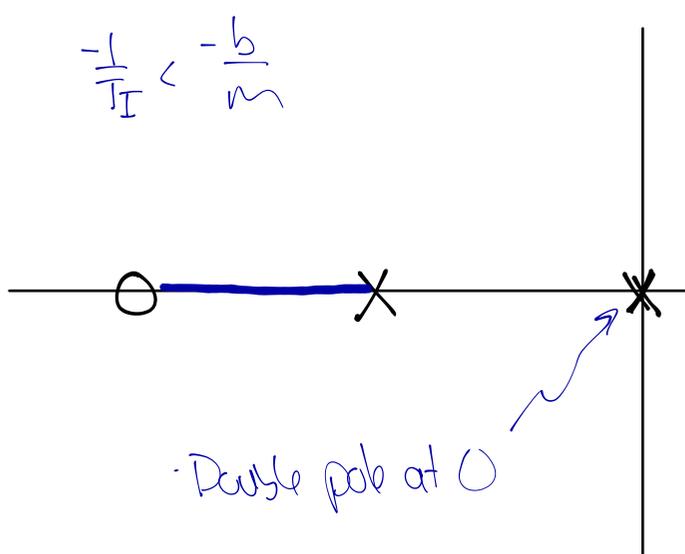


$$G_c G_p = \left[\frac{k_p (T_I s + 1)}{T_I s} \right] \left[\frac{1}{ms^2 + bs} \right] = \frac{k_p (T_I s + 1)}{T_I s (ms^2 + bs)}$$

\leftarrow CL zero at $-1/T_I$
 \leftarrow CL poles at $0, 0, -b/m$

CL TF

$$\frac{X}{X_d} = \frac{G_c G_p}{1 + G_c G_p} = \frac{k_p (T_I s + 1)}{T_I s (ms^2 + bs) + k_p (T_I s + 1)} = \frac{k_p (T_I s + 1)}{m T_I s^3 + b T_I s^2 + k_p T_I s + k_p}$$



Number of separate loci = 3

PI Control of a Mass with Viscous Friction (cont.)

Asymptotes:

$$\sigma_A = \frac{\sum(p) - \sum(z_c)}{n-m} = \frac{(0+0-\frac{b}{m}) - (-\frac{1}{T_I})}{3-1} = \frac{-\frac{b}{m} + \frac{1}{T_I}}{2}$$

IF $\frac{1}{T_I} > \frac{b}{m}$ what happens???

RHP asymptote \rightarrow can be unstable

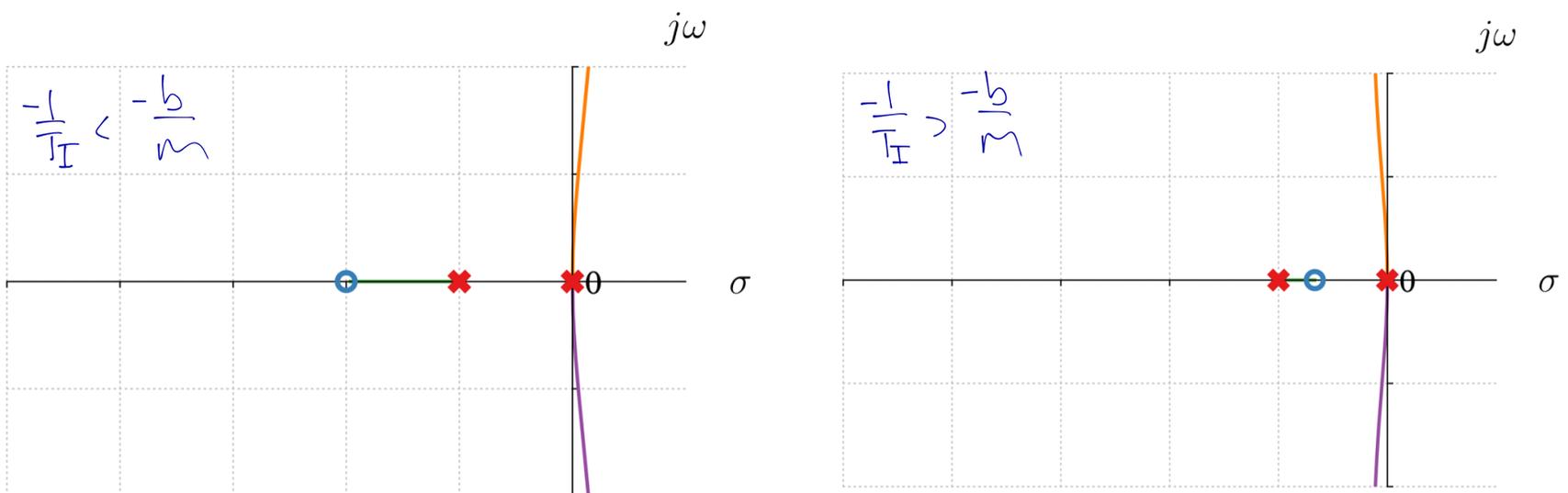
$$\phi_A = \left(\frac{2k+1}{n-m}\right) 180^\circ = \left(\frac{2k+1}{2}\right) 180^\circ \quad k=0,1 \rightarrow 90^\circ, 270^\circ \quad (+90^\circ, -90^\circ)$$

Breakaway

Q: Where is the only place a breakaway could happen on this locus?

Must happen between two poles on the real axis $\rightarrow 0$

Especially given this case



In this case, we go unstable immediately!

Ziegler-Nichols Setup for this example

1) Set $k_I = 0$ and increase $k_p \rightarrow$ unstable (unsamped oscillation)

At this point define $k_u = k_p$ and $P_u =$ period for this gain k_p

2) Use Table to select

$$k_p = 0.45 k_u \quad \text{and} \quad k_I = \frac{0.54 k_u}{P_u}$$