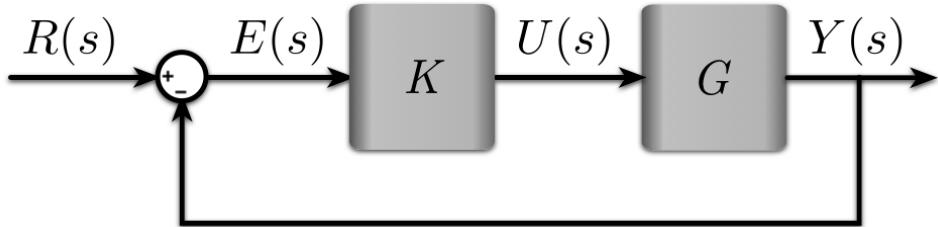


Example 7.4



$$G = \frac{1}{s^4 + 12s^3 + 64s^2 + 128s}$$

Q: What is the open-loop transfer function?

$$KG = \frac{K}{s^4 + 12s^3 + 64s^2 + 128s}$$

Q: What is the closed-loop TF?

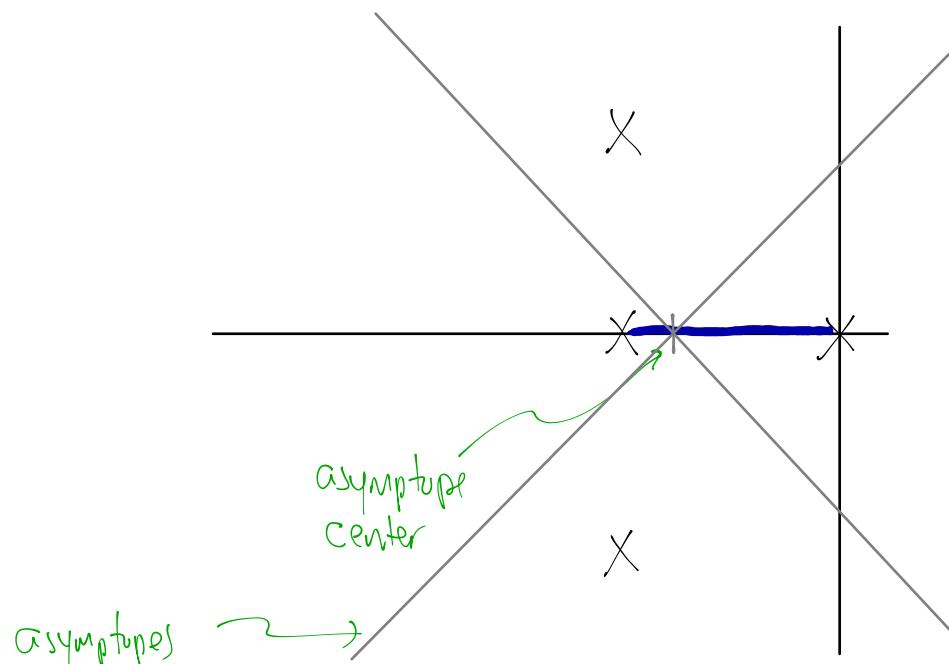
$$\frac{Y}{R} = \frac{KG}{1+KG} = \frac{K}{s^4 + 12s^3 + 64s^2 + 128s + K}$$

Q: Open-loop poles and zeros?

no zeros, poles at roots of $s^4 + 12s^3 + 64s^2 + 128s = 0$

$$s(s+4)(s^2 + 8s + 32) = 0 \quad \left. \begin{array}{l} \text{poles at } 0, -4, -4+4i, -4-4i \end{array} \right\}$$

This means well how
4 separate loci



To find asymptotes

centred at:

$$\sigma_A = \frac{\sum(-p_j) - \sum(-z_i)}{n-m}$$

$$= \frac{(0-4-4+4/-4-4i/-4+4i)}{4} = -12 = -3$$

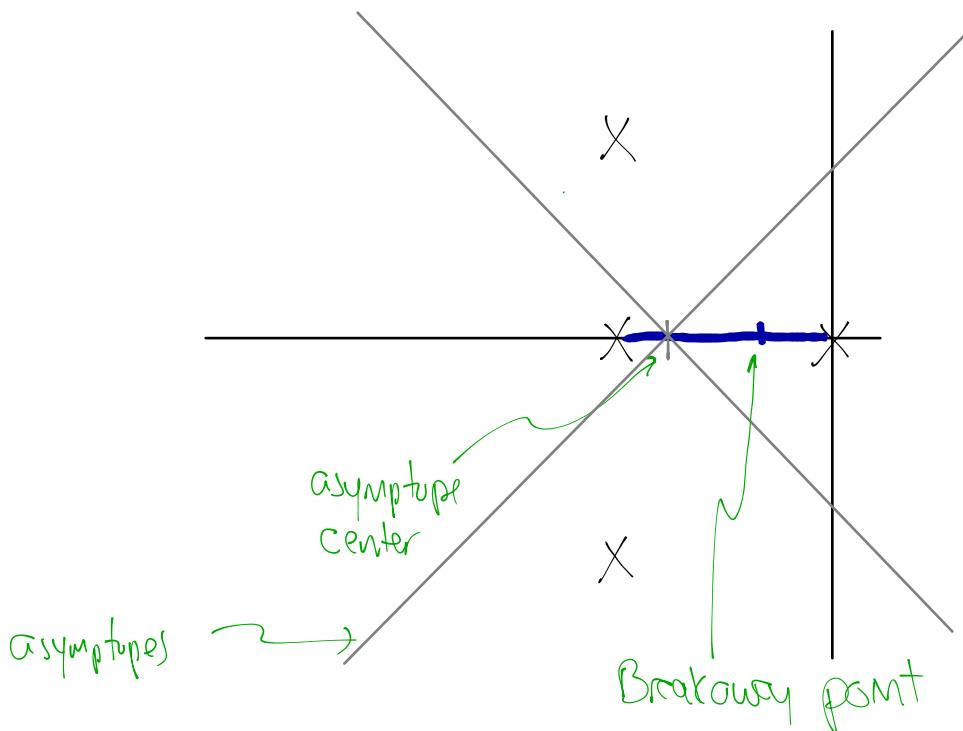
at angle

$$\phi_A = \left(\frac{2k+1}{n-M} \right) 180^\circ$$

$$k = 0, 1, \dots (n-M-1)$$

$$\phi_A = \left(\frac{2k+1}{4} \right) 180^\circ = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

Example 7.4 (cont.)



Now we need to find the breakaway point

$$K = -(s^4 + 12s^3 + 64s^2 + 128s)$$

$$\frac{dK}{ds} = -4s^3 - 36s^2 - 128s + 128 = 0$$

$$\text{Best candidate } s \approx -1.58$$

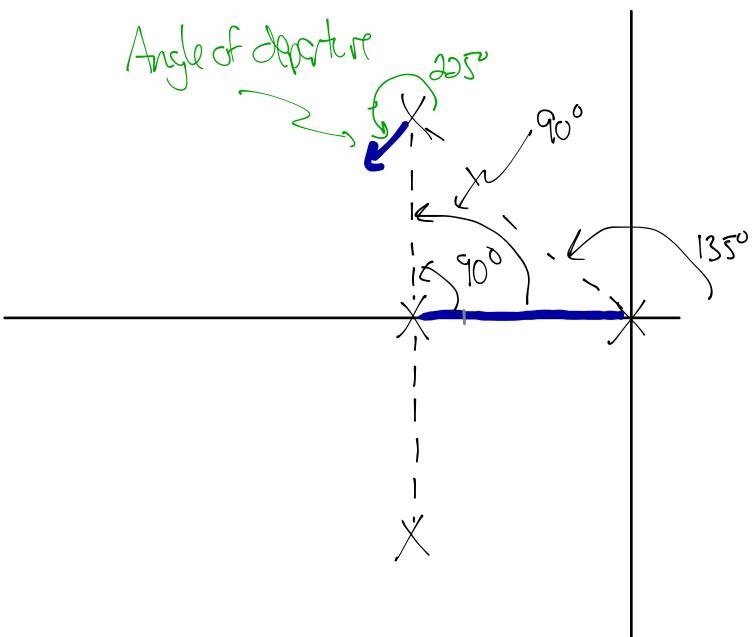
Q: What's left?

- Angle of departure from complex pole
- Crossing point of Im axis

Finding the angle of departure

Uses the phase-angle condition - $\angle KG = 180^\circ + k360^\circ \quad k=0, \pm 1, \pm 2, \dots$

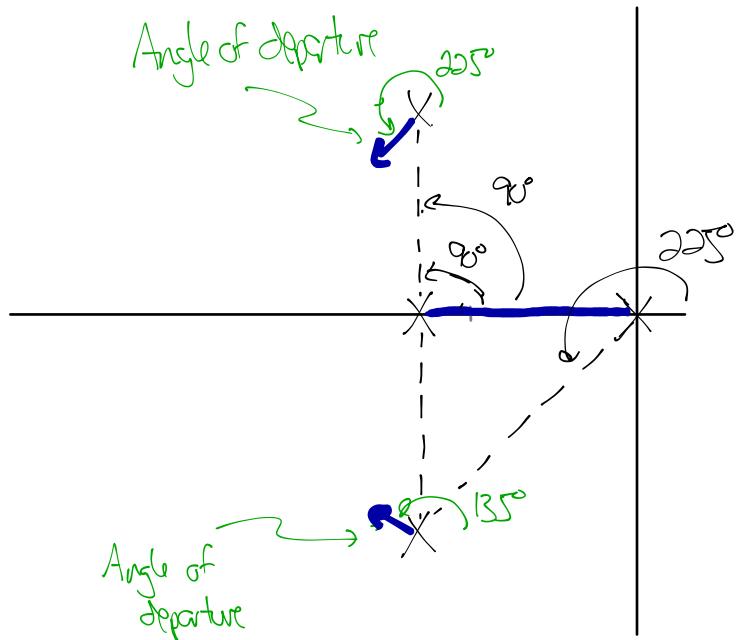
$$\begin{aligned} \text{Angle of departure} &= 180^\circ - (\sum \text{angles of vectors to all other poles}) \\ &\quad + (\sum \text{angles of vectors to all zeros}) \end{aligned}$$



Consider $-4+4i$

$$\begin{aligned} \text{Angle} &= 180^\circ - 90^\circ - 90^\circ - 135^\circ \\ &= -135^\circ = +225^\circ \end{aligned}$$

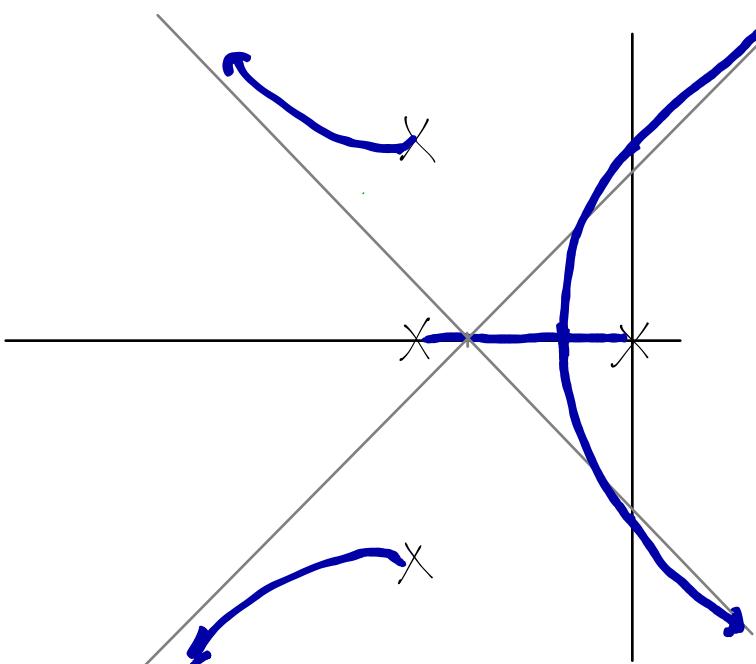
Example 7.4 (cont.)



Consider $-4-4i$

$$\text{Angle} = 180^\circ - 90^\circ - 90^\circ - 225^\circ = -225^\circ$$

$$\text{Angle} = -225^\circ = 135^\circ$$



Q: How do we find Im-axis crossing?

1-way, plug in (iw) for s and solve

$$s^4 + 12s^3 + 64s^2 + 128s + K = 0$$

$$(iw)^4 + 12(iw)^3 + 64(iw)^2 + 128(iw) + K = 0$$

$$\omega^4 + (-12\omega^3 i) + (-64\omega^2) + 128i\omega + K = 0$$

$$(\omega^4 - 64\omega^2 + K) + i(-12\omega^3 + 128\omega) = 0$$

$$\text{Need } \omega^4 - 64\omega^2 + K = 0 \quad \text{and} \quad -12\omega^3 + 128\omega = 0$$

$$\left(\frac{128}{12}\right)^2 - 64\left(\frac{128}{12}\right) + K = 0$$

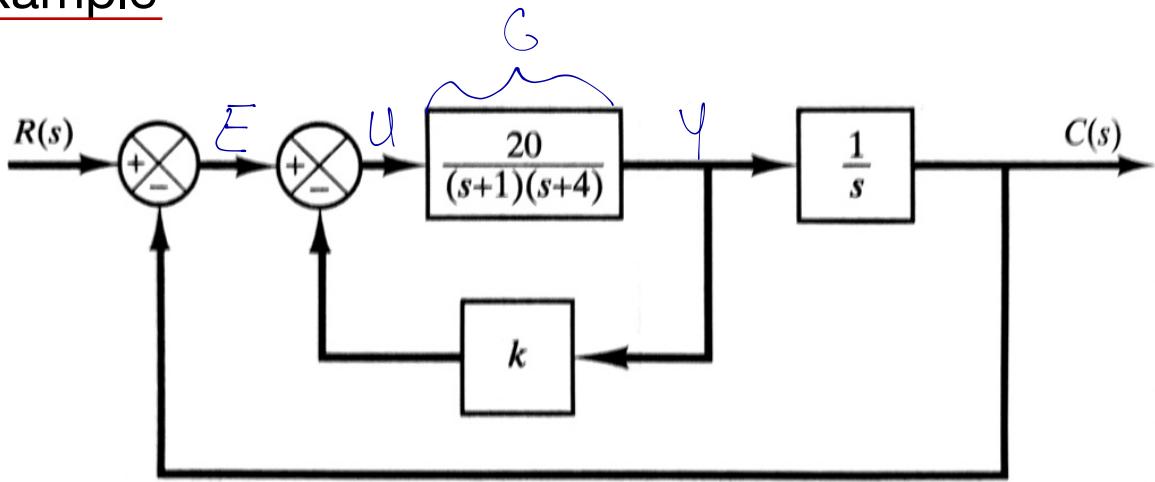
$$K = 568.89$$

$$\omega(-12\omega^2 + 128) = 0$$

$\omega=0, K=0$ is one solution

$$\omega = \sqrt{\frac{128}{12}} \approx \pm 3.264$$

Example



Draw the root locus for this system

$$Y = GU, \quad U = E - KY \rightarrow Y = G(E - KY) \rightarrow (1 + KG)Y = GE$$

$$\frac{Y}{E} = \frac{G}{1 + KG}$$

$$Y = \frac{G}{1 + KG} E = \frac{G}{1 + KG} (R - C) = SC$$

$$\left(S + \frac{G}{1 + KG} \right) C = \frac{G}{1 + KG} R$$

$$(S + SKG + G)C = GR$$

$$\frac{C}{R} = \frac{G}{S + SKG + G} = \frac{20}{S(S+1)(S+4) + 20KS + 20} \quad \} \text{closed-loop TF}$$

Open-loop TF is without outer feedback loop

$$\frac{Y}{E} = \frac{SC}{R} = \frac{20}{(S+1)(S+4) + 20K} \rightarrow \frac{C}{R} = \frac{20}{S(S+1)(S+4) + 20KS} \quad \} \text{open-loop TF}$$

↑
no feedback $E = R - (0)$

Example (cont.)

$$\frac{C}{R} = \frac{20}{s(s+1)(s+4) + 20ks}$$

} open-loop TF

Q: What's the problem with this?

Notice that K doesn't appear in the "right" place

We need the characteristic equation to "look" like

$1 + KG$ where K is the parameter we're varying

Define $K = 20k$, so the char eq becomes

$$s^3 + 5s^2 + 4s + Ks + 20 = 0$$

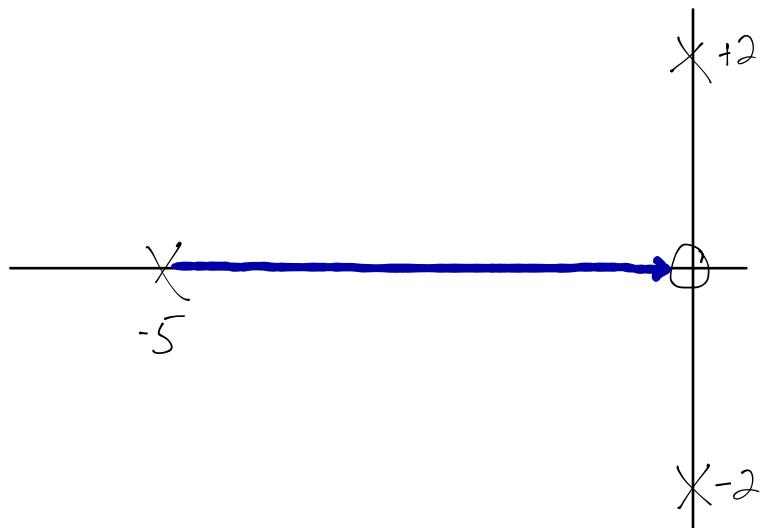
Now divide by all terms without K:

$$1 + \frac{Ks}{s^3 + 5s^2 + 4s + 20} = 0$$

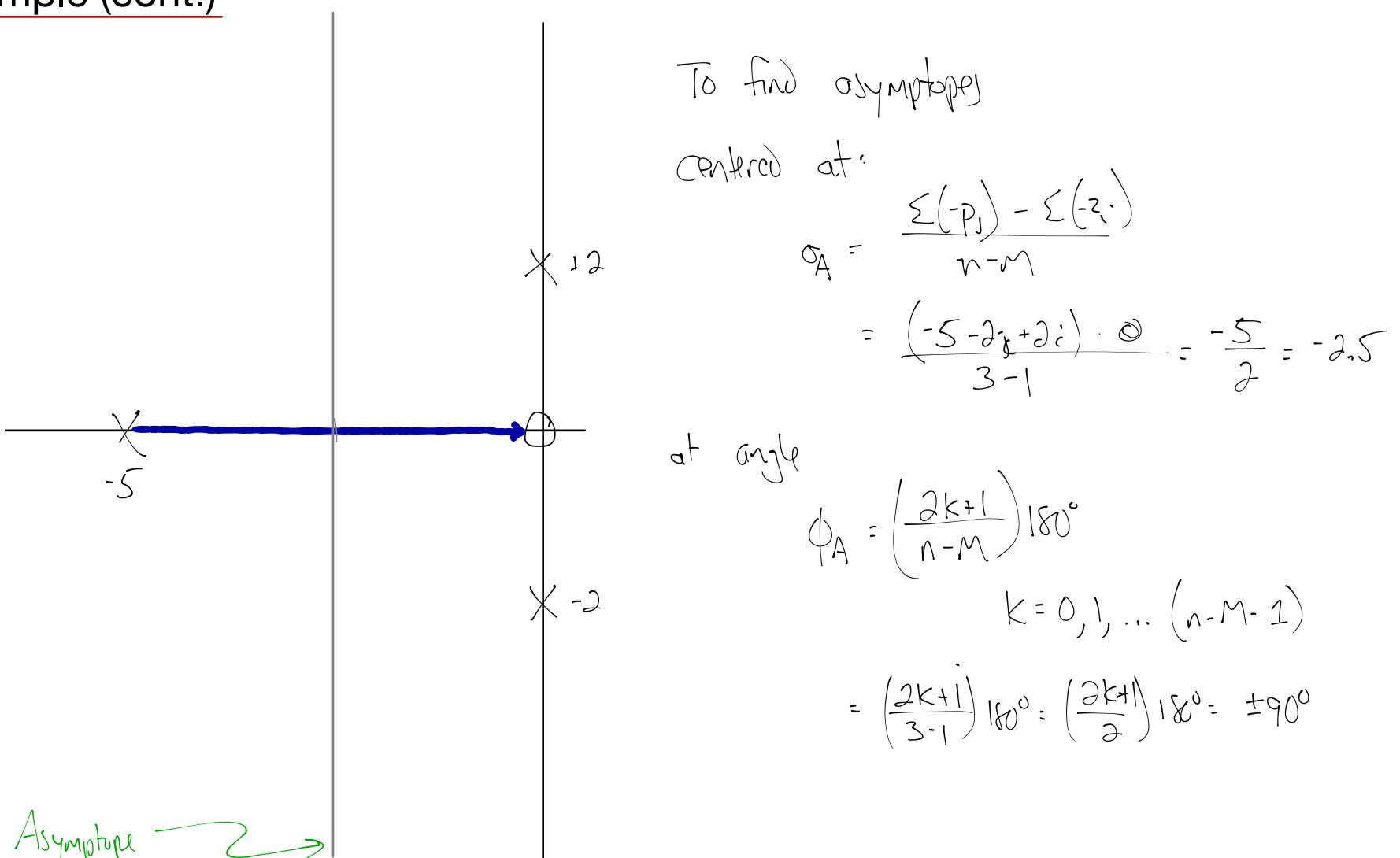
} Now, the TF is in the "right" form and we can proceed.

$$1 + \frac{Ks}{(s+2i)(s-2i)(s+5)} = 0$$

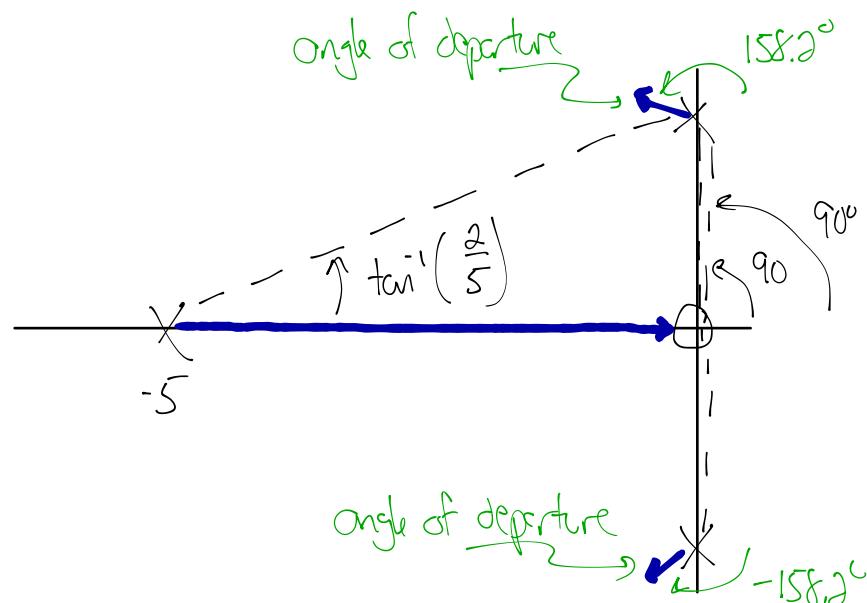
1 open-loop 0
3 open-loop poles



Example (cont.)



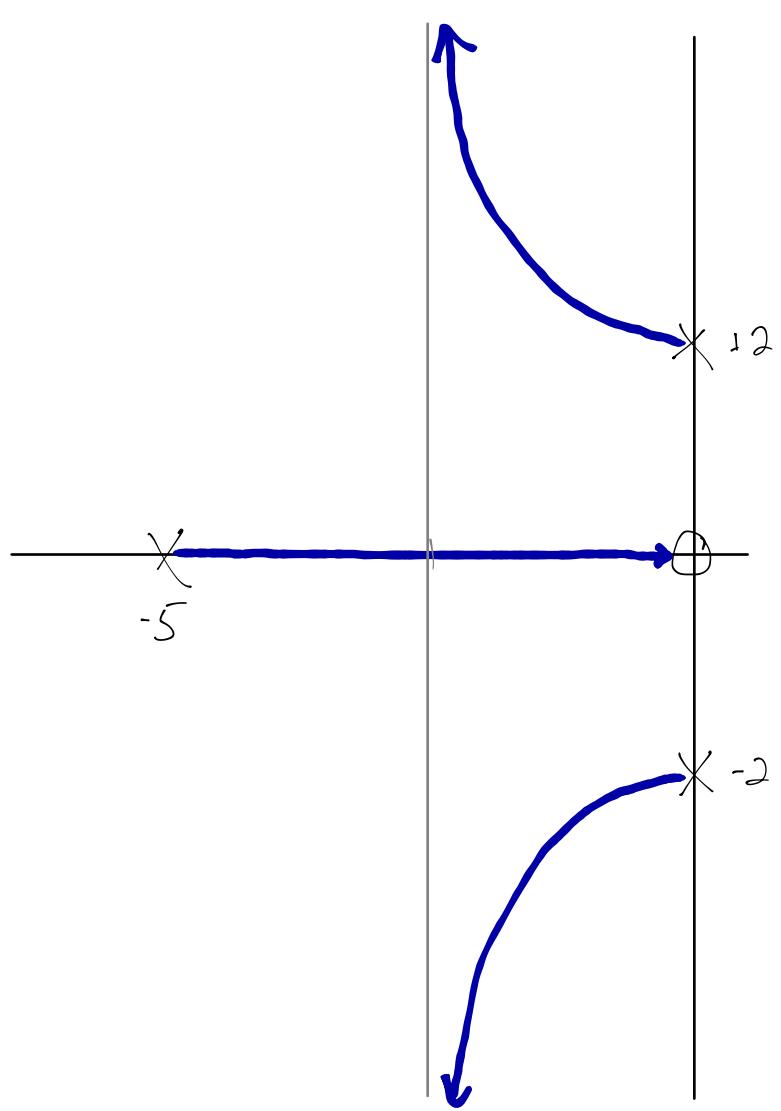
$$\text{Angle of Departure} = 180^\circ - (\sum \text{angles of vectors to all other poles}) \\ + (\sum \text{angles of vectors to all zeros})$$



$$180^\circ - 90^\circ - 21.8^\circ + 90^\circ \\ \approx 158.2^\circ$$

Tip: You don't need to calculate the $-2i$ case because the locus is symmetric

Example (cont.)



Now, we can use this Root Locus to choose a "good" value for k