Effects of a Third Pole and a Zero on a 2nd-order System Response (Sec. 5.4)

The responses and perf measures in the last section are exact only for 2nd-order systems.

But they can provide info on other systems as many systems behave approximately like a 2nd-order system.

Adding a pole

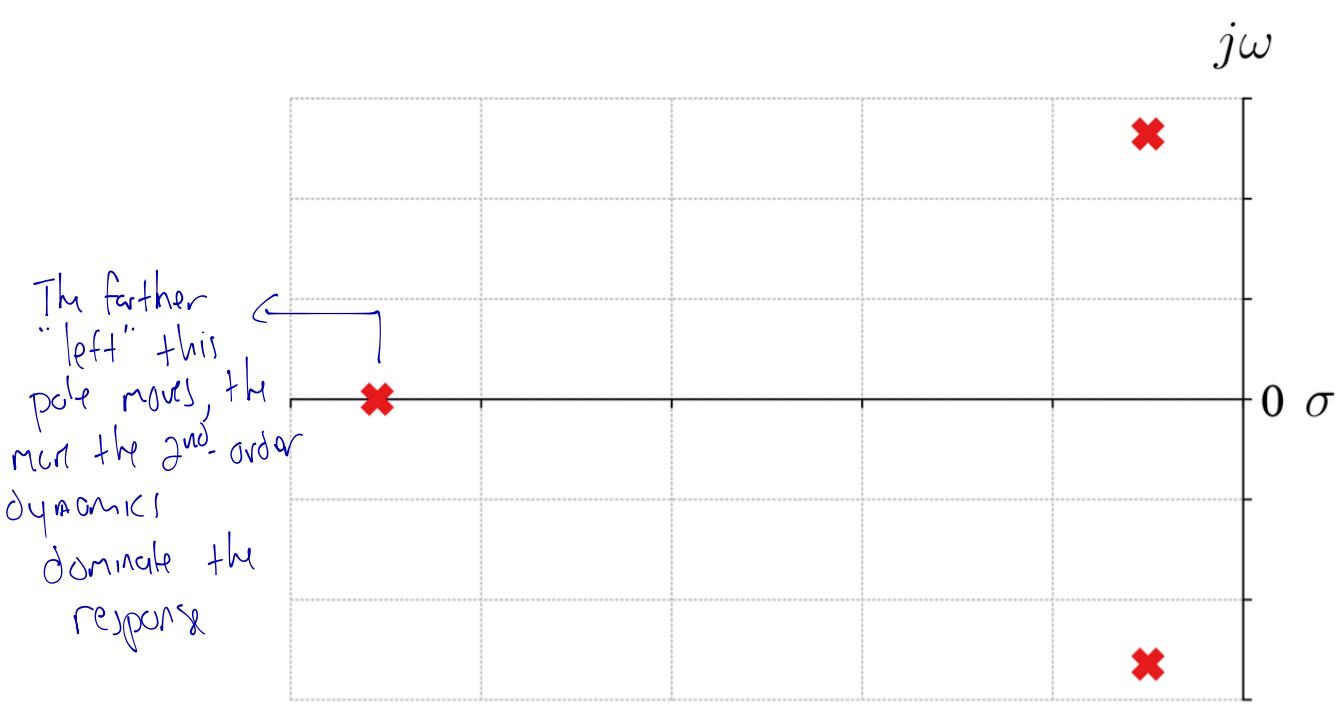
Let's look 1st at 3rd-order systems (adding a pole)

$$T(s) = \frac{1}{(s^2 + 2\zeta\omega_n s + \omega_n^2)(\gamma s + 1)}$$

How this pole affects the system can be characterized by how dominant the roots of the 2nd-order term are. Generally, a 2nd-order system is a good approx when:

$$\underbrace{|1/\gamma| \geq 10|\zeta\omega_n|}$$

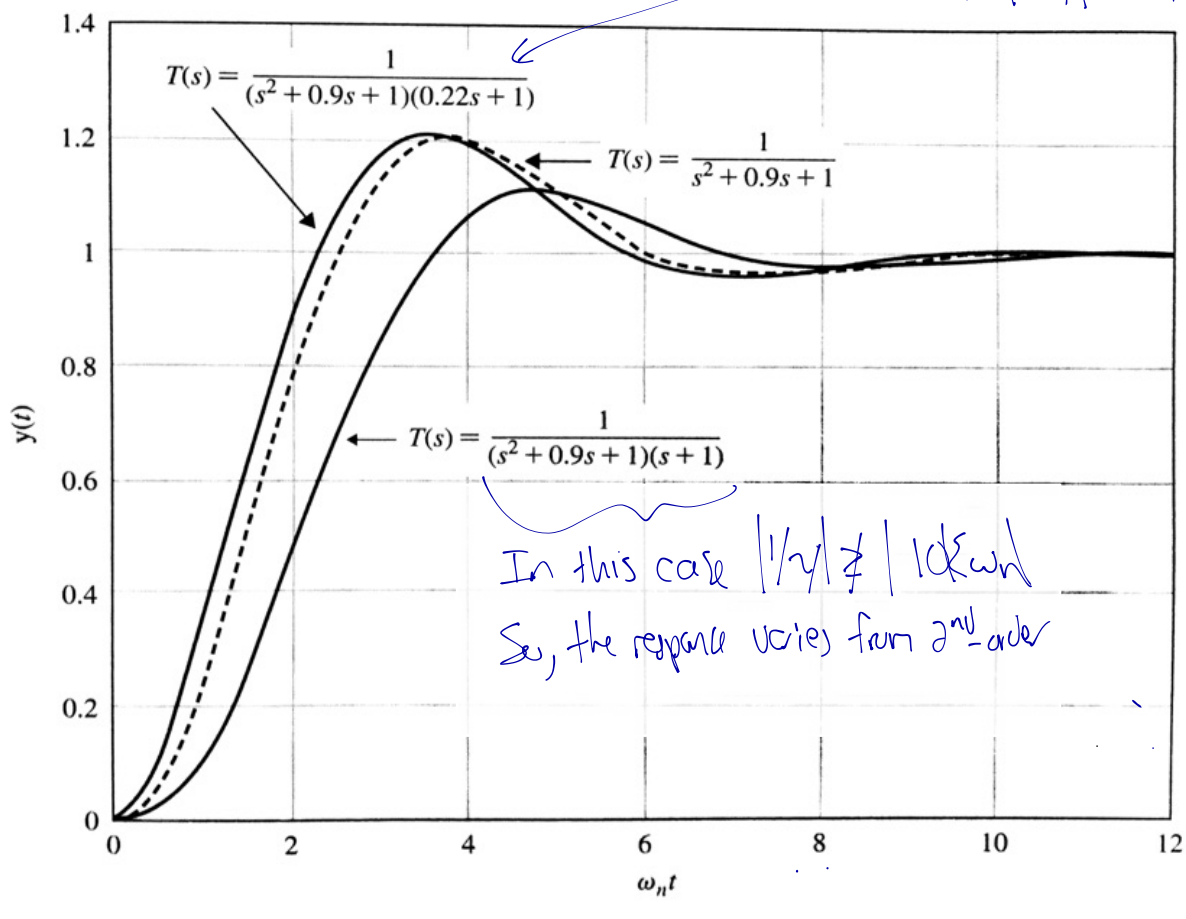
When this holds, we say that the system response can be approximated by the dominant roots of the 2nd order sys



The farther "left" this pole moves, the more the 2nd-order dynamics dominate the response

Effects of a Third Pole and a Zero on a 2nd-order System Response (cont.)

In this case $|1/\gamma| \geq 10|\zeta\omega_n|$, so a 2nd-order response closely approximates it.



In this case $|1/\gamma| \ngeq |10\zeta\omega_n|$
So, the response varies from 2nd-order

FIGURE 5.12
Comparison of two third-order systems with a second-order system (dashed line) illustrating the concept of dominant poles when $|1/\gamma| \geq 10\zeta\omega_n$.

Effects of a Third Pole and a Zero on a 2nd-order System Response (cont.)

Adding a zero

$$T(s) = \frac{\omega_n^2/a\,(s+a)}{s^2+2\zeta\omega_n s+\omega_n^2}$$

The book includes this to normalize the results

The parameter $a/\zeta\omega_n$ will tell us how much the zero affects the response.

As $a/\zeta\omega_n$ increase, the response approaches a 2nd-order sys.

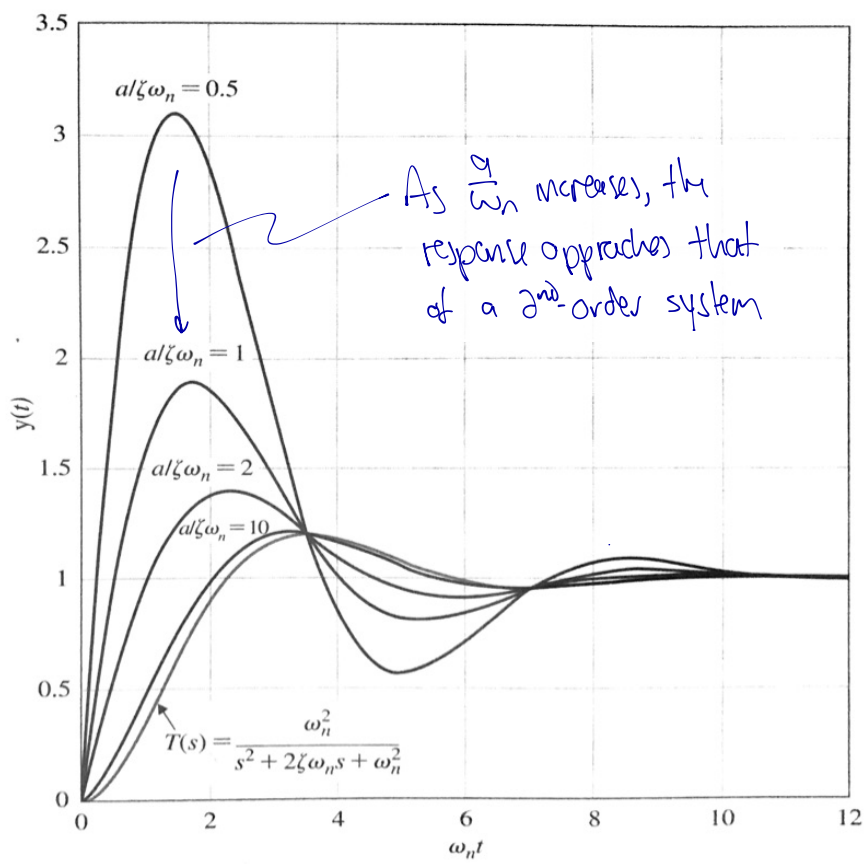
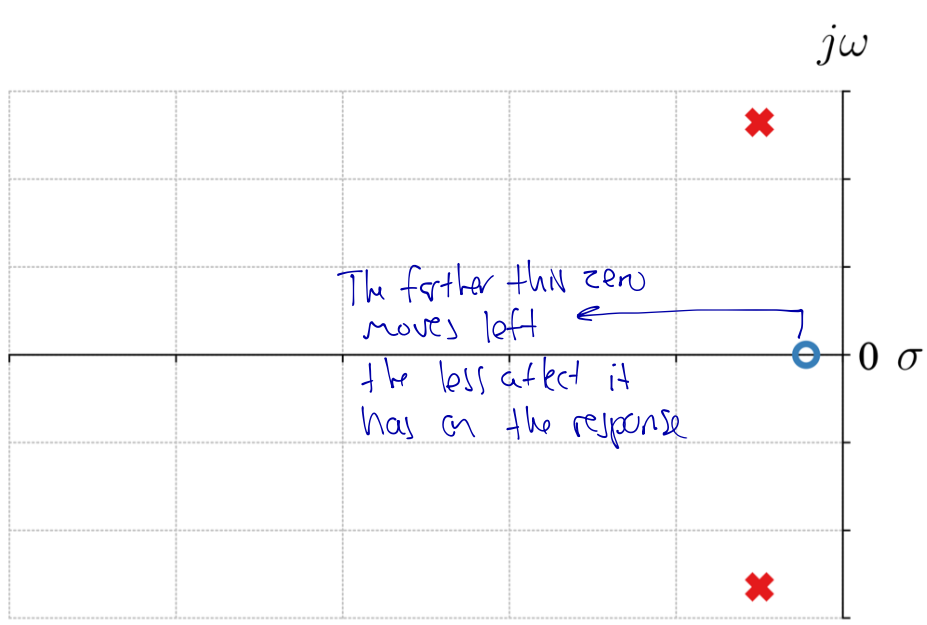


FIGURE 5.13 The response for the second-order transfer function with a zero for four values of the ratio $a/\zeta\omega_n = 0.5, 1, 2,$ and 10.0 when $\zeta = 0.45$.

Looking at the pole-zero map



Example 5.2

$$T(s) = \frac{1.6(s+2.5)}{(s^2+6s+25)(0.16s+1)}$$

Q: What is the DC gain of this TF?

plug in $s=0$

$$T(s) = \frac{1.6(2.5)}{(25)(1)} = 0.16$$

Note: The book says the DC gain is 1. It's wrong.

We'd like to approximate this system as 2nd-order.

Q: What 2nd-order system is appropriate?

$$T(s) = \frac{1.6(s+2.5)}{(s^2+6s+25)(0.16s+1)} = \frac{\omega_n^2/a\,(s+a)}{(s^2+2\zeta\omega_n s+\omega_n^2)(1+\tau s)}$$

Want

$$T(s) \approx \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

By matching terms, we can see:

$\omega_n^2 = 25$, $\zeta\omega_n = 3$, $a = 2.5$, and $\tau = 0.16$

So, $\omega_n = 5$ rad/s and $\zeta = 0.6$

Q: What needs to be true for this approximation to be "okay"?

$|1/\tau| \geq 10|\zeta\omega_n|$ or more generally $\tau \ll \frac{1}{\zeta\omega_n}$ } The 3rd pole is much "faster" than the flexible poles

$\frac{a}{\zeta\omega_n}$ is large or more generally $a \gg \zeta\omega_n$

In this case, $a \not\gg \zeta\omega_n$ <u>and</u> $\tau \not\ll \frac{1}{\zeta\omega_n}$ so the approx. is not a good one.

The s-plane Root Location and the Transient Response (Sec. 5.5)

We've seen that the root location can tell us a lot about the response (particularly for 2nd order systems)

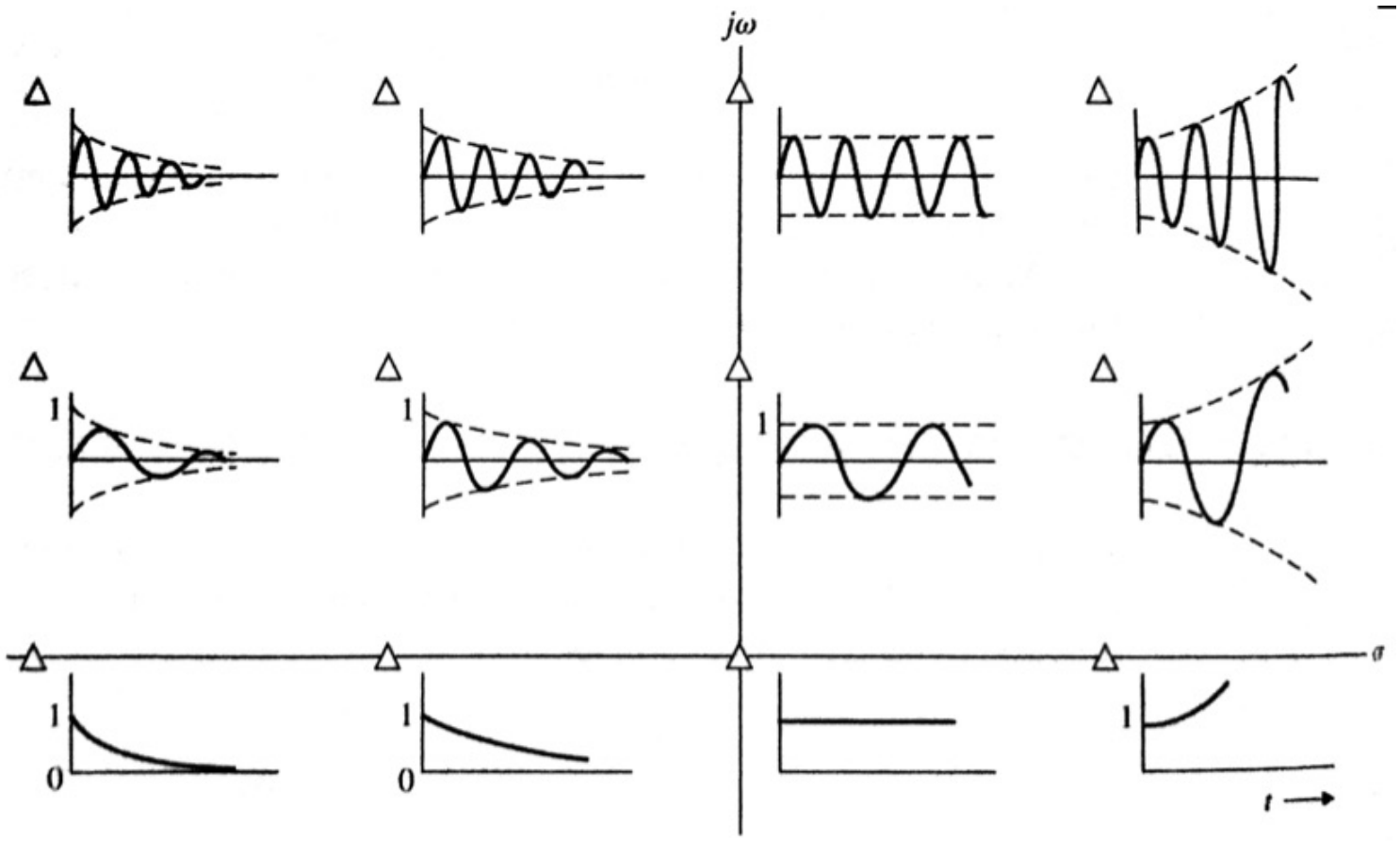


FIGURE 5.17 Impulse response for various root locations in the s-plane. (The conjugate root is not shown.)

Poles in the LHP are stable

Poles with 0 real part are undamped (sometimes called marginally stable)

Poles in the RHP (positive real part) are unstable

Problem P.17

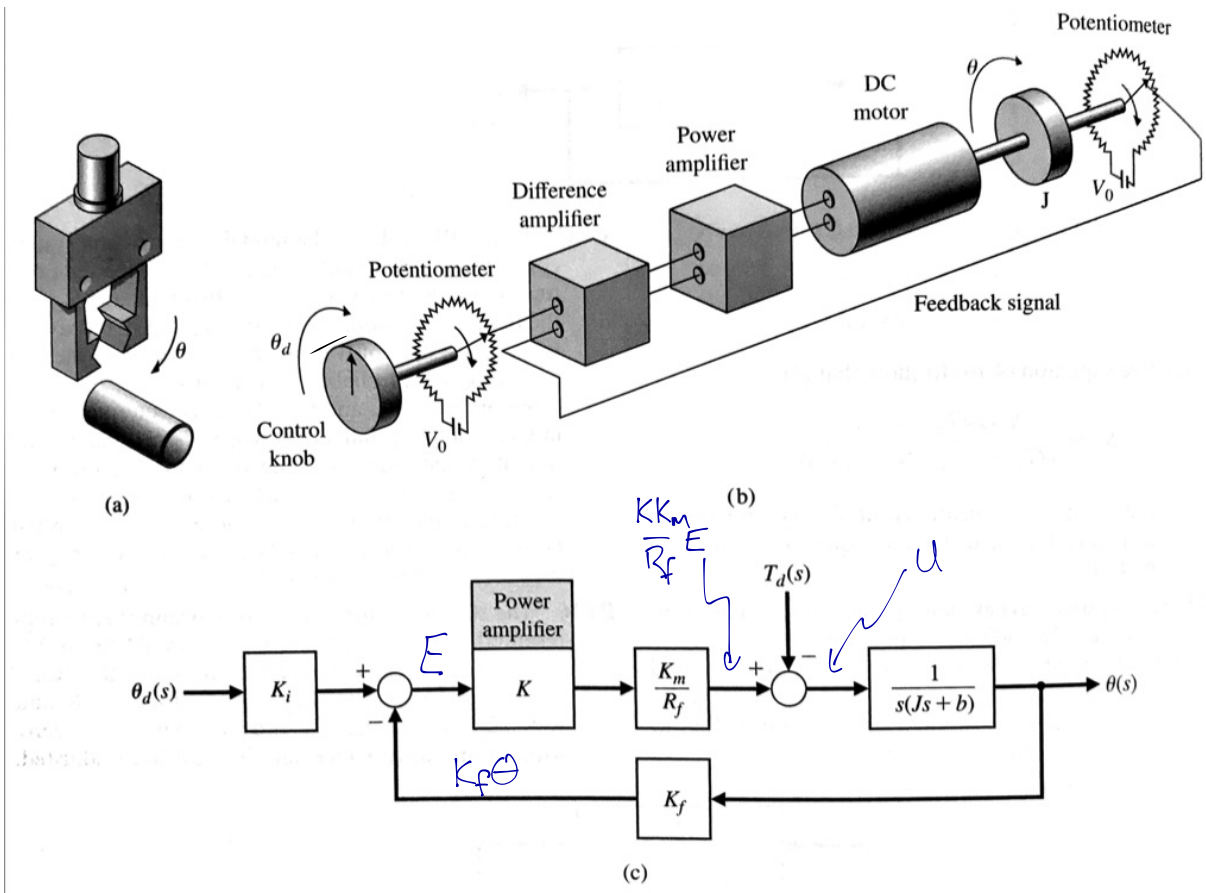


FIGURE P4.17 Robot gripper control.

a) What is $\theta(t)$ for a unit step in $\theta_d(t)$ when $K=20$?

b) Assume $\theta_d(t)=0$, find response to load described by $T_d(s) = A/s$

c) Determine steady-state error for a ramp input in $\theta_d(t)$. Assume $T_d = 0$

We're given:

$K_m = 60$ $\quad R_f = 2\Omega$ $\quad K_f = K_I = 1$ $\quad J = 0.2$ $\quad b = 1$

First, let's find the closed-loop TF for the system

$$\theta = \frac{1}{s(Js+b)}u = \frac{1}{s(Js+b)}\left[\frac{KK_m}{R_f}E - T_d\right]$$

$$E = K_i\theta_d - K_f\theta$$

$$\theta = \frac{1}{s(Js+b)}\left[\frac{KK_m}{R_f}\left(K_i\theta_d - K_f\theta\right) - T_d\right]$$

$$\theta = \frac{KK_mK_i}{sR_f(Js+b)}\theta_d - \frac{KK_mK_i}{sR_f(Js+b)}\theta - \frac{1}{s(Js+b)}T_d$$

$$\left(sR_f(Js+b) + KK_mK_i\right)\theta = KK_mK_i\theta_d - R_fT_d$$

Problem P.17 (cont.)

$$\left(sR_f(Js+b) + KK_mK_i\right)\Theta = KK_mK_i\Theta_d - R_fT_d$$

$$\Theta = \frac{KK_mK_i}{sR_f(Js+b)+KK_mK_i}\Theta_d - \frac{1}{s(Js+b)+KK_mK_i}T_d$$

with $K_m=60$, $R_f=2\Omega$, $K_f=K_I=1$, $J=0.2$, $b=1$

This becomes

$$\Theta = \frac{60K}{2s(0.2s+1)+60K}\Theta_d - \frac{1}{s(0.2s+1)+60K}T_d$$

a) When $K=20$, what is the response to a step in Θ_d? $(T_d=0)$

$$\Theta = \frac{1200}{2s(0.2s+1)+1200}\Theta_d = \frac{600}{0.2s^2+s+600}\Theta_d$$

For a step in $\Theta_d(t) \rightarrow \Theta_d(s)=\frac{1}{s}$

$$\boxed{\Theta = \frac{600}{s(0.2s^2+s+600)}}$$

To get the time response $\Theta(t)$ take the inverse Laplace transform of this

b) When $K=20$, what is the response to disturbance $T_d(s)=\frac{A}{s}$? $(\Theta_d=0)$

$$\Theta = \frac{-1}{s(0.2s+1)+600}T_d = \frac{-1}{0.2s^2+s+600}T_d$$

For $T_d=\frac{A}{s}$

$$\boxed{\Theta = \frac{-A}{s(0.2s^2+s+600)}}$$

To get the time response $\Theta(t)$ take the inverse Laplace transform of this

Problem P.17 (cont.)

We can also look at the steady-state resp. to this disturbance using the Final Value Theorem

$$\boxed{\lim_{t\to\infty}\theta(t) = \lim_{s\to 0} s\Theta(s) = \frac{\cancel{s}(-A)}{\cancel{s}(0.2s^2+s+600)} = \frac{A}{600}}$$

Q: What does that result mean physically?

- For a step disturbance of size A, the steady state response is $\frac{1}{600}(A)$
- Disturbances are attenuated by $\frac{1}{600}$

c) When $K=20$, what is the steady-state error to $\theta_d(t)=t$, $t>0$? (Assume $T_d=0$)

$$\Theta = \frac{600}{0.2s^2+s+600}\Theta_d \qquad \theta_d(t)=t \longrightarrow \Theta_d(s)=\frac{1}{s^2}$$

$$E = K_i\Theta_d - K_f\Theta = \Theta_d - \Theta = \frac{1}{s^2} - \frac{600}{s^2(0.2s^2+s+600)}$$

$$= \frac{0.2s^2+s+600}{s^2(0.2s^2+s+600)} - \frac{600}{s^2(0.2s^2+s+600)} = \frac{0.2s^2+s}{s^2(0.2s^2+s+600)}$$

Use the Final Value Theorem to calculate the error

$$\boxed{\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \frac{\cancel{s}(0.2s^2+s)}{s^{\cancel{2}}(0.2s^2+s+600)} = \frac{1}{600}}$$