Disturbance Signals in a Feedback Control System (Sec. 4.4)

Just more detail and examples of what we just discussed. We'll come back to it later.

Control of Transient Response (Sec. 4.5)

Q: What is the transient response?

response for all time prior to steady-state

But first... Example 2.5 – Transfer Function of a D.C. Motor

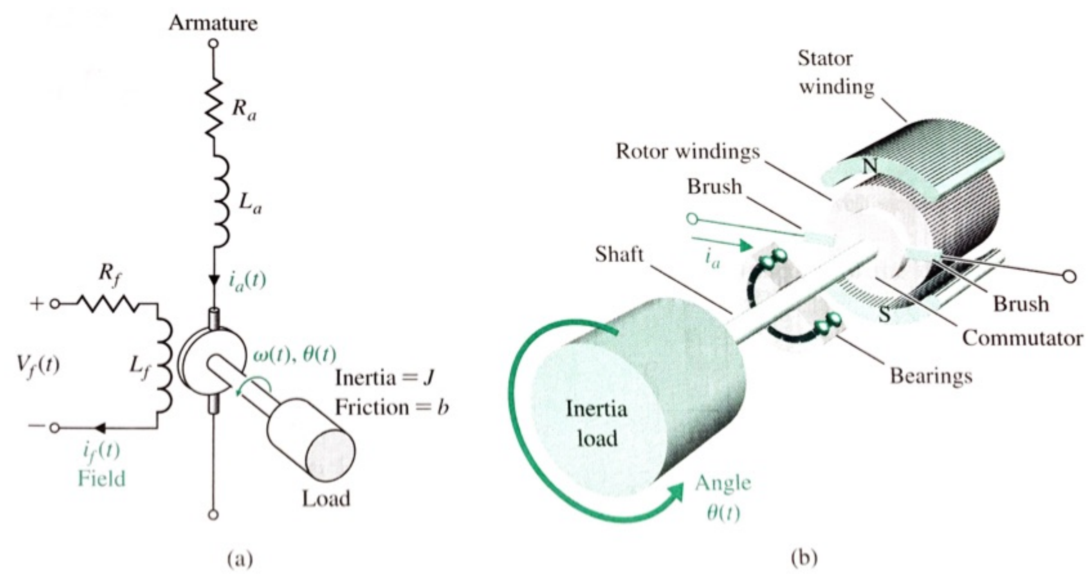


FIGURE 2.18
A DC motor (a) electrical diagram and (b) sketch.

Air gap flux $\phi(t) = K_f i_f(t)$ ← proportional to field current $i_f(t)$

Torque of motor varies linearly with $\phi(t)$:

$$T_M(t) = K_1 \phi(t) i_a(t) = K_1 K_f i_f(t) i_a(t)$$

Q: What must happen for this to be linear?

either $i_f(t)$ or $i_a(t)$ is constant while the other is treated as the input

Example 2.5 (cont.)

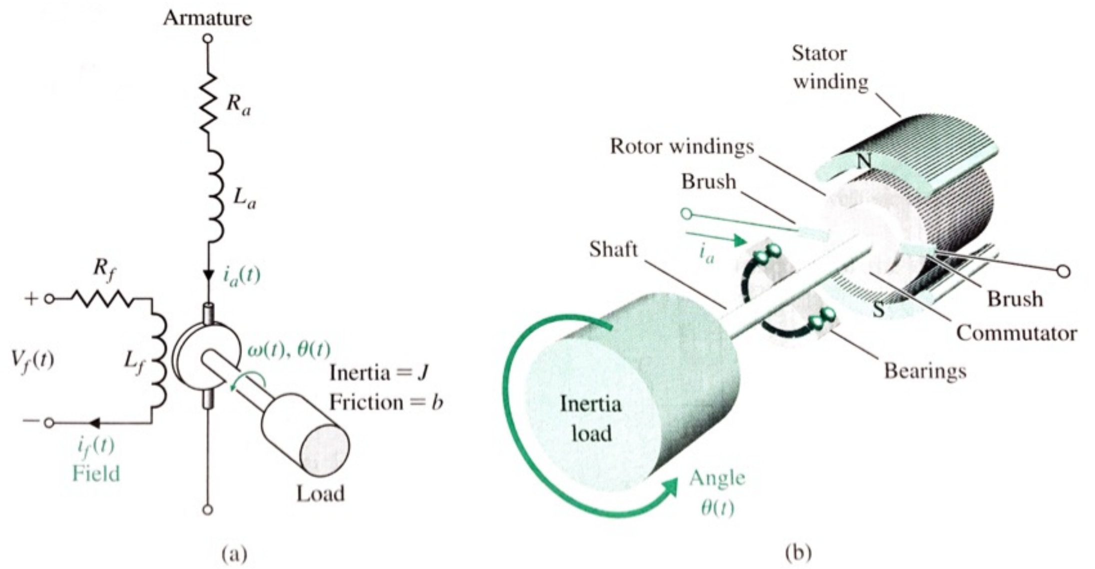


FIGURE 2.18
A DC motor (a) electrical diagram and (b) sketch.

Field Current Controlled DC Motor

$$T_m = K_1 K_f i_a i_f(t) \quad \leftarrow \quad i_a = \text{constant}$$

$$T_M(s) = \underbrace{(K_1 K_f I_a)}_{K_m \equiv \text{ motor constant}} I_f(s)$$

The field current is related to applied field voltage, $V_f(t)$, by

$$V_f(s) = (R_f + L_f s) I_f(s)$$

$R_f \equiv$ field resistance
$L_f \equiv$ field inductance

So $$\frac{I_f}{V_f} = \frac{1}{R_f + L_f s}$$

Motor torque equiv to torque at the load

$$T_m(s) = T_L(s) + T_d(s)$$

$T_L \equiv$ load torque
$T_d \equiv$ disturbance torque

The load torque for a rotating inertia is just

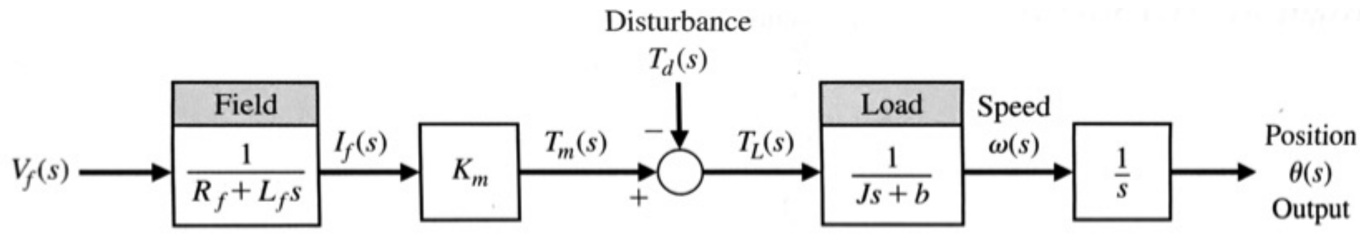
$$T_L(s) = (Js^2 + bs)\,\Theta(s)$$

$J \equiv$ moment of inertia of load about motor axis
$b \equiv$ "damping" losses in system

So $$\frac{\Theta(s)}{T_L(s)} = \frac{1}{Js^2 + bs}$$

Example 2.5 (cont.)

FIGURE 2.19 Block diagram model of field-controlled DC motor.



The TF from field voltage input to load position is then

$$\frac{\Theta(s)}{V_f(s)} = \frac{K_m}{s(Js+b)(L_f s + R_f)}$$

We can also write this in terms of motor time constants

$$\tau_f = \frac{L_f}{R_f} \quad \text{and} \quad \tau_L = \frac{J}{b}$$

So,

$$\frac{\Theta(s)}{V_f(s)} = \frac{K_m/(bR_f)}{s(\tau_f s + 1)(\tau_L s + 1)}$$

Usually $\tau_L \gg \tau_f$, so we can often ignore τ_f effects

Armature-Controlled DC Motor

Now, field current is constant and we control the armature current.

$$T_m(s) = (K_1 K_f I_f)\, I_a(s) = K_m I_a(s)$$

If a permanent magnet is used (which is common), K_m is a function of the magnet prop.

The relationship between armature current, $i_a(t)$, and the input voltage is

$$V_a(s) = (R_a + L_a s)\, I_a(s) + V_b(s) \longrightarrow \frac{I_a}{[V_a + V_b]} = \frac{1}{R_a + L_a s}$$

where

$$V_b(s) \equiv \text{back emf} \leftarrow \text{prop to motor speed}$$

so we can write

$$V_b(s) = K_b \omega(s) = K_b (s\Theta(s))$$

Example 2.5 (cont.)

The armature current can then be written as:

$$I_a(s) = \frac{V_a(s) - V_b(s)}{R_a - L_a s} = \frac{V_a(s) - K_b \omega(s)}{R_a - L_a s}$$

As before, we can write the load torque as

$$T_L(s) = (Js^2 + bs)\Theta(s) = T_m(s) - T_d$$

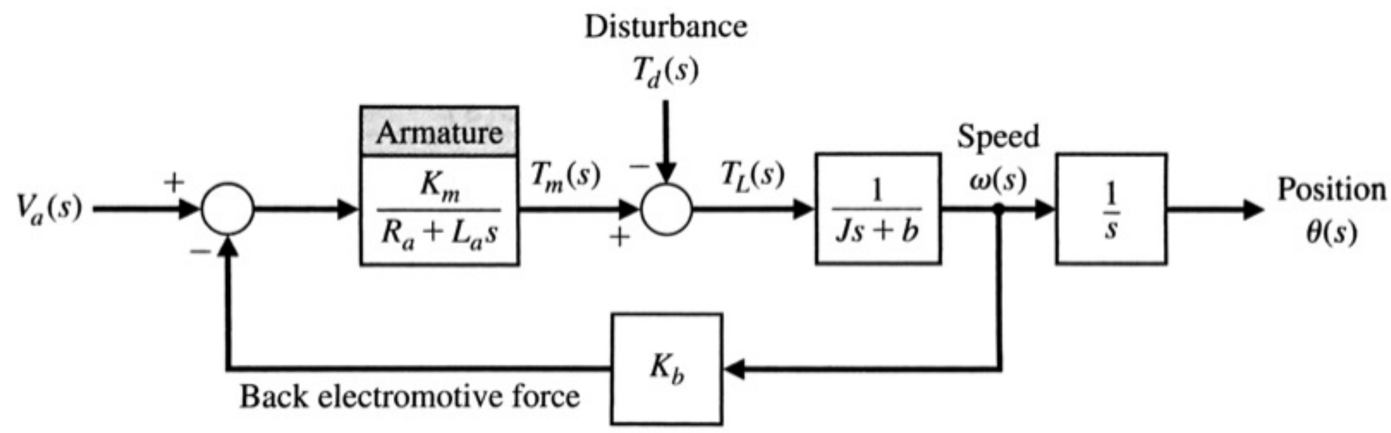


FIGURE 2.20 Armature-controlled DC motor.

Assume $T_d(s) = 0$ (for now)

$$\frac{\Theta(s)}{V_a(s)} = \frac{K_m}{s\left[(R_a + L_a s)(Js + b) + K_b K_m\right]} = \frac{K_m}{s\left[L_a J s^2 + (R_a J + L_a b)s + (R_a b + K_b K_m)\right]}$$

$$= \frac{K_m / L_a J}{s\underbrace{\left[s^2 + \left(\frac{R_a J + L_a b}{L_a J}\right)s + \frac{R_a b + K_b K_m}{L_a J}\right]}} = \frac{K_m / L_a J}{s\left[s^2 + 2\zeta\omega_n s + \omega_n^2\right]}$$

Q: Does the form of this part of the TF look familiar?

where $2\zeta\omega_n = \frac{R_a J + L_a b}{L_a J}$

$\omega_n^2 = \frac{R_a b + K_b K_m}{L_a J}$

The response can be oscillatory!

Usually $\tau_a = \frac{L_a}{R_a}$ is very small, (or equiv $L_a \ll R_a$) so we can write

$$\frac{\Theta}{V_a} = \frac{K_m}{s\left[R_a(Js + b) + K_B K_m\right]} = \frac{K_m}{s\left[R_a J s + (K_B K_m + R_a b)\right]}$$

Define time constant τ_1

$$\tau_1 \equiv \frac{R_a J}{K_B K_m + R_a b}$$

so

$$\frac{\Theta}{V_a} = \frac{K_m / (R_a b + K_B K_m)}{s(\tau_1 s + 1)}$$

Also note that $K_m = K_b$

Okay… back to:

Control of Transient Response (Sec. 4.5)

Imagine our armature-controlled motor subject to a large inertia load (J is large)

$$\frac{\Theta}{V_a} = \frac{K_1}{s(\tau_1 s + 1)} \qquad \text{where } K_1 = \frac{K_m}{R_a b + K_B K_m}$$

$$\tau_1 \equiv \frac{R_a J}{K_B K_m + R_a b}$$

We want to maintain a constant speed $\omega(s) = s\Theta(s)$.

Q: What's the TF between input voltage V_a and $\omega(s)$?

Just mult position TF by s (remember that this is like taking the deriv)

$$\Theta = \frac{K_1}{s(\tau_1 s + 1)} V_a \qquad \text{mult by } s \rightarrow \quad s\Theta = \frac{\cancel{s} K_1}{\cancel{s}(\tau_1 s + 1)} V_a$$

$$\frac{s\Theta}{V_a} = \frac{\omega(s)}{V_a(s)} = \frac{K_1}{\tau_1 s + 1}$$

Q: If we give a step-change in voltage, what's the response?

$$\omega(s) = \frac{K_1}{\tau_1 s + 1} V_a(s) = \frac{K_1}{\tau_1 s + 1}\left(\frac{1}{s}\right)$$

$$\omega(s) = \frac{K_1}{\tau_1 s^2 + s}$$

$$\omega(s) = \frac{a_1}{s} + \frac{a_2}{s + 1/\tau_1} \qquad \leftarrow \text{partial fraction expansion}$$

$$a_1 = s\left(\frac{K_1}{(\tau_1 s + 1)}\right)\Bigg|_{s=0} = K_1 \qquad a_2 = \left(\cancel{s + 1/\tau_1}\right)\left(\frac{K_1/\tau_1}{s(\cancel{s + 1/\tau_1})}\right)\Bigg|_{s=-1/\tau_1} = \frac{K_1/\tau_1}{-1/\tau_1} = -K_1$$

$$\omega(s) = \frac{K_1}{s} + \frac{(-K_1)}{s + 1/\tau_1} \qquad \leftarrow \text{Look up in Inverse Laplace Table}$$

$$\boxed{\omega(t) = K_1 - K_1 e^{-t/\tau_1}}$$

Control of Transient Response (cont.)

Q: What does the response of $\omega(t) = K_1 - \underbrace{K_1 e^{-t/\tau_1}}$ look like?

as $t \to \infty$, this term goes to 0

So, we exponentially approach K_1 given a unity magnitude step in V_a

Q: What dictates how fast we approach this setpoint?

$$\tau_1 = \frac{R_a J}{R_a D + K_b K_m}$$

Q: What of this can we control/change?

They're all physical parameters.

We may be able to reduce J (load moment of inertia) but most others are not easily changed.

Q: So, what can we do if we want a faster response from our system?

Feedback control - compare actual speed to the desired speed and adjust V_a to limit the difference between them.

We'll come back to this example later, once we know more about Feedback Control design.