

## Chapter 4 - Feedback Control System Characteristics

Closed-loop system - use measurement(s) of system output/state to decide input

open-loop system - no measurement(s) of system are used to inform the input

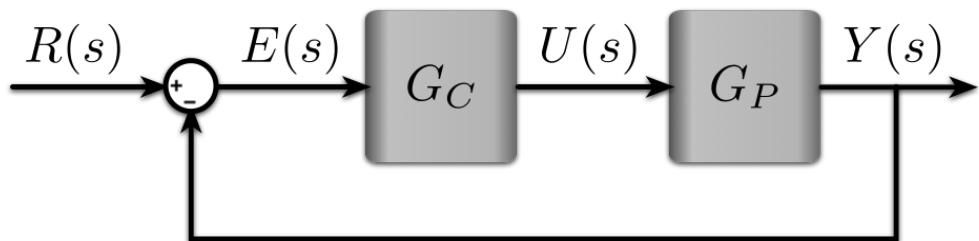
Q: What are some advantages to closed-loop feedback control systems?

- adjust/complement for changes in sys. parameters
- reject disturbance to the system
- reduce steady-state error

But more complex than open-loop sys.

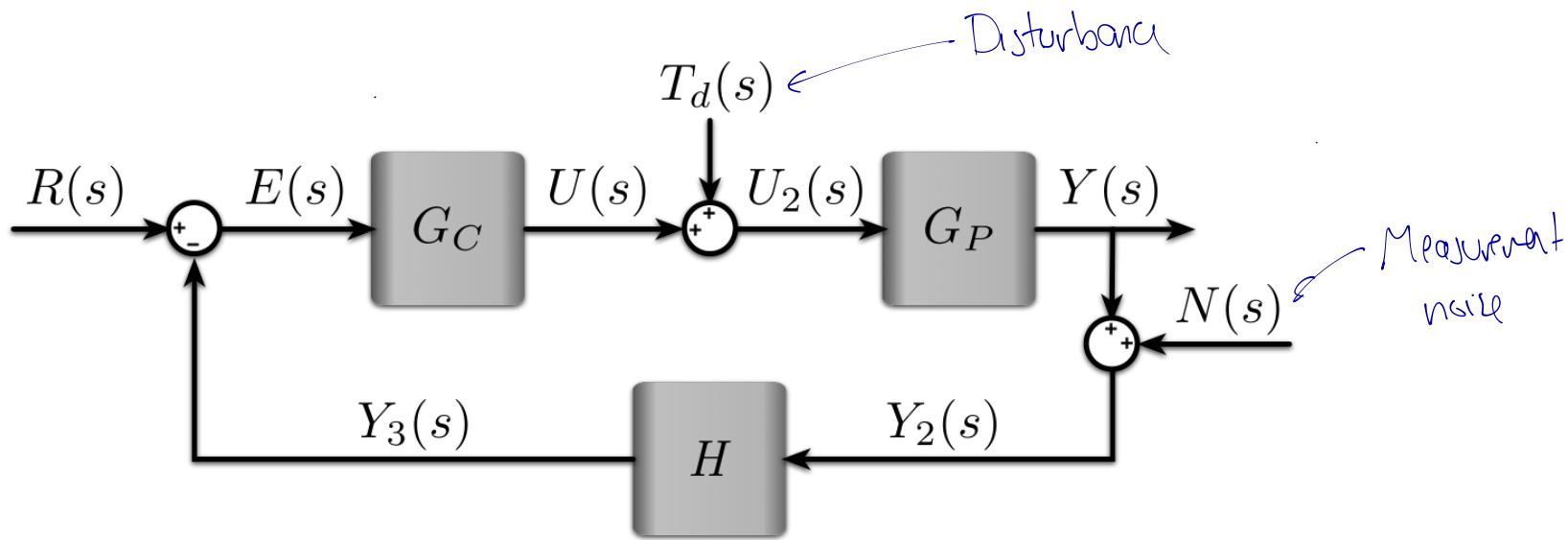
## Error Signal Analysis (Sec. 4.2)

generally output compared to some reference command



$$E(s) = R(s) - Y(s) \quad \leftarrow \text{error between reference and output}$$

## Error Signal Analysis (cont.)



We want to look at how sensitive the system perf. is to disturbances and noise

Q: How?

Look at the TF between these inputs and the tracking error,  $E(s)$

Let  $H(s)=1$  and start by writing  $\Psi(s)$

$$\Psi(s) = G_p U_2 = G_p (U + T_d) = G_p (G_c E + T_d) = G_p (G_c (R - \Psi_3) + T_d)$$

$$\Psi_3 = \Psi_2 = \Psi + N$$

$$\Psi = G_p [ G_c [ R - (\Psi + N) ] + T_d ] = G_c G_p R - G_c G_p \Psi - G_c G_p N + G_p T_d$$

$$(1 + G_c G_p) \Psi = G_c G_p R - G_c G_p N + G_p T_d$$

$$\Psi = \frac{G_c G_p}{1 + G_c G_p} R - \frac{G_c G_p}{1 + G_c G_p} N + \frac{G_p}{1 + G_c G_p} T_d$$

$$E = R - \Psi = \frac{1 + G_c G_p}{1 + G_c G_p} R - \left[ \frac{G_c G_p}{1 + G_c G_p} R - \frac{G_c G_p}{1 + G_c G_p} N + \frac{G_p}{1 + G_c G_p} T_d \right]$$

$$E = \frac{1}{1 + G_c G_p} R + \frac{G_c G_p}{1 + G_c G_p} N - \frac{G_p}{1 + G_c G_p} T_d$$

## Error Signal Analysis (cont.)

$$E = \frac{1}{1+G_c G_p} R + \frac{G_c G_p}{1+G_c G_p} N - \frac{G_p}{1+G_c G_p} T_d$$

Define  $L(s) = G_c(s)G_p(s)$  ← Loop Gain - we'll later see that this can tell us a lot about our system

$$E = \frac{1}{1+L} R + \frac{L}{1+L} N - \frac{G_p}{1+L} T_d$$

Define  $F(s) = 1+L(s)$  and

Sensitivity Function

$$S(s) = \frac{1}{F(s)} = \frac{1}{1+L(s)}$$

Complementary Loop Function

$$C(s) = \frac{L(s)}{1+L(s)}$$

In terms of  $C(s)$  and  $S(s)$ , the error is:

$$E = SR + CN - SG_p T_d$$

Q: So, how can we minimize the tracking error?

We only have control over  $S$  and  $C$ . Making both small would be ideal.

But,

$$S(s) + C(s) = \frac{1}{1+L(s)} + \frac{L(s)}{1+L(s)} = 1 \quad \leftarrow \text{We must navigate some type of design compromise}$$

Q: What does it mean for a TF to be "small"?

We'll see in more detail soon when we look at frequency response.

For now, just think of it as  $|TF|$  over some frequency range

How much is the input amplified or attenuated if its content is in this freq. range?

## Error Signal Analysis (cont.)

$$E = SR + CN - SG_p T_d$$

Let's look 1<sup>st</sup> at the influence of the disturbance:

$$E_{T_d} = -SG_p T_d = \left( \frac{1}{1+L} \right) G_p T_d \quad \leftarrow \text{Want } L \text{ to be large so } \frac{1}{1+L} \text{ is small}$$

This means we want  $G_c G_p$  to be large

We usually only have control over  $G_c$  ( $G_p$  is the physical system. For now, assume it's fixed)

So, we need  $G_c$  to be large to attenuate the effects of  $T_d$

Now, let's look at the effects of measurement noise

$$E_N = CN = \left( \frac{L}{1+L} \right) N$$

We want this term to be small

So, we need  $L = G_c G_p$  to be small, meaning

we need  $G_c$  to be small to attenuate the effects of  $N$

## Sensitivity of Control Systems to Parameter Variations (Sec. 4.3)

Another cause of error in system control is changes in the system.

Define a small change in the plant TF  $\Delta G_p(s)$

This results in a small change in error  $\Delta E(s)$  such that

$$E + \Delta E = \frac{1}{1 + G_c(G_p + \Delta G_p)} R \quad \leftarrow \text{For now, ignore } N \text{ and } T_d$$

So the change in tracking error is

$$\begin{aligned} \Delta E &= \frac{1}{1 + G_c(G_p + \Delta G_p)} R - E \\ &= \frac{1}{1 + G_c(G_p + \Delta G_p)} R - \frac{1}{1 + G_c G_p} R = \\ &= \left[ \frac{\frac{1 + G_c G_p}{(1 + G_c G_p + G_c \Delta G_p)(1 + G_c G_p)}} - \frac{(1 + G_c G_p + G_c \Delta G_p)}{(1 + G_c G_p + G_c \Delta G_p)(1 + G_c G_p)} \right] R \end{aligned}$$

$$\Delta E = \frac{-G_c \Delta G_p}{(1 + G_c G_p + G_c \Delta G_p)(1 + G_c G_p)}$$

$G_c G_p$  is usually "larger" than  $G_c \Delta G_p$  so

$$\Delta E \approx \frac{-G_c \Delta G_p}{(1 + G_c G_p)^2} R = \frac{-G_c \Delta G_p}{(1 + L)^2} R \quad \leftarrow \text{So, we want } L(s) \text{ to be large to reduce the effects of changes in } G_p$$

So, we want  $G_c$  to be large to limit the effects of changes (or error) in the plant.

## Sensitivity of Control Systems to Parameter Variations (cont.)

Q: How can we define / quantify sensitivity?

To change in system TF

To change in plant TF

Define total system TF as

$$T = \frac{Y(s)}{R(s)}$$

So, the sensitivity is:

$$S = \left. \frac{\partial T/T}{\partial G_p/G_p} \right\} \text{for infinitesimal change} \rightarrow S = \frac{\partial \ln T}{\partial \ln G} = \frac{\partial \ln T}{\partial \ln G}$$

Q: What is the open-loop system sensitivity?

1 ← changes in the plant result in equivalent changes to the sys. TF

Let's look at our "normal" feedback system

$$T = \frac{G_c G_p}{1 + G_c G_p} \quad \frac{\partial T}{\partial G_p} = \frac{G_c}{(1 + G_c G_p)^2}$$

So

$S_T^T$  ← Sensitivity of T  
 $S_{G_p}^T$  ← To changes in  $G_p$

$$S_{G_p}^T = \frac{\partial T/T}{\partial G_p/G_p} = \frac{\frac{\partial T}{\partial G_p}}{T} = \frac{G_c}{(1 + G_c G_p)^2} \left[ \frac{G}{G_c/(1 + G_c G_p)} \right]$$

$$\boxed{S_{G_p}^T = \frac{1}{1 + G_c G_p}}$$

← matches our earlier definition  $S(s)$

← So if  $G_c$  is large, sensitivity to plant changes is minimized

We can follow a similar process to look at the sensitivity to parameters within the TF.