MCHE 474: Control Systems
Fall 2017 – Homework 5

Assigned:	Sunday, November 5th
Due:	Tuesday, November 7th, 9pm

Assignment: From *Modern Control Systems (13th Edition)* by Richard Dorf and Robert Bishop, solve problems:
E7.6, E7.9, E7.13, E7.17 – just sketch the loci, E7.22

Submission: Submission is not required. Solutions will be posted shortly after the due date listed above.

Problem E7.6

E7.6 One version of a space station is shown in Figure E7.6 [28]. It is critical to keep this station in the proper orientation toward the Sun and the Earth for generating power and communications. The orientation controller may be represented by a unity feedback system with an actuator and controller, such as

$$L(s) = G_c(s)G(s) = \frac{15K}{s(s^2 + 15s + 75)}.$$

Sketch the root locus of the system as K increases. Find the value of K that results in an unstable system.

Answers: $K = 75$

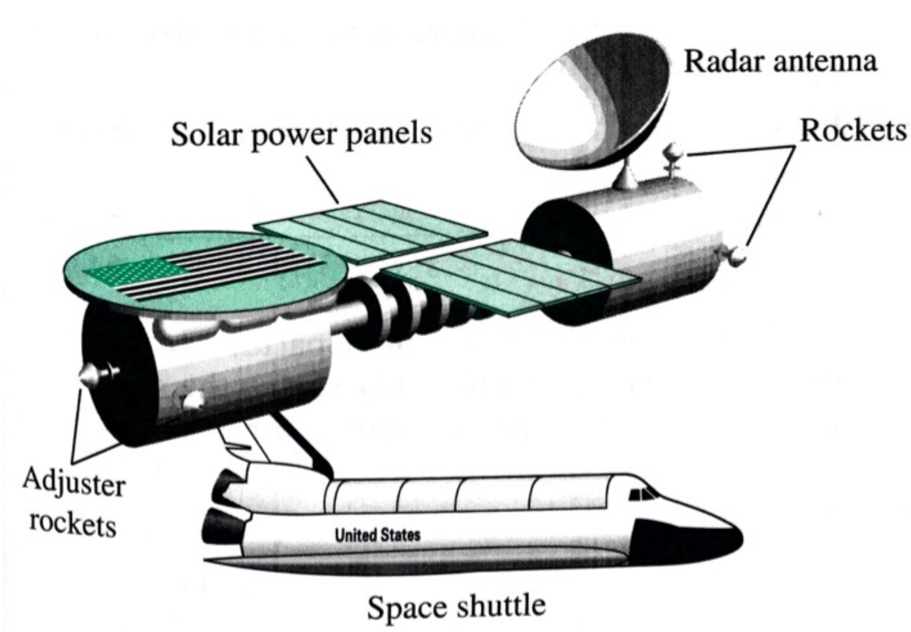


FIGURE E7.6 Space station.

OL poles at $s=0$

$$s = \frac{-15 \pm \sqrt{225-4(75)}}{2}$$

$$s = \frac{-15 \pm \sqrt{-75}}{2} = \frac{-15 \pm 5\sqrt{3}}{2}$$

Separate loci - 3

$$\sigma_A = \frac{\sum(-p_j) + \sum(-z_i)}{n-m} = \frac{-15}{3} = -5$$

$$\phi_A = \left(\frac{2k+1}{3}\right)180^\circ \quad k=0,1,2 \quad = 60^\circ, 180^\circ, 300^\circ$$

Breakaway point:

$$s(s^2+15s+75) + 15K = 0$$

$$K = -\frac{s^3+15s^2+75s}{15}$$

$$\frac{\partial K}{\partial s} = \frac{-(3s^2+30s+75)}{15} = 0$$

↑ $s=-5$

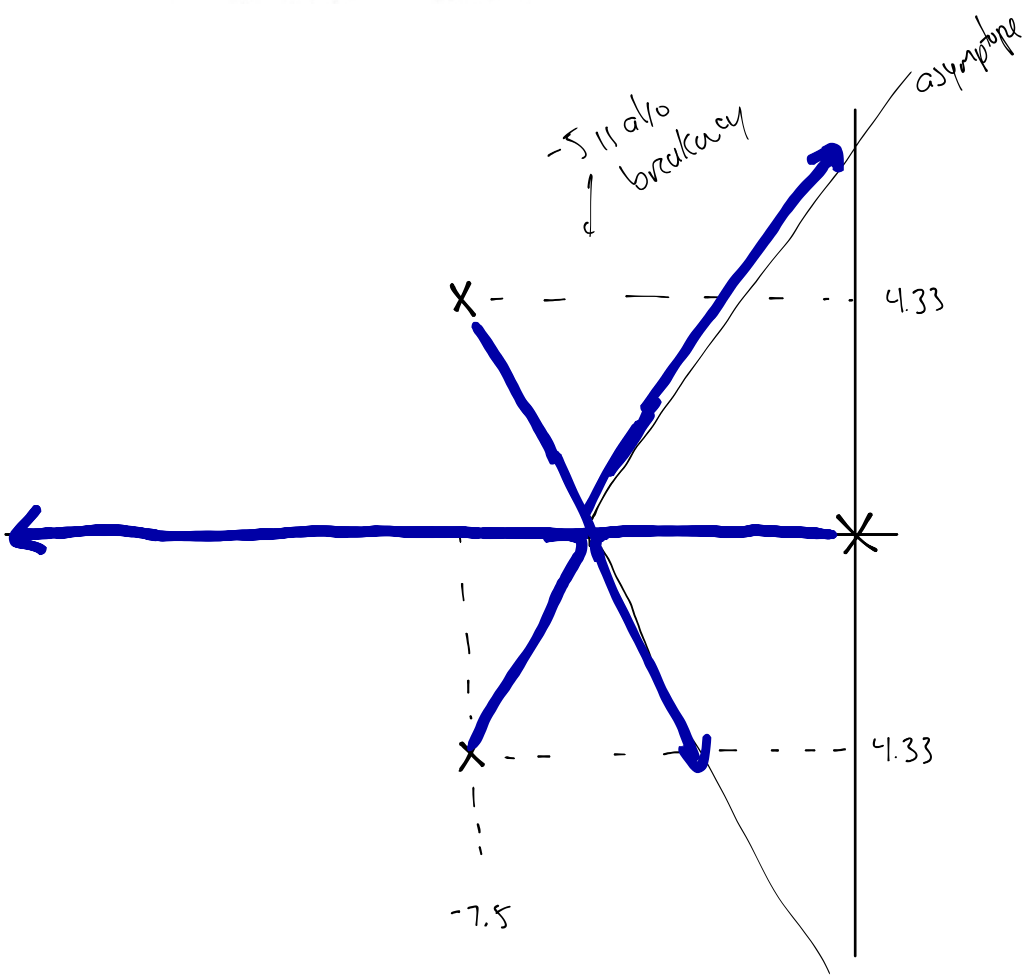
Angle of departure:

for $s = -7.5 + \frac{5}{2}\sqrt{3}$

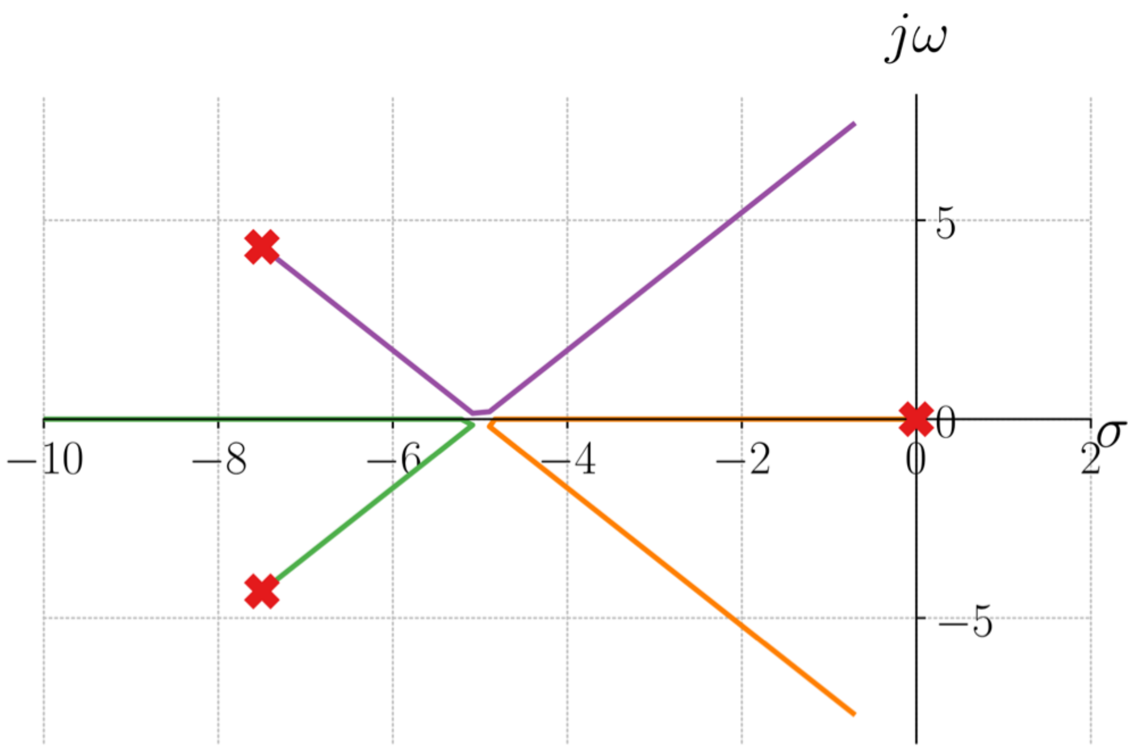
$$180^\circ - \left(\sum \text{angles to poles}\right) + \sum\left(\text{angles to zeros}\right)$$

$$180^\circ - \left(90^\circ + \left(180 - \tan^{-1}\left(\frac{5\sqrt{3}/2}{7.5}\right)\right)\right)$$

$$-90 + 30 = 60^\circ$$



Problem E7.6 (cont.)



Problem E7.9

E7.9 The primary mirror of a large telescope can have a diameter of 10 m and a mosaic of 36 hexagonal segments with the orientation of each segment actively controlled. Suppose this unity feedback system for the mirror segments has the loop transfer function

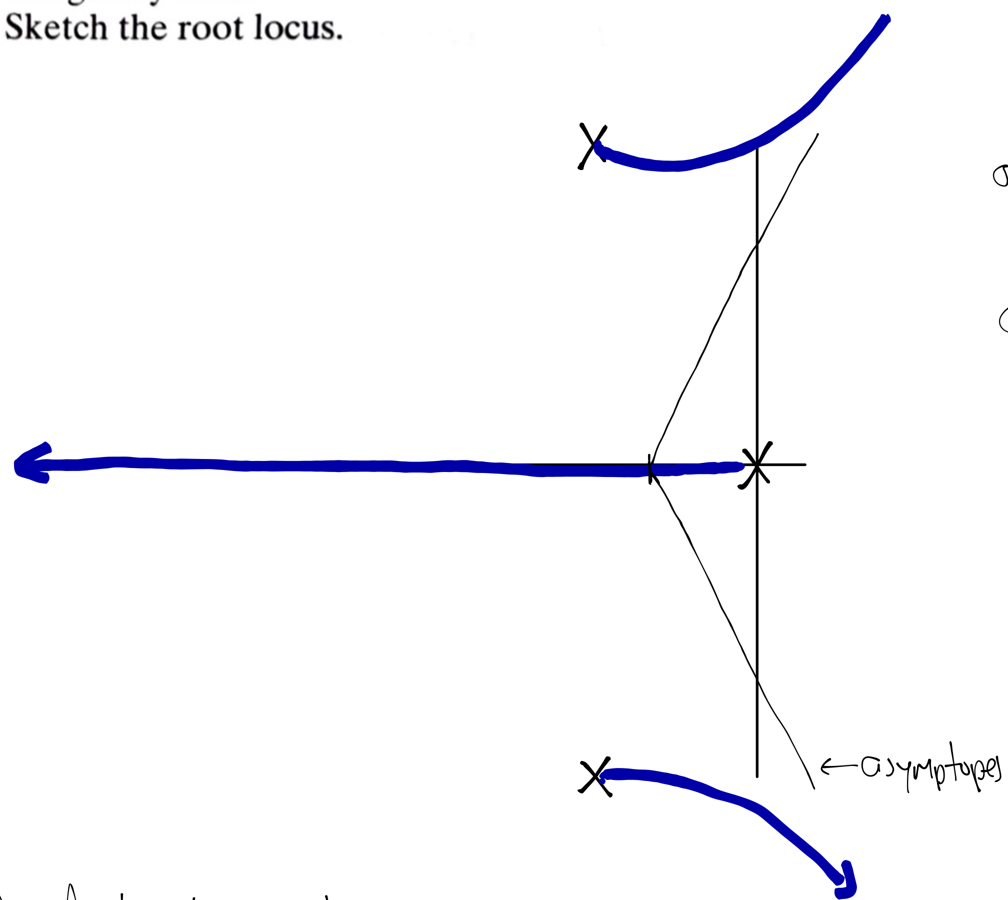
$$L(s) = G_c(s)G(s) = \frac{K}{s(s^2 + 2s + 5)}.$$

(a) Find the asymptotes and sketch them in the s-plane.
(b) Find the angle of departure from the complex poles.
(c) Determine the gain when two roots lie on the imaginary axis.
(d) Sketch the root locus.

poles at $s=0$ and $s = \frac{-2 \pm \sqrt{4-4(5)}}{2}$

$s = -1 \pm 2i$

a)



$$\sigma_A = \frac{\Sigma(-p_i) + \Sigma(-z_i)}{n-n} = \frac{-2}{3}$$

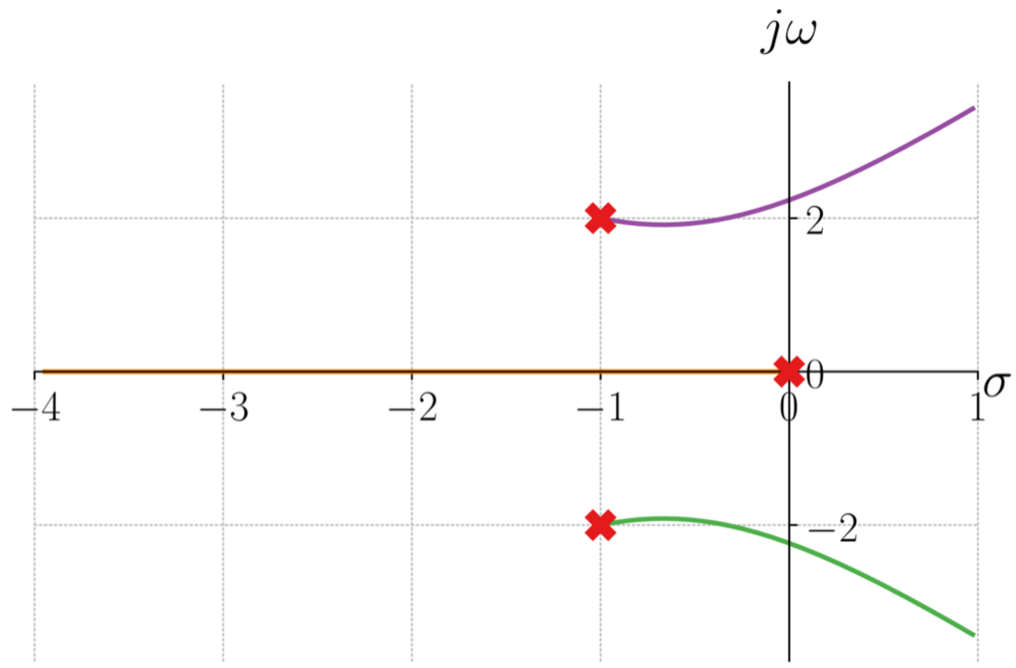
$$\phi_A = \left(\frac{2k+1}{n-m}\right) 180^\circ \quad k = 0,1,2$$

$$\phi_A = \pm 60^\circ,\ 180^\circ$$

b) Angle of departure

$$180^\circ - \Sigma(\text{angles to other poles}) = 180^\circ - \left(90 + \left(180^\circ - \tan^{-1}\left(\frac{2}{1}\right)\right)\right) = -26.57^\circ$$

d)



Problem E7.13

E7.13 A unity feedback system has a loop transfer function

$$L(s) = G_c(s)G(s) = \frac{4(s+z)}{s(s+1)(s+3)}.$$

(a) Draw the root locus as z varies from 0 to 100. (b) Using the root locus, estimate the percent overshoot and settling time (with a 2% criterion) of the system at $z = 0.6$, 2, and 4 for a step input. (c) Determine the actual overshoot and settling time at $z = 0.6$, 2, and 4.

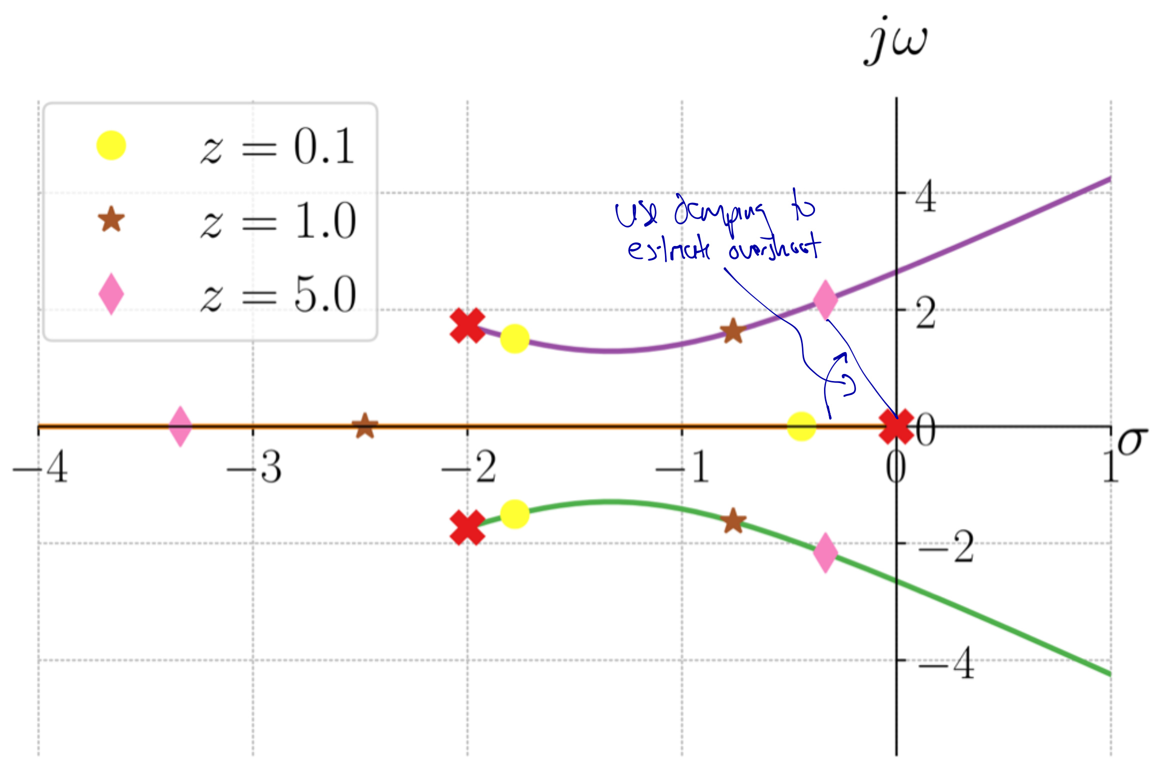
a) Write the CL TF

$$\frac{L}{1+L} = \frac{4(s+z)}{s(s+1)(s+3)+4(s+z)} \qquad \text{Char eq: } s(s+1)(s+3)+4(s+z)$$

Divide the char eq by all "non-z" terms $[s(s+1)(s+3)+4s]$

$$\frac{s(s+1)(s+3)+4s+4z}{s(s+1)(s+3)+4s} = 1 + \underbrace{\frac{4z}{s(s+1)(s+3)+4s}}_{\text{we draw the locus for this}}$$

$$\frac{4z}{s(s^2+4s+3)+4s} = \frac{4z}{s^3+4s^2+7s}$$



use $\zeta\omega_n$ distance to estimate settling time

Problem E7.17

E7.17 A control system, as shown in Figure E7.17, has process

$$G(s) = \frac{1}{s(s-2)}.$$

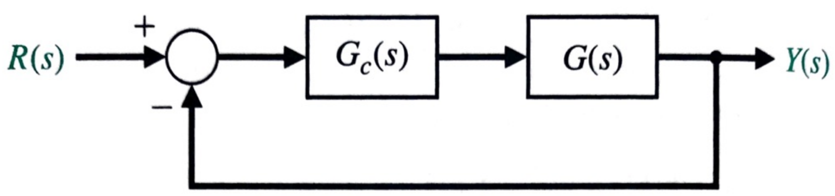


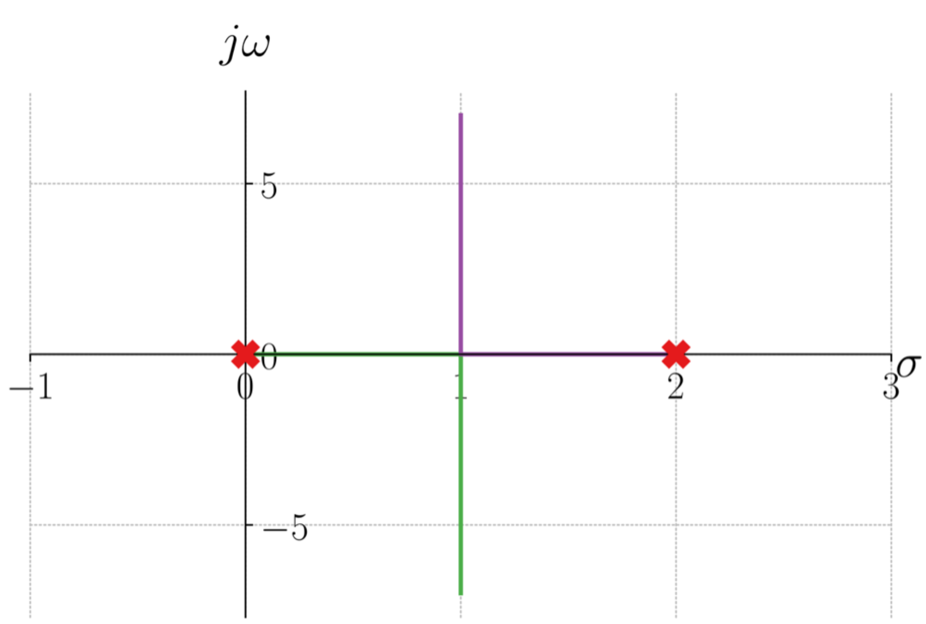
FIGURE E7.17 Feedback system.

(a) When $G_c(s) = K$, show that the system is always unstable by sketching the root locus. (b) When

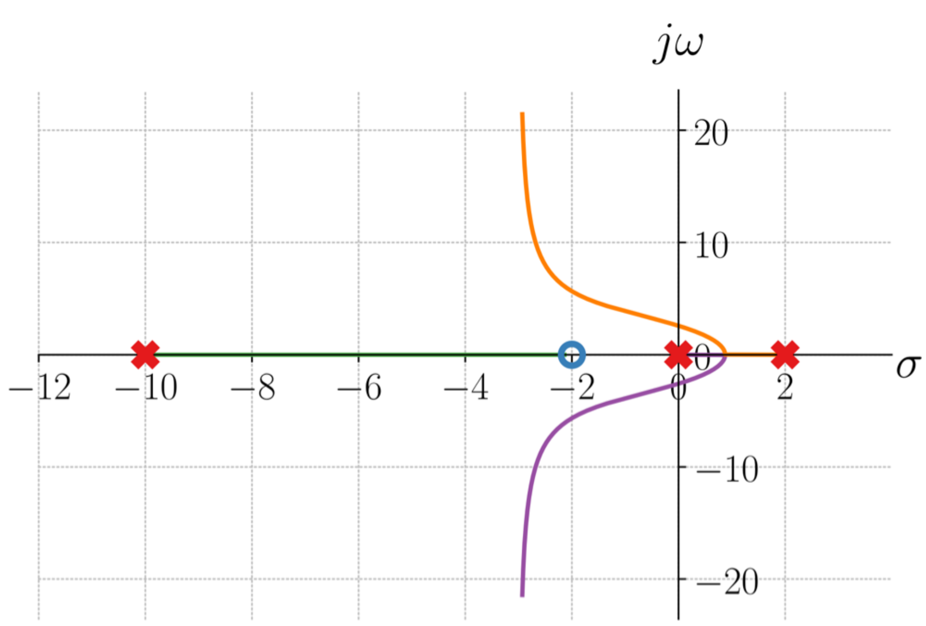
$$G_c(s) = \frac{K(s+2)}{s+10},$$

sketch the root locus and determine the range of K for which the system is stable. Determine the value of K and the complex roots when two roots lie on the $j\omega$-axis.

a)



b)



Problem E7.22

E7.22 A high-performance missile for launching a satellite has a unity feedback system with a loop transfer function

$$L(s) = G_c(s)G(s) = \frac{K(s^2 + 18)(s + 2)}{(s^2 - 2)(s + 12)}.$$

Sketch the root locus as K varies from $0 < K < \infty$.