MCHE 474: Control Systems
Fall 2017 – Homework 4

Assigned: Wednesday, October 11th
Due: Wednesday, October 18th, 5pm

Assignment: From *Modern Control Systems (13th Edition)* by Richard Dorf and Robert Bishop, solve problems:
E4.14, E4.15, P4.4, P4.5, P4.16, E5.7, E5.8, E5.19

Submission: Emailed *single* pdf document:

- to joshua.vaughan@louisiana.edu
- with subject line `CLID-MCHE474-HW4`, where `CLID` is replaced with your CLID
- and that has a *single* pdf attached with filename `CLID-MCHE474-HW4`, where `CLID` is replaced with your CLID
- *Note:* Submissions with incorrect filenames or submitted as multiple images/pdfs will be rejected.

Problem E4.14

E4.14 Consider the unity feedback system shown in Figure E4.14. The system has two parameters, the controller gain K and the constant K_1 in the process.

(a) Calculate the sensitivity of the closed-loop transfer function to changes in K_1.

(b) How would you select a value for K to minimize the effects of external disturbances, $T_d(s)$?

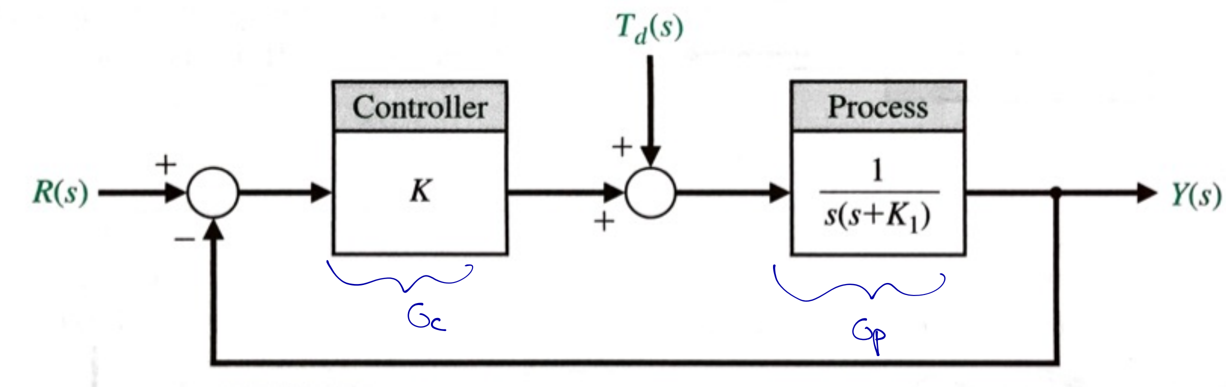


FIGURE E4.14 Closed-loop feedback system with two parameters, K and K_1.

a) The closed-loop TF is:

$$\frac{Y}{R} = T(s) = \frac{G_c G_p}{1 + G_c G_p} = \frac{K}{s(s+K_1) + K} = \frac{K}{s^2 + K_1 s + K}$$

The sensitivity is:

$$S^T_{K_1} = \frac{\partial T / T}{\partial K_1 / K_1} = \frac{\partial T}{\partial K_1} \frac{K_1}{T} = \frac{\partial T}{\partial K_1} \frac{(s^2 + K_1 s + K) K_1}{K}$$

$$\frac{\partial T}{\partial K_1} = K(s^2 + K_1 s + K)^{-1} = K\left(\frac{s}{(s^2 + K_1 s + K)^2}\right)$$

$$\frac{\partial T}{\partial K_1} \frac{K_1}{T} = \frac{sK}{(s^2 + K_1 s + K)^2} \frac{(s^2 + K_1 s + K) K_1}{K} \approx \boxed{\frac{s K_1}{s^2 + K_1 s + K}}$$

b) $Y = G_p T_d + G_c G_p (R - Y)$ ← make $R = 0$

$Y = G_p T_d - K G_p (R - Y)$

$(1 + K G_p) Y = G_p T_d$

$$\boxed{\frac{Y}{T_d} = \frac{G_p}{1 + K G_p}}$$

← so increase K to limit the effects of T_d

Problem E4.15

E4.15 Reconsider the unity feedback system discussed in E4.14. This time select $K = 100$ and $K_1 = 50$. The closed-loop system is depicted in Figure E4.15.

(a) Calculate the steady-state error of the closed-loop system due to a unit step input, $R(s) = 1/s$, with $T_d(s) = 0$. Recall that the tracking error is defined as $E(s) = R(s) - Y(s)$.

(b) Calculate the steady-state response, $y_{ss} = \lim_{t \to \infty} y(t)$, when $T_d(s) = 1/s$ and $R(s) = 0$.

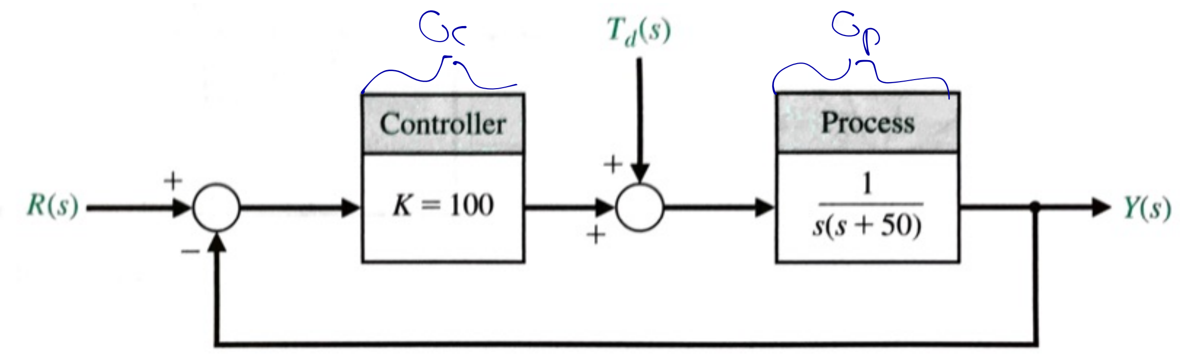


FIGURE E4.15 Closed-loop feedback system with $K = 100$ and $K_1 = 50$.

a) The closed-loop TF is:

$$\frac{Y}{R} = T(s) = \frac{G_c G_p}{1 + G_c G_p} = \frac{K}{s(s+K_1) + K} = \frac{K}{s^2 + K_1 s + K} = \frac{100}{s^2 + 50s + 100}$$

$$Y = \left[\frac{100}{s^2 + 50s + 100}\right] R$$

$$E = R - Y = R - \left[\frac{100}{s^2 + 50s + 100}\right] R = \left[\frac{s^2 + 50s}{s^2 + 50s + 100}\right] R$$ ← if R is a step $R(s) = \frac{1}{s}$

$$E = \left[\frac{s^2 + 50s}{s^2 + 50s + 100}\right]\left[\frac{1}{s}\right]$$

$$\boxed{\lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \cancel{s}\left[\frac{s^2 + 50s}{\cancel{s}(s^2 + 50s + 100)}\right] = 0}$$

b) In E4.14, we found

$$\frac{Y}{T_d} = \frac{G_p}{1 + KG_p} = \frac{1}{s^2 + 50s + K}$$ so $$Y(s) = \left[\frac{1}{s^2 + 50s + 100}\right] T_d$$ ← For a step $T_d = 1/s$

$$\boxed{\lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s) = \lim_{s \to 0} \cancel{s}\left[\frac{1}{\cancel{s}(s^2 + 50s + 100)}\right] = \frac{1}{100}}$$

Problem P4.4

P4.4 A control system has two forward paths, as shown in Figure P4.4. (a) Determine the overall transfer function $T(s) = Y(s)/R(s)$. (b) Calculate the sensitivity, S_G^T, using Equation (4.16). (c) Does the sensitivity depend on $U(s)$ or $M(s)$?

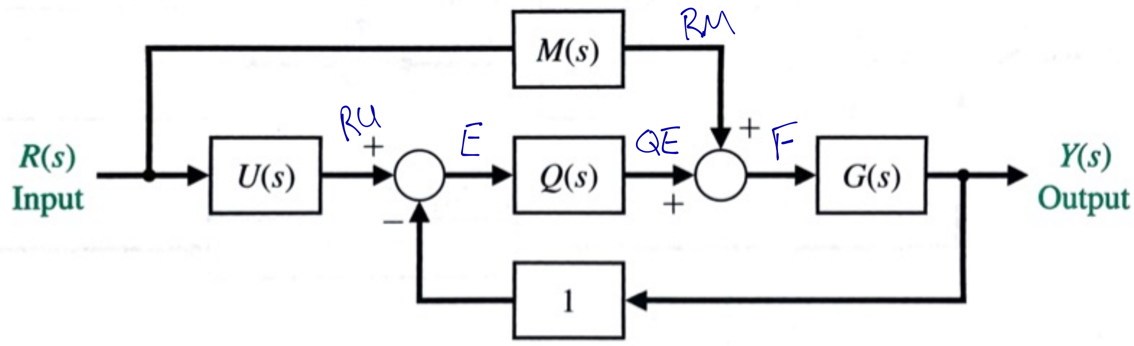


FIGURE P4.4
Two-path system.

a) $Y = GF = G(RM + QE)$ $\qquad E = RU - Y$

$= G(RM + Q(RU - Y))$

$= GRM + GQ(RU - Y)$

$Y = GRM + GQRU - GQY$

$(1 + GQ)Y = (GM + GQU)R$

$$\boxed{\frac{Y}{R} = \frac{GM + GQU}{1 + GQ}}$$

b) $S_G^T = S_G^N - S_G^D =$ $\qquad N = GM + GQU$, $D = 1 + GQ$

$S_G^N = \dfrac{\partial N/N}{\partial G/G} = \dfrac{\partial N}{\partial G} \cdot \dfrac{G}{N}$

$\dfrac{\partial N}{\partial G} = M + QU \qquad \dfrac{G}{N} = \dfrac{1}{M + QU} \longrightarrow S_G^N = 1$

$S_G^D = \dfrac{\partial D/D}{\partial G/G} = \dfrac{\partial D}{\partial G} \cdot \dfrac{G}{D}$

$\dfrac{\partial D}{\partial G} = Q \qquad \dfrac{G}{D} = \dfrac{G}{1 + GQ} \rightarrow S_G^D = \dfrac{GQ}{1 + GQ}$

$$\boxed{S_G^T = 1 - \frac{GQ}{1 + GQ} = \frac{1}{1 + GQ}}$$

c) From b) the sens. does not depend on either U or M

Problem P4.5

P4.5 Large microwave antennas have become increasingly important for radio astronomy and satellite tracking. A large antenna with a diameter of 60 ft, for example, is susceptible to large wind gust torques. A proposed antenna is required to have an error of less than 0.10° in a 35 mph wind. Experiments show that this wind force exerts a maximum disturbance at the antenna of 200,000 ft lb at 35 mph, or the equivalent to 10 volts at the input $T_d(s)$ to the amplidyne. One problem of driving large antennas is the form of the system transfer function that possesses a structural resonance. The antenna servosystem is shown in Figure P4.5. The transfer function of the antenna, drive motor, and amplidyne is approximated by

$$G(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)},$$

where $\zeta = 0.707$ and $\omega_n = 15$. The transfer function of the power amplifier is approximately

$$G_1(s) = \frac{k_a}{\tau s + 1},$$

where $\tau = 0.15$ s. (a) Determine the sensitivity of the system to a change of the parameter k_a. (b) The system is subjected to a disturbance $T_d(s) = 1/s$. Determine the required magnitude of k_a in order to maintain the steady-state error of the system less than 0.10° when the input $R(s)$ is zero. (c) Determine the error of the system when subjected to a disturbance $T_d(s) = 10/s$ when it is operating as an open-loop system ($k_s = 0$) with $R(s) = 0$.

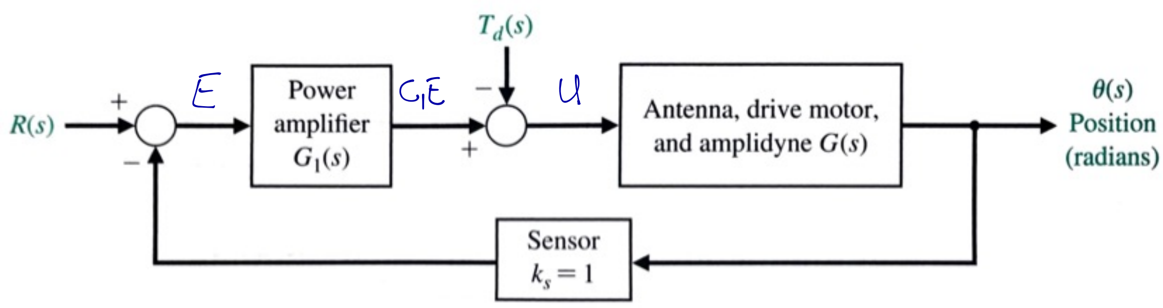


FIGURE P4.5 Antenna control system.

a) Closed-loop TF, $T(s)$

$$T(s) = \frac{G_1 G}{1 + G_1 G} = \frac{k_a \omega_n^2}{(\tau s + 1)[s(s^2 + 2\zeta\omega_n s + \omega_n^2)] + k_a \omega_n^2}$$

Define $G_2 = \frac{1}{\tau s + 1}$ so $G_1 = k_a G_2$

$$T(s) = \frac{k_a G_2 G}{1 + k_a G_2 G}$$

$$S^T_{k_a} = \frac{\partial T / T}{\partial k_a / k_a} = \frac{\partial T}{\partial k_a}\frac{k_a}{T}$$

$$\frac{\partial T}{\partial k_a} = \left(\frac{G_2 G}{1 + k_a G_2 G}\right) - \frac{(k_a G_2 G) G_2 G}{(1 + k_a G_2 G)^2} \qquad \frac{k_a}{T} = \frac{1 + k_a G_2 G}{G_2 G}$$

$$\boxed{S^T_{k_a} = \left[\frac{(1 + k_a G_2 G)\cancel{G_2 G} - (k_a G_2 G)\cancel{G_2 G}}{(1 + k_a G_2 G)^{\cancel{2}}}\right]\left[\frac{\cancel{1 + k_a G_2 G}}{\cancel{G_2 G}}\right] = \frac{1}{1 + k_a G_2 G} = \frac{1}{1 + G_1 G}}$$

Problem P4.5 (cont.)

b)

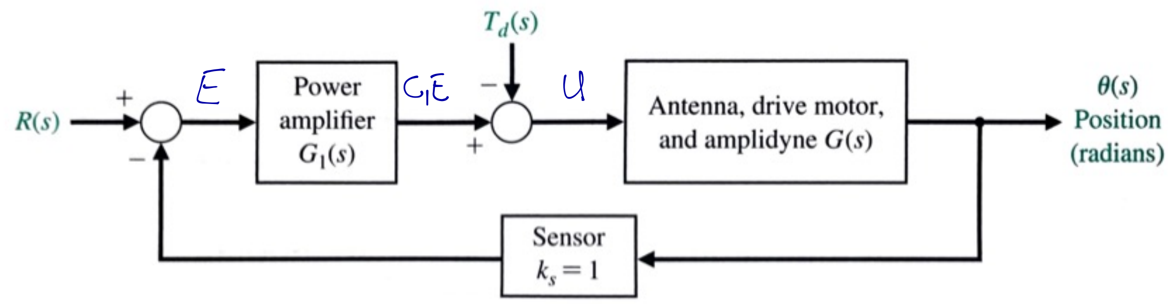


FIGURE P4.5 Antenna control system.

$$\Theta = GU = G(-T_d + G_1E) = G(-T_d + G_1(\cancel{R}^{\,0} - \Theta)) = -GT_d - GG_1\Theta$$

$$(1 + GG_1)\Theta = -GT_d$$

$$\frac{\Theta}{T_d} = \frac{-G}{1 + G_1G}$$

So $\Theta = \frac{-G}{1+G_1G} T_d$ $\rightarrow$ Error is $R(s) - \Theta(s)$ $\rightarrow$ $R(s) = 0$ $E(s) = -\Theta(s)$

$$\lim_{t\to\infty} -\Theta(t) = \lim_{s\to 0} -s\Theta = \lim_{s\to 0} \cancel{s}\left[\left(\frac{+G}{1+G_1G}\right)\frac{1}{\cancel{s}}\right] = \lim_{s\to 0} \frac{+G}{1 + K_aG_2G} = \frac{+1}{K_a}$$

Need $\frac{1}{K_a} < (0.10^\circ)\left(\frac{\pi}{180}\right)$

$$\frac{1}{K_a} < 0.0017 \longrightarrow \boxed{K_a > 572.9}$$

c) Open-loop TF from T_d to Θ is

$$\frac{\Theta}{T_d} = -G \qquad E = -\Theta \rightarrow \boxed{\lim_{s\to 0} \cancel{s}\left(-G\left(\frac{10}{\cancel{s}}\right)\right) = \lim_{s\to 0} \frac{10\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)} = \infty}$$

$$\Theta = -GT_d$$

$$= -G\left(\frac{10}{s}\right)$$

Problem P4.16

P4.16 The steering control of a modern ship may be represented by the system shown in Figure P4.16 [16, 20]. (a) Find the steady-state effect of a constant wind force represented by $T_d(s) = 1/s$ for $K = 8$ and $K = 22$. Assume that the rudder input $R(s)$ is zero, without any disturbance, and has not been adjusted. (b) Show that the rudder can then be used to bring the ship deviation back to zero.

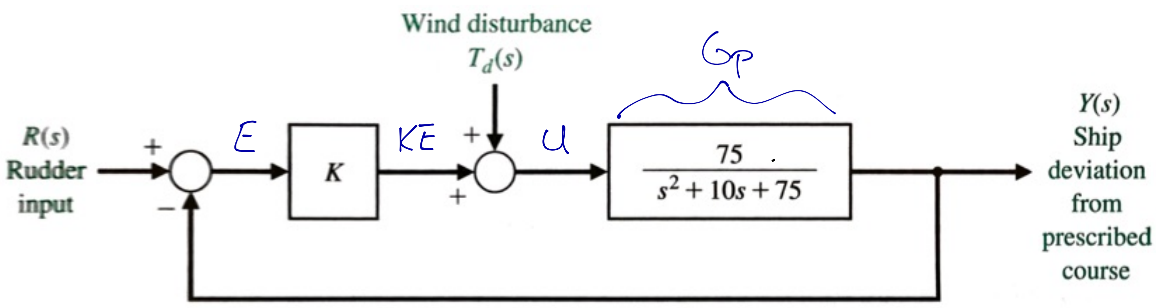


FIGURE P4.16 Ship steering control.

a) $Y = G_p U = G_p(T_d + KE) = G_p(T_d + K(\cancel{R}^{\,0} - Y)) = G_p T_d + (-K G_p Y)$

$$(1 + KG_p)Y = G_p T_d$$

$$\frac{Y}{T_d} = \frac{G_p}{1+KG_p} \rightarrow Y = \frac{G_p}{1+KG_p}\left[\frac{1}{s}\right]$$

$$\boxed{\lim_{t \to 0} y(t) = \lim_{s \to 0} sY(s) = \frac{G_p}{1+KG_p} = \frac{1}{1+K} \qquad \text{for } K = 8 \quad Y_{ss} = \frac{1}{9} \qquad \text{for } K = 22 \quad Y_{ss} = \frac{1}{23}}$$

b) $Y = KG_p(R - Y)$

$$(1 + KG_p)Y = KG_p R$$

$$\frac{Y}{R} = \frac{KG_p}{1+KG_p}$$

Problem E5.7

E5.7 Effective control of insulin injections can result in better lives for diabetic persons. Automatically controlled insulin injection by means of a pump and a sensor that measures blood sugar can be very effective. A pump and injection system has a feedback control as shown in Figure E5.7. Calculate the suitable gain K so that the percent overshoot of the step response due to the drug injection is $P.O. = 7\%$. $R(s)$ is the desired blood-sugar level and $Y(s)$ is the actual blood-sugar level.

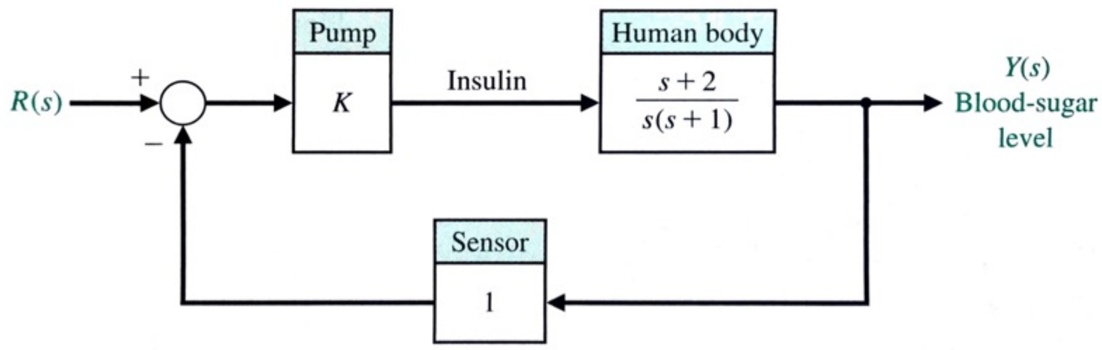


FIGURE E5.7
Blood-sugar level control.

CL Transfer function is

$$\frac{Y}{R} = \frac{K(s+2)}{s(s+1)+K(s+2)} = \frac{K(s+2)}{s^2+(1+K)s+2K} \leftarrow 2\zeta\omega_n = 1+K \quad \text{and} \quad \omega_n^2 = 2K$$

$$\%\text{ overshoot} = 100\exp\left[\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right] \rightarrow \text{need } \zeta \doteq 0.646$$

$$2\zeta\omega_n = 1+K$$

$$2\zeta\sqrt{2K} = 1+K$$

$$(4\zeta^2)(2K) = (1+K)^2$$

$$8K\zeta^2 = 1+2K+K^2$$

$$K^2 + (2-8\zeta^2)K + 1 = 0$$

Problem E5.8

E5.8 A control system for positioning the head of a floppy disk drive has the closed-loop transfer function

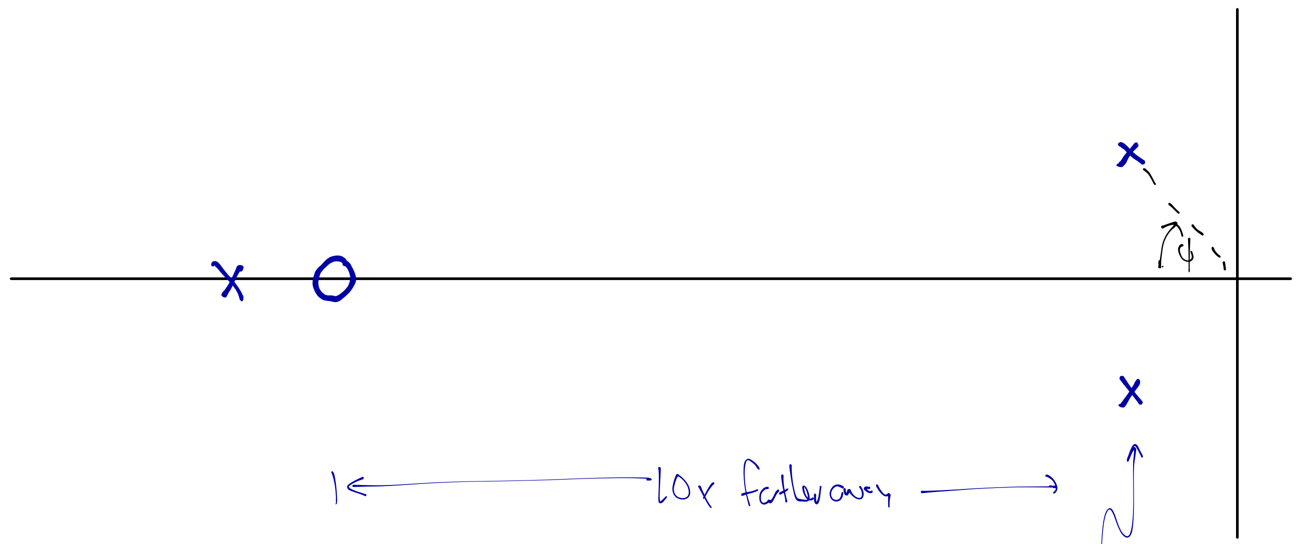
$$T(s) = \frac{11.1(s+18)}{(s+20)(s^2+4s+10)}.$$

Plot the poles and zeros of this system and discuss the dominance of the complex poles. What percent overshoot for a step input do you expect?

zero at $s = -18$

poles at -20 and $\frac{-4 \pm \sqrt{16-4(1)(10)}}{2} = \frac{-4 \pm \sqrt{-24}}{2}$

$-2 \pm i\sqrt{6} \approx -2 \pm i2.45$



These poles are much closer to Im axis, so they dominate the response

For these two dominant poles

$\zeta\omega_n = 2$ so $t_s = \frac{4}{2} \approx 2s$

$\phi = \cos^{-1}(\zeta) = \tan^{-1}\left(\frac{\omega_d}{\zeta\omega_n}\right) = \tan^{-1}\left(\frac{2.45}{2}\right) = 0.886$

$\cos^{-1}(\zeta) = 0.886 \rightarrow \zeta \approx 0.63$

$$\boxed{\%\ \text{overshoot} = 100 \exp\left[\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right] \approx 7.69\%}$$

Problem E5.19

E5.19 A second-order system has the closed-loop transfer function

$$T(s) = \frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{7}{s^2 + 3.175s + 7}.$$

(a) Estimate the percent overshoot $P.O.$, the time to peak T_p, and the settling time T_s of the unit step response.

(b) Obtain the system response to a unit step and verify the results in part (a).

a) $2\zeta\omega_n = 3.175$ $\qquad \omega_n^2 = 7 \qquad \omega_n = \sqrt{7}$

$\zeta = \dfrac{3.175}{2\omega_n}$

$\omega_d = \omega_n\sqrt{1-\zeta^2} = 2.12 \ \frac{rad}{s}$

$\zeta = 0.6$

$\% \text{ overshoot} = 100 \exp\left[\dfrac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right] \approx 9.5\%$

$T_p = \dfrac{\pi}{\omega_d} = 1.48s$

$T_s = \dfrac{4}{\zeta\omega_n} = 2.52s$

b)