MCHE 474: Control Systems
Fall 2017 – Homework 1

Assigned: Monday, August 28
Due: Friday, September 1, 5pm

Assignment: The problems included in this document.

Submission: Emailed *single* pdf document:

- to joshua.vaughan@louisiana.edu
- with subject line `CLID-MCHE474-HW1`, where `CLID` is replaced with your CLID
- and that has a *single* pdf attached with filename `CLID-MCHE474-HW1`, where `CLID` is replaced with your CLID
- *Note:* Submissions with incorrect filenames or submitted as multiple images/pdfs will be rejected.

Problem 1

Figure 1 shows a planar representation of a walking robot of body mass m. You may assume the legs are massless. The robot is on an incline, but the center of mass is still centered between its legs with respect to the inclined plane.

a. Identify all the forces acting on the system. Draw the free body diagram.
b. Assume a rigid configuration as drawn in Figure 2. Sum moments about the lower leg's ground-contact location.
c. What has to happen for the robot to fall down the hill? Use your answer from part b as support.
d. What do these moments suggest about how the robot should be oriented on uneven terrain?¶

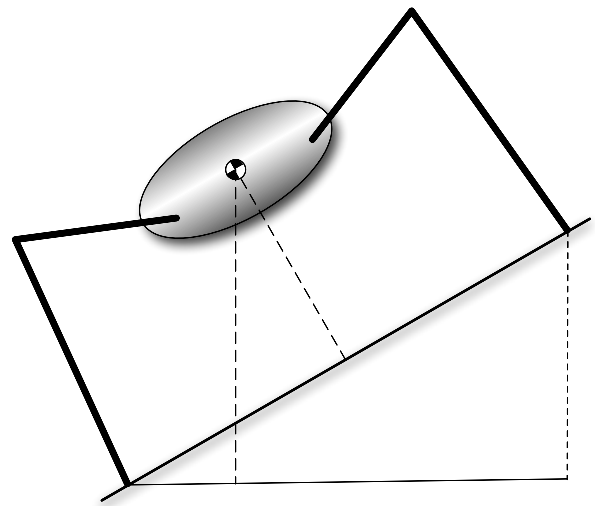
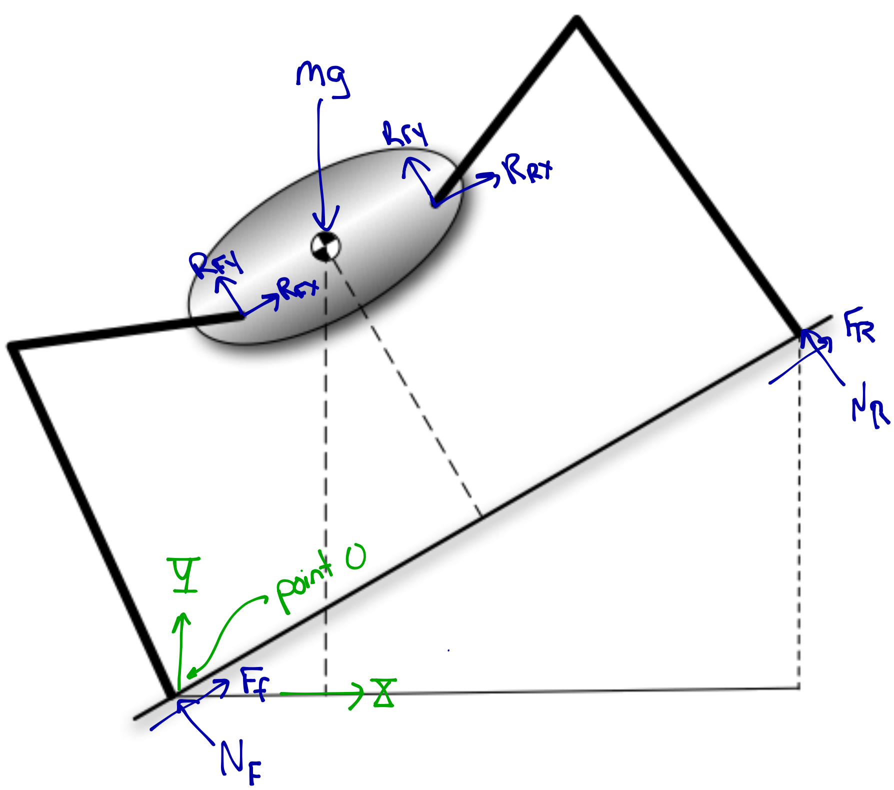


Figure 2: Walking Robot on an Incline

a)

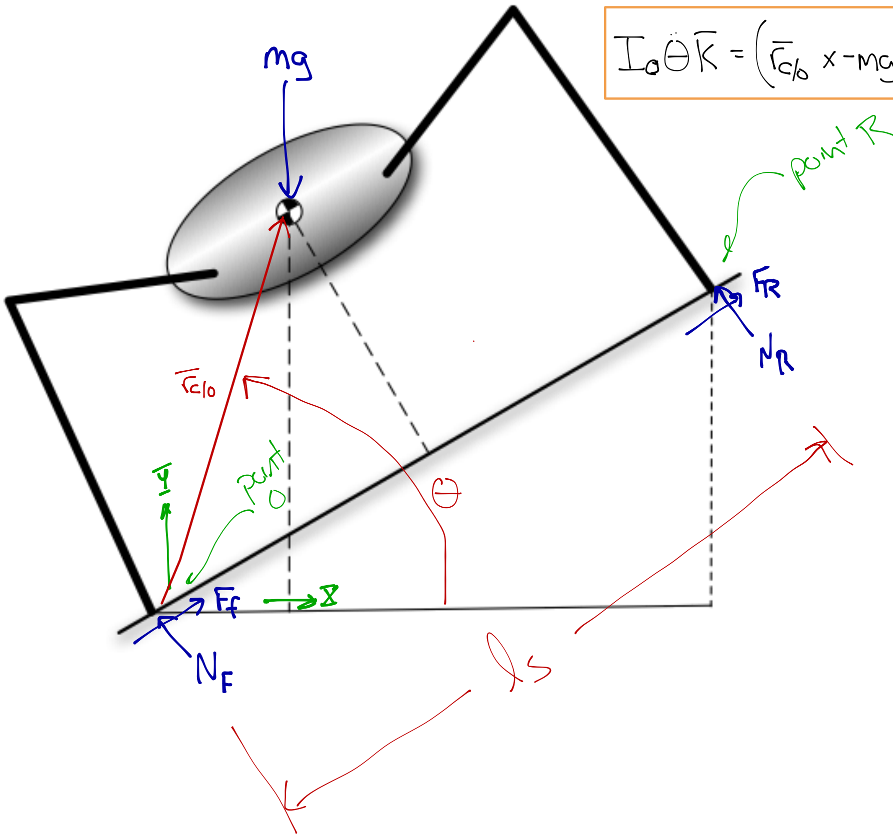


Problem 1 (cont)

b. Sum moments about point 0. A key insight to this problem is treating this configuration as a single, rigid body. This allows us to ignore the reaction forces. So, the free body diagram reduces to:

$$\sum \bar{M}_O = \left(\bar{r}_{C/O} \times -mg\bar{J}\right) + \left[\bar{r}_{R/O} \times \left(\bar{F}_R + \bar{N}_R\right)\right]$$

$$\boxed{I_O \ddot{\theta} \bar{K} = \left(\bar{r}_{C/O} \times -mg\bar{J}\right) + \left[\bar{r}_{R/O} \times \left(\bar{F}_R + \bar{N}_R\right)\right]}$$



c) To stay stable, we want moments pushing θ back to θ_0. This is the $-\bar{K}$ direction. For the robot to fall down the hill, the COM must shift to "outside" of point O. When that happens, gravity creates a positive moment about O, which "wants" to make the the hill.

d) The COM should be kept inside the leg contact points (which define the support polygon)

Problem 2

Figure 2 shows a robotic arm. Assume that the elbow joint is held rigid, such that θ_2 is constant. The two links, l_1 and l_2, are both rigid and massless. There is a point mass, m, at the endpoint. There is a pure torque, T, acting at the pin joint.

a. Identify all the forces acting on the system. Draw the free body diagram.
b. How many degrees-of-freedom does the system have?
c. Write the position of the endpoint mass, m, relative to the pin.
d. Write the equations of motion for this system.
e. What torque, T, is required to maintain a configuration at angle θ_1?

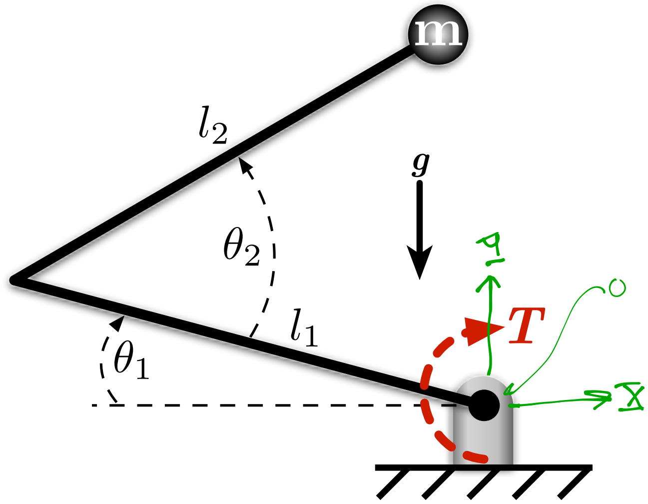
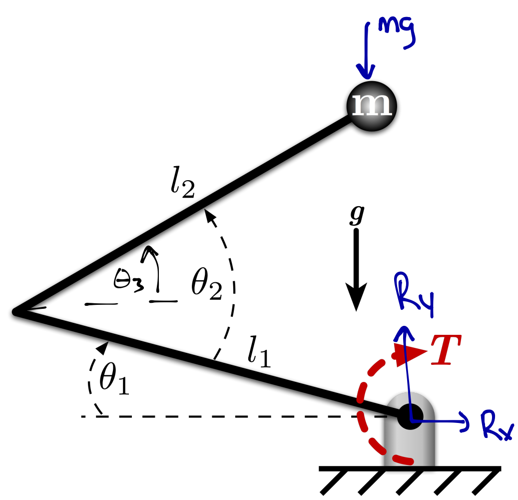


Figure 2: A Robotic Arm

a)



Define $\theta_3 = \theta_2 - \theta_1$

b) The system has one degree of freedom.

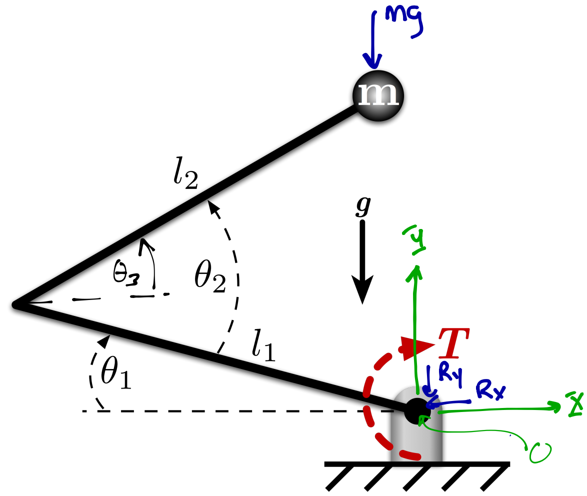
c) $\bar{r}_{m/o} = \left(-l_1 \cos\theta_1 + l_2 \cos\theta_3\right)\bar{I} + \left(l_1 \sin\theta_1 + l_2 \sin\theta_3\right)\bar{J}$

d) Sum moments about point O (the pin).

$$I_O \bar{\alpha} = \bar{r}_{m/o} \times -mg\bar{J} = \left[\left(-l_1\cos\theta_1 + l_2\cos\theta_3\right)\bar{I} + \left(l_1\sin\theta_1 + l_2\sin\theta_3\right)\bar{J}\right] \times \left(-mg\bar{J}\right) - T\bar{K}$$

$$I_O \bar{\alpha} = \left[-mg\left(-l_1\cos\theta_1 + l_2\cos\theta_3\right) - T\right]\bar{K}$$

Problem 2 (cont.)



$$I_O \bar{\alpha} = \left[-mg\left(-l_1\cos\theta_1 + l_2\cos\theta_3\right) - T\right]\bar{K}$$

$$\bar{\alpha} = -\ddot{\theta}_1 \bar{K} \leftarrow$$ positive accel of θ_1 is in the negative $\bar{K}$ direction

$$-I_O\ddot{\theta} = -mg\left(-l_1\cos\theta_1 + l_2\cos\theta_3\right) - T \leftarrow \text{all in } \bar{K}$$

$$\boxed{\underbrace{I_O\ddot{\theta} - mg\left(-l_1\cos\theta_1 + l_2\cos\theta_3\right)}_{\text{Notice that this is only stable in certain configurations. (This term needs to be positive)}} = T}$$

$I_O = m\left(\bar{r}_{m/O}\right)^T\left(\bar{r}_{m/O}\right)$ - remember $\bar{r}_{m/O}$ represents the distance from O to m

e) For a fixed θ_1, the torque and the moment created by gravity must balance.

One "trick" is to set all the motion terms to zero.

$$\cancel{I_O\ddot{\theta}}^{\,\to 0} - mg\left(-l_1\cos\theta_1 + l_2\cos\theta_3\right) = T$$

So $\boxed{T = -mg\left(-l_1\cos\theta_1 + l_2\cos\theta_3\right)}$

Problem 3

In Figure 3, a block of mass m, height h, and width w rests on an incline place of angle $\boldsymbol{\theta}$. The coefficient of friction between the plane and the block is μ.

a. Draw the free body diagram for this system.
b. How many degrees of freedom does this system have if it is assumed that it does not tip over?
c. How many degrees of freedom does this system have if it's possible that it tips over?
d. Write the equations of motion for this system.
e. At what combination of angle and coefficient of friction will the block begin to slide?

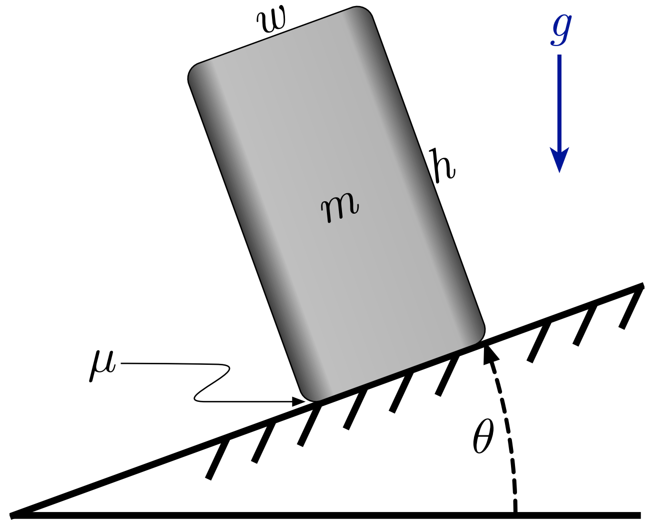
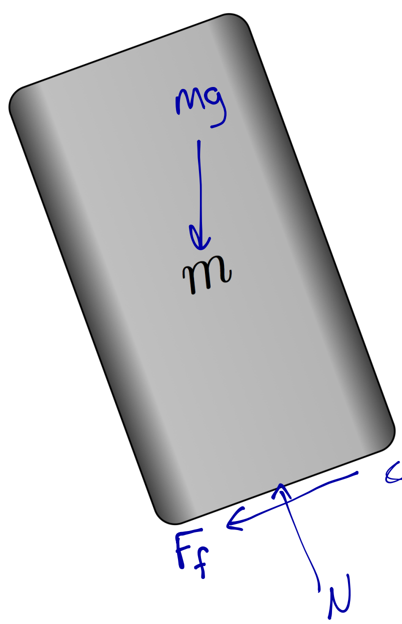


Figure 3: Block on an Incline Plane

a)



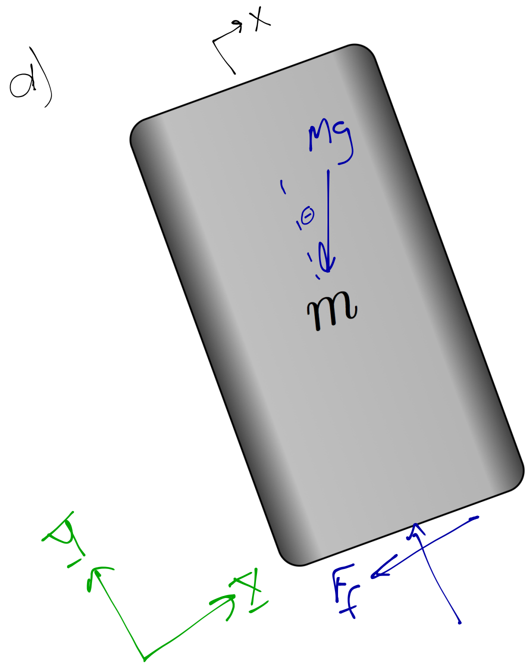
I'm going to assume a positive "$\ddot{x}$" up the slope, so if a move in that direction occurs, Ff will point down the slope, opposing it.

b) The system has 1 DOF. The block only slides down the incline.

c) If the box can tip over, then there are 2 DOF. We can use the COM location along the incline and the angle of tipping relative to the incline.

Problem 3 (cont)

d)



Assume the box will not tip over. It only slides down the incline.

$$\Sigma \bar{F} = \left[-F_f - mg\sin\theta\right]\bar{I} + \underbrace{\left[N - mg\cos\theta\right]}\bar{J}$$

$F_f = \mu N \, sgn(\dot{x})$

No motion in $\bar{J}$, so $N = mg\cos\theta$

$$sgn(\dot{x}) = \begin{cases} -1 & \dot{x} < 0 \\ 0 & \dot{x} = 0 \\ 1 & \dot{x} > 0 \end{cases}$$

So,

$$m\ddot{x} = -\mu N \, sgn(\dot{x}) - mg\sin\theta$$

$$m\ddot{x} = -\mu (mg\cos\theta) \, sgn(\dot{x}) - mg\sin\theta$$

$$\boxed{\ddot{x} = -\mu g\cos\theta \, sgn(\dot{x}) - g\sin\theta}$$

e) To determine this, look at the condition were no motion occurs. Find the boundary where that expression is no longer true.

$$\cancel{\ddot{x}}^{\,0} = -\mu g\cos\theta \, sgn(\dot{x}) - g\sin\theta \longrightarrow -\mu\cos\theta \, sgn(\dot{x}) = \sin\theta$$

Now if the mass does slide, $\dot{x} < 0$ so $sgn(\dot{x}) = -1$

$$\mu\cos\theta = \sin\theta$$

$$\mu = \frac{\sin\theta}{\cos\theta} = \tan\theta \leftarrow \text{This is the minimum } \mu \text{ to prevent sliding}$$

So $\boxed{\text{If } \mu < \tan\theta \text{ the block will slide.}}$

Problem 4

Figure 4 shows a system of a uniform rod, length l and mass m rotating about a perfect pin. An always-horizontal force is acts on the end of the rod.

a. Draw the free body diagram for this system.

b. Write the equations of motion for this system.

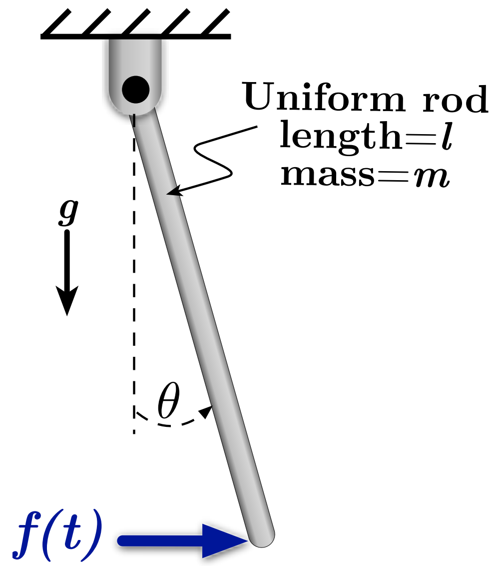
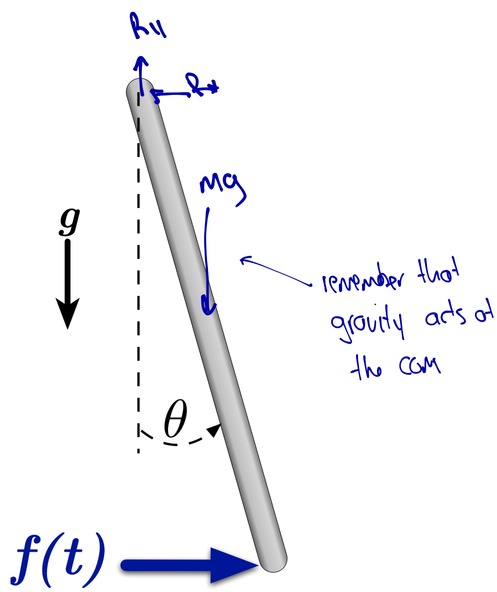


Figure 4: A Bar Pendulum with Force at End

a)



b) Sum moments about point O

$$I_O \bar{\alpha} = \sum \bar{M}_O = \left(\bar{r}_{C/O} \times -mg\bar{J}\right) + \left(\bar{r}_{P/O} \times f\bar{I}\right)$$

$$= \left[\left(\frac{l}{2}\sin\theta\,\bar{I} - \frac{l}{2}\cos\theta\,\bar{J}\right) \times \left(-mg\bar{J}\right)\right] + \left[\left(l\sin\theta\,\bar{I} - l\cos\theta\,\bar{J}\right) \times f\bar{I}\right]$$

$$\frac{1}{3}ml^2\ddot{\theta}\,\bar{K} = -mg\frac{l}{2}\sin\theta\,\bar{K} + fl\cos\theta\,\bar{K}$$

$$\boxed{\frac{1}{3}ml^2\ddot{\theta} + mg\frac{l}{2}\sin\theta = fl\cos\theta}$$ ← we can linearize by assuming small angles about $\theta = 0$ ($\sin\theta \approx \theta$ and $\cos\theta \approx 1$)

$$\boxed{\frac{1}{3}ml^2\ddot{\theta} + mg\frac{l}{2}\theta = fl}$$ ← Linearized Eq of Motion