Stability

MCHE 470: Robotics

Dr. Joshua Vaughan

Rougeou 225

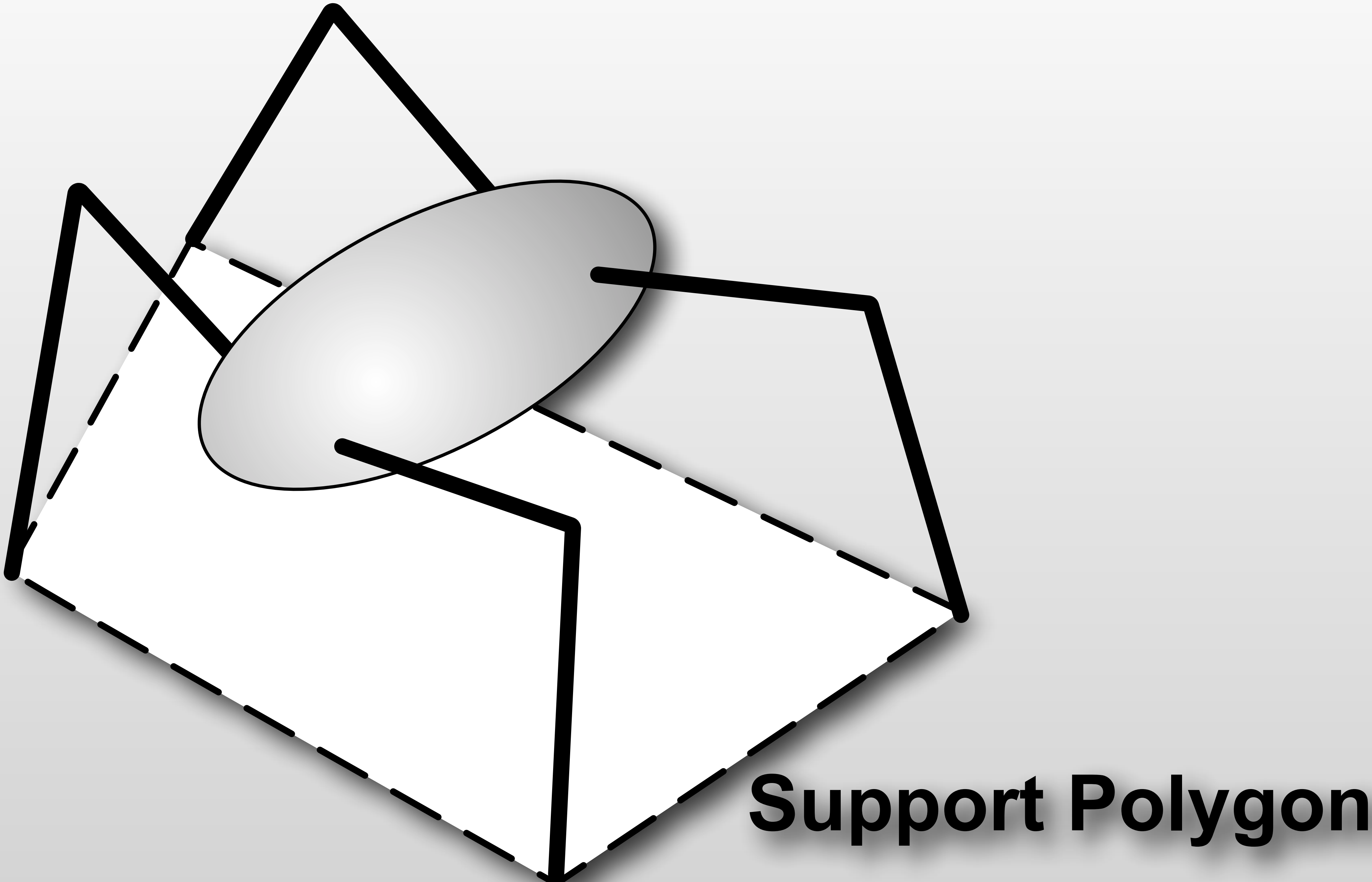
joshua.vaughan@louisiana.edu

@Doc_Vaughan

Agenda

- Terminology
- Static Stability
  - Geometric methods
  - Energy methods
- Utilization of chosen method
- Dynamic Stability Overview

Support Polygon



Stability Margins

- Measure of how likely "machine" is to tip over
- Divided into static and dynamic methods
- Nearly all identical on ideal, even surfaces
- Can be used during both mechanical design and motion planning

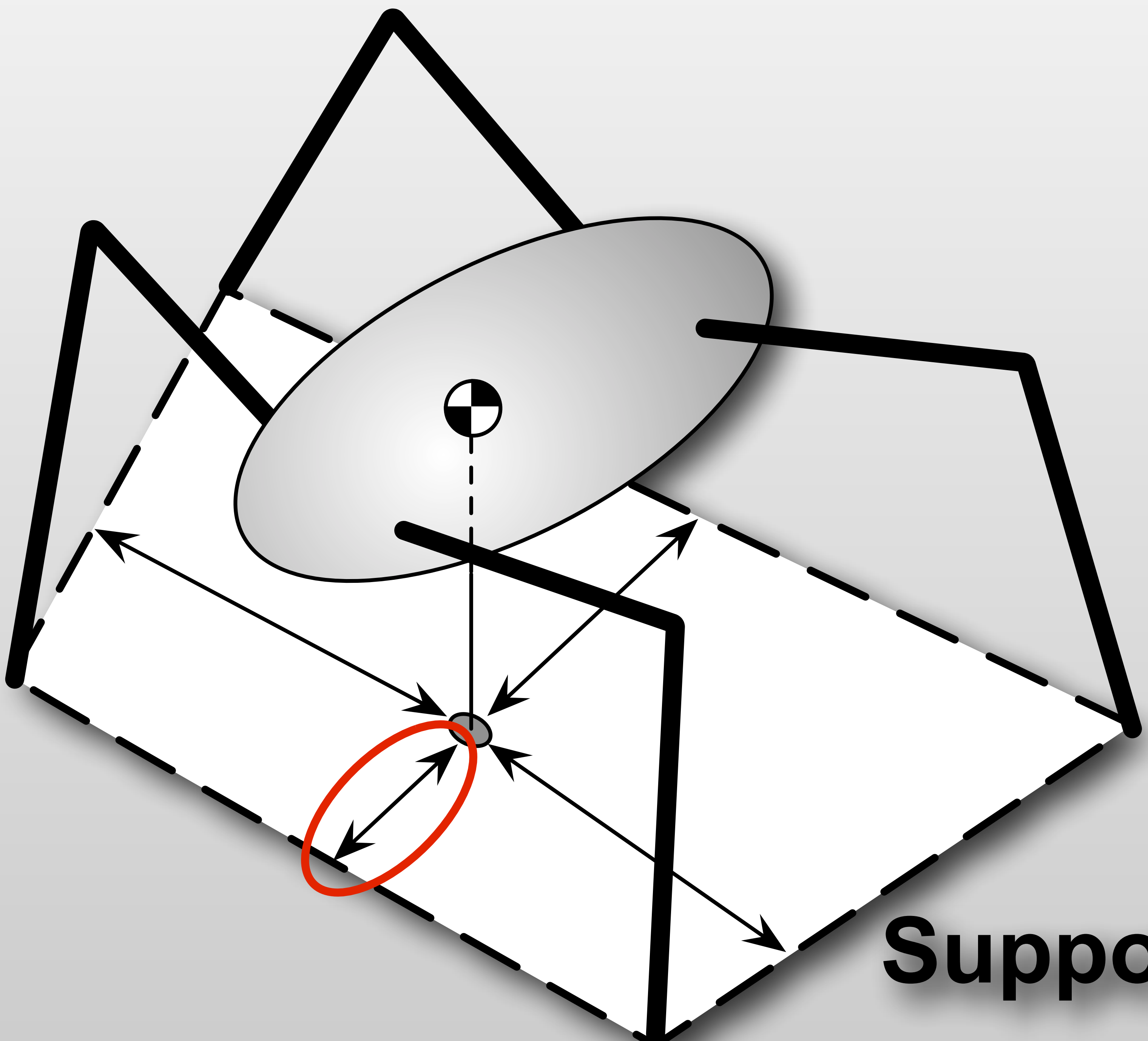
Static Stability Margin

- Center of Gravity Projection Method (McGhee and Frank, 1968)

- If the projection of the COM is within the support polygon, then the system is stable

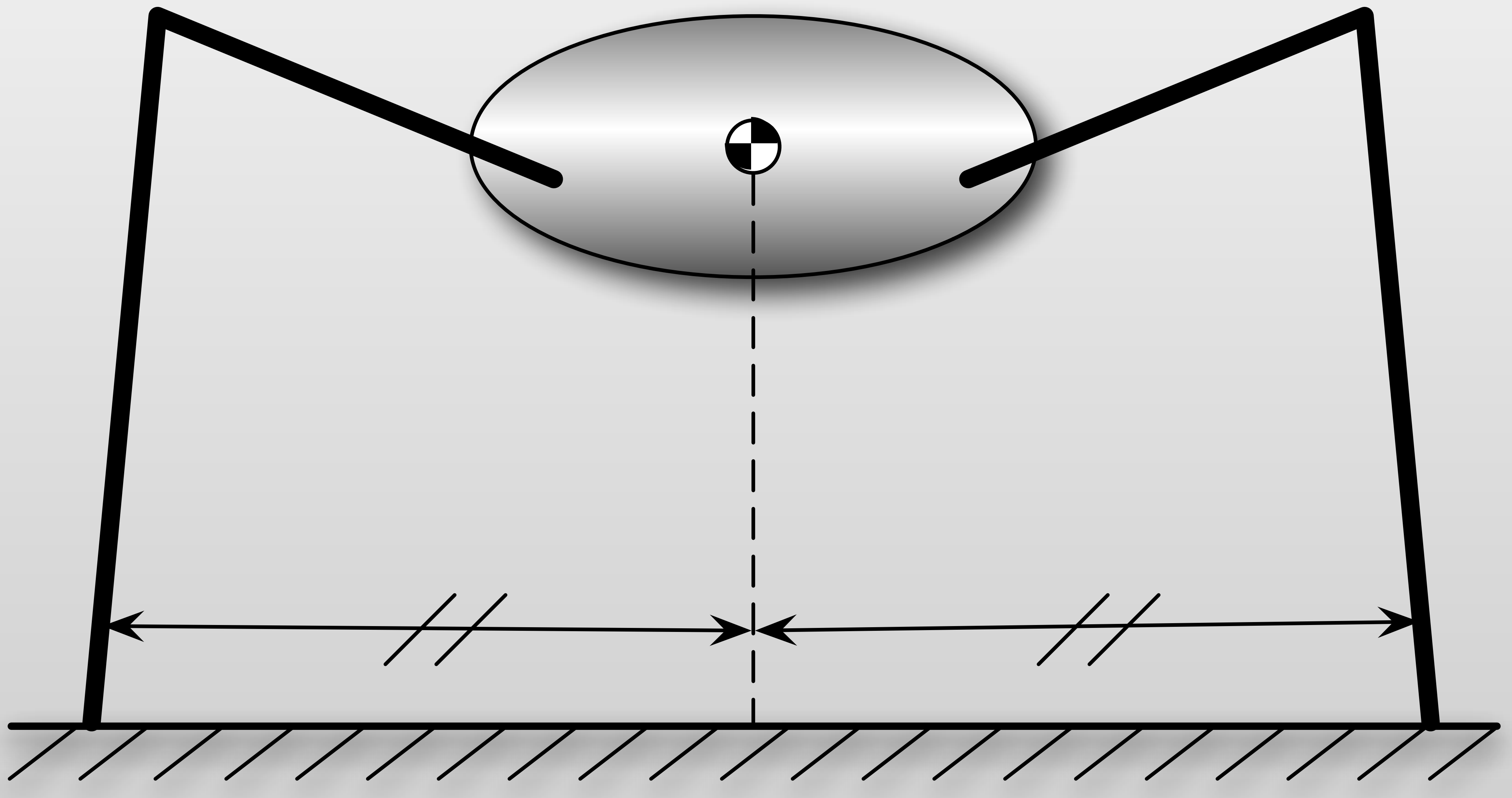
Static Stability Margin

- Smallest distance from COM projection to support polygon



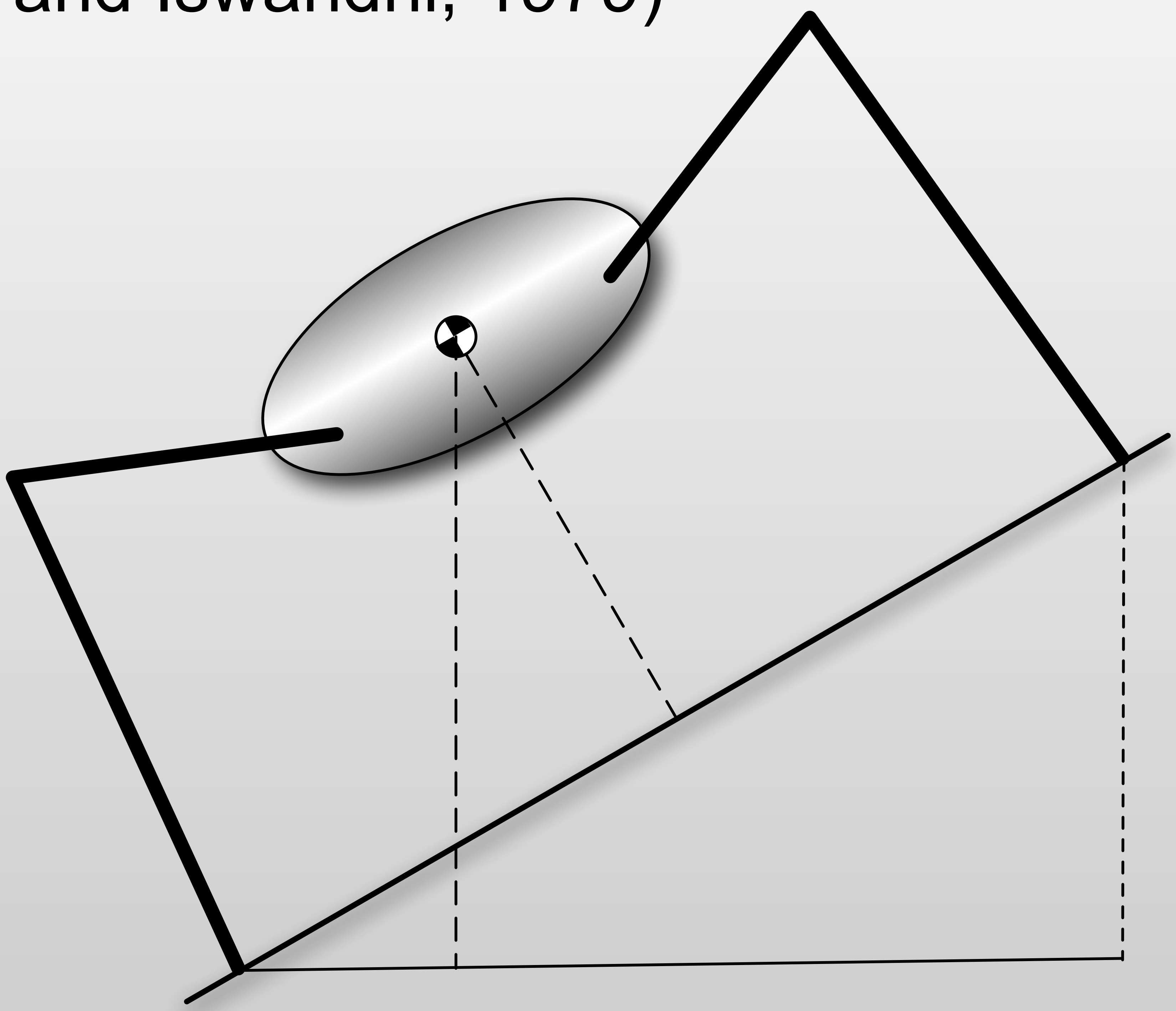
Planar example

- Optimal COM location centered between support points

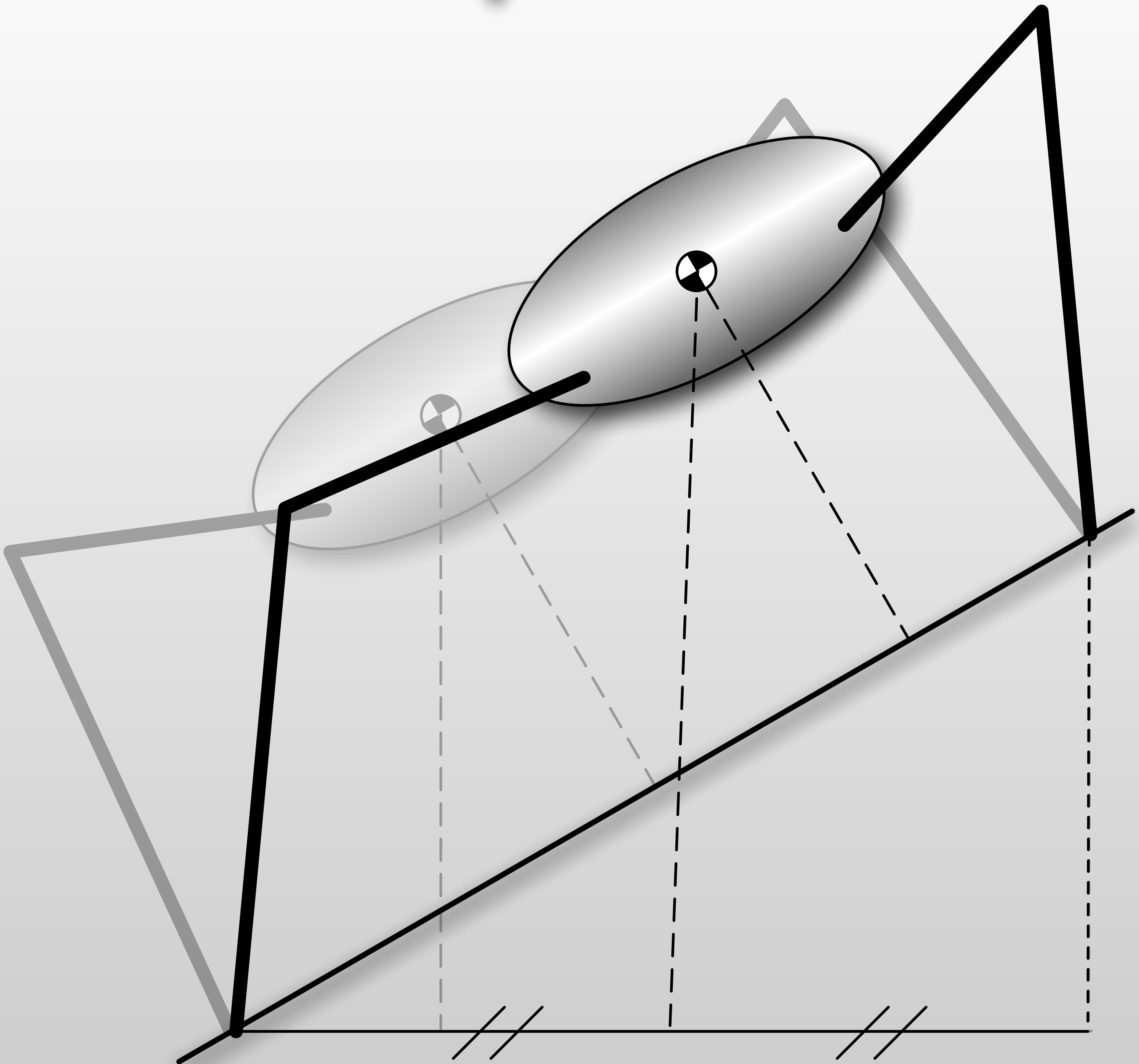


Planar Example - Incline

- Use horizontal projection of support polygon (McGhee and Iswandhi, 1979)

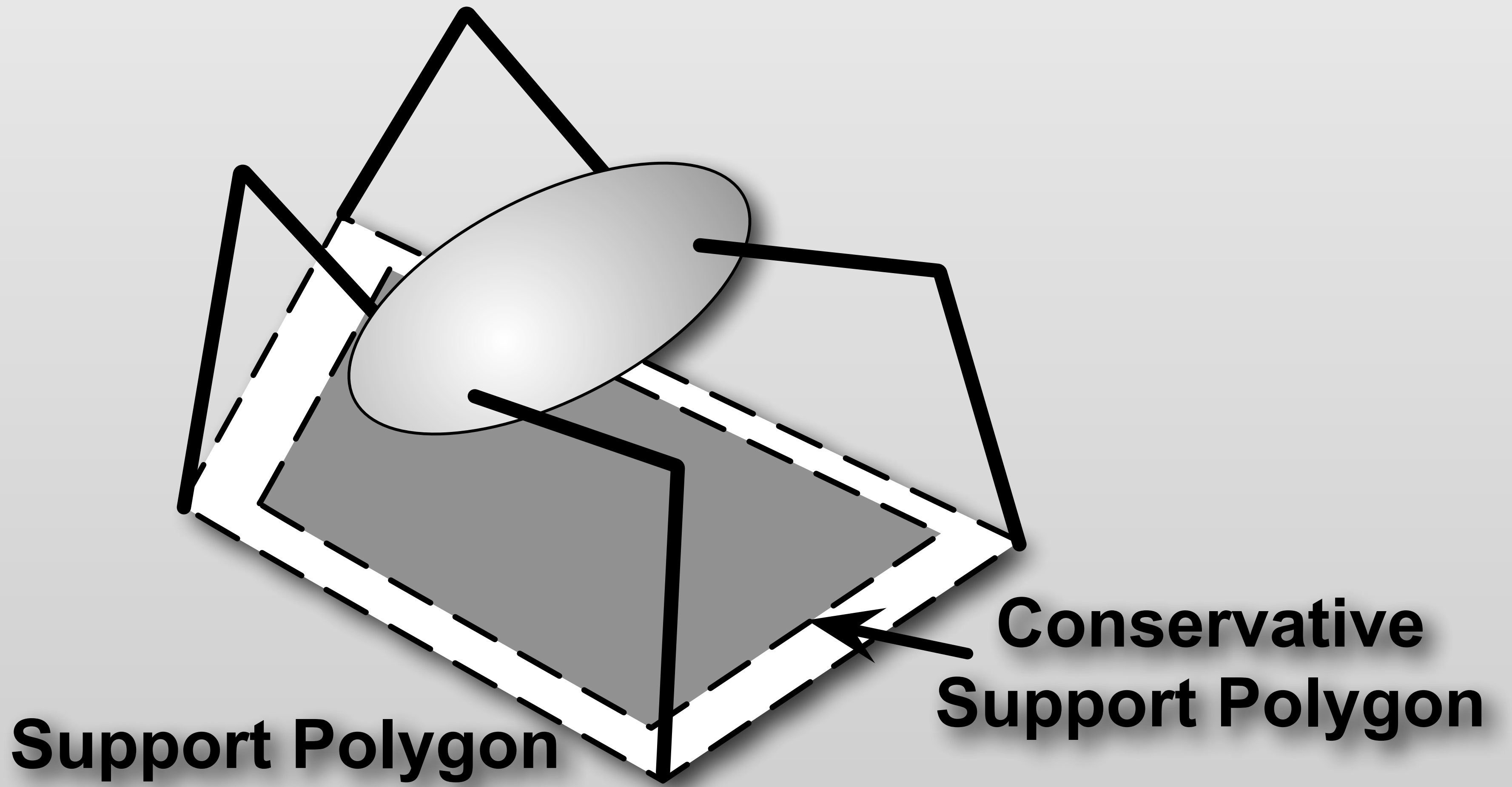


Planar Example - Incline

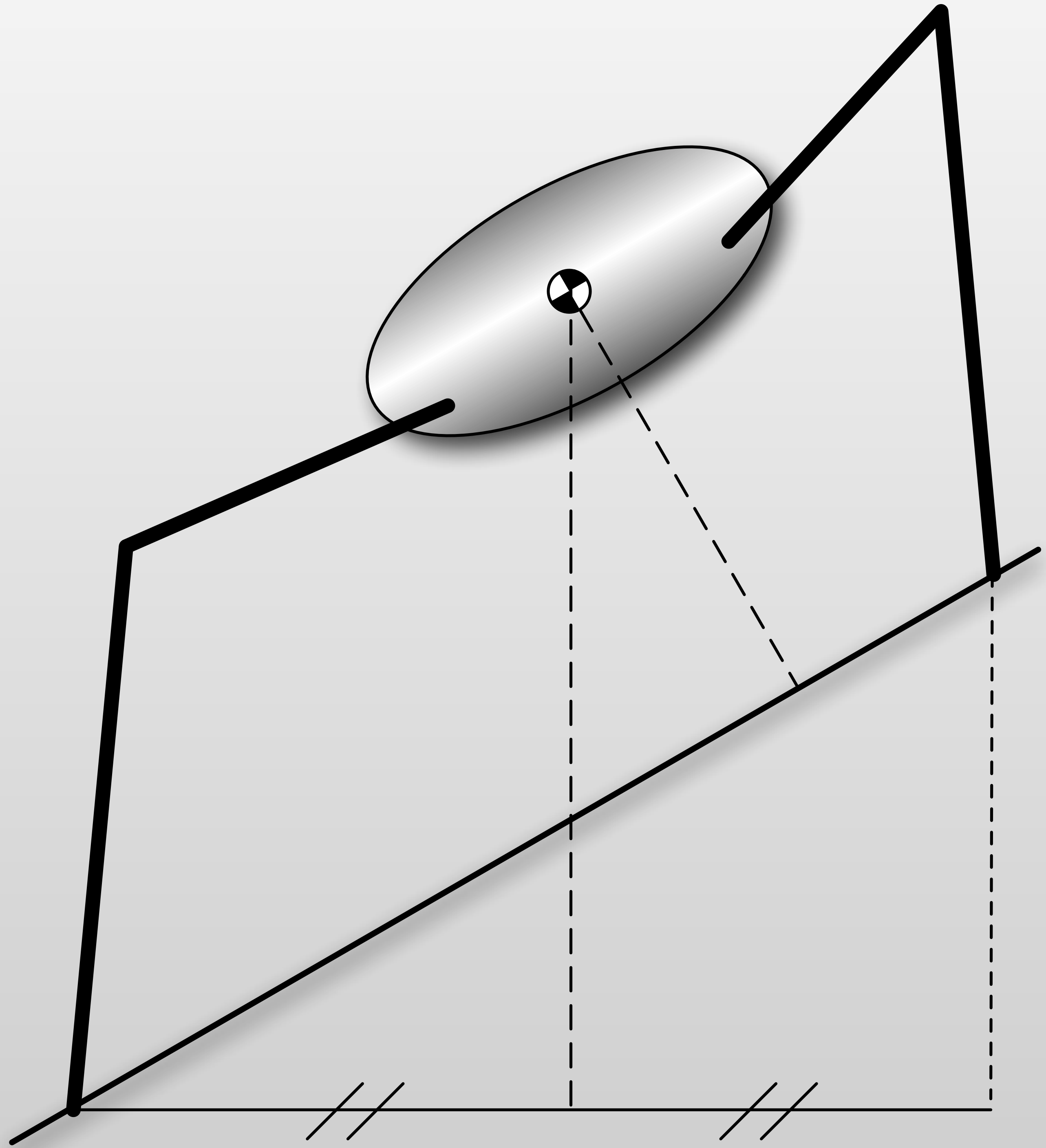


Simplifications/Modifications

- Only look at most likely tipping axes
- Look at front and rear axes in direction of motion
- Artificially shrink support polygon in calculation: Conservative Support Polygon

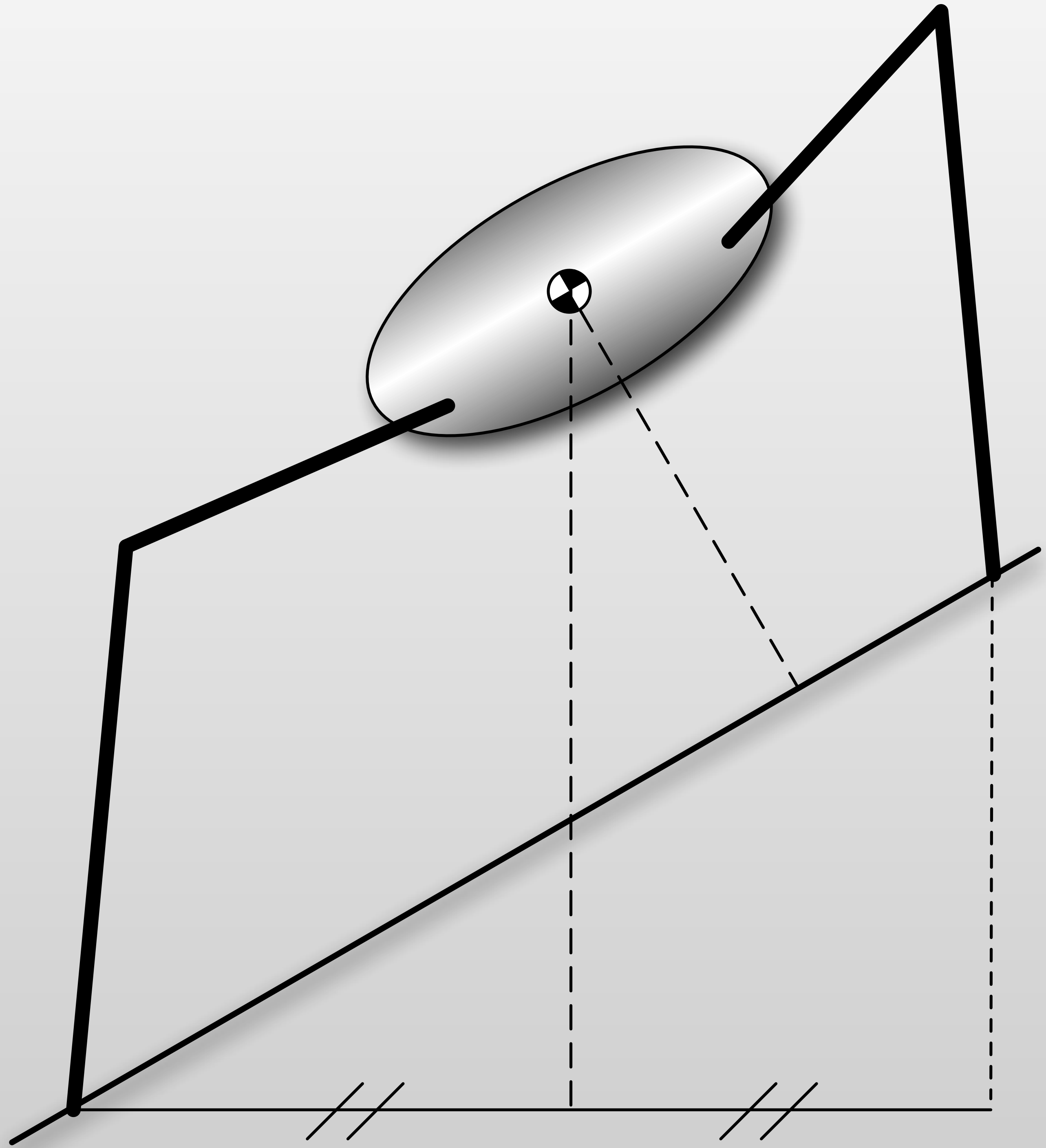


Problems? (other than ignoring dynamics)



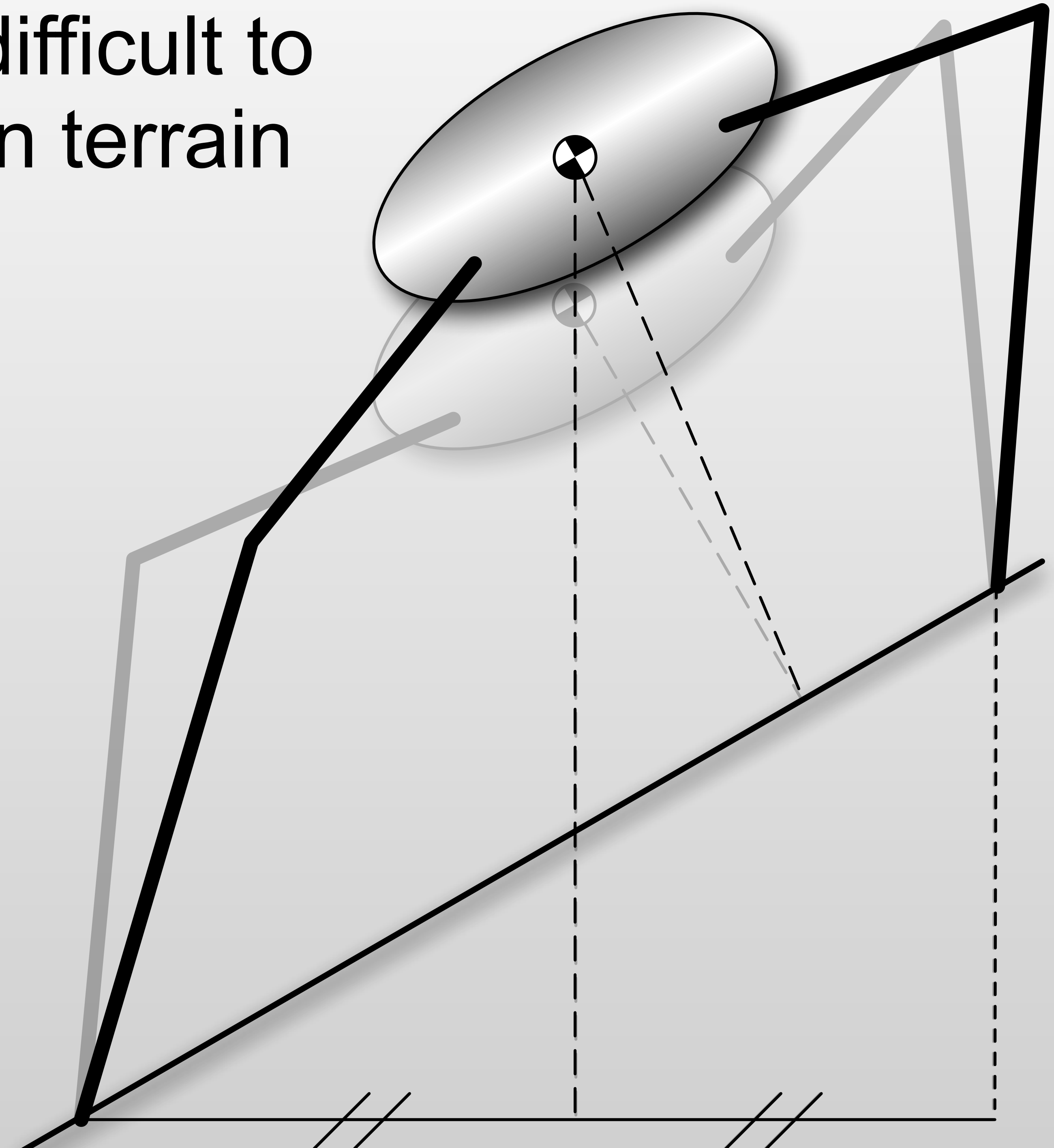
Problems? (other than ignoring dynamics)

- Ignores COM height



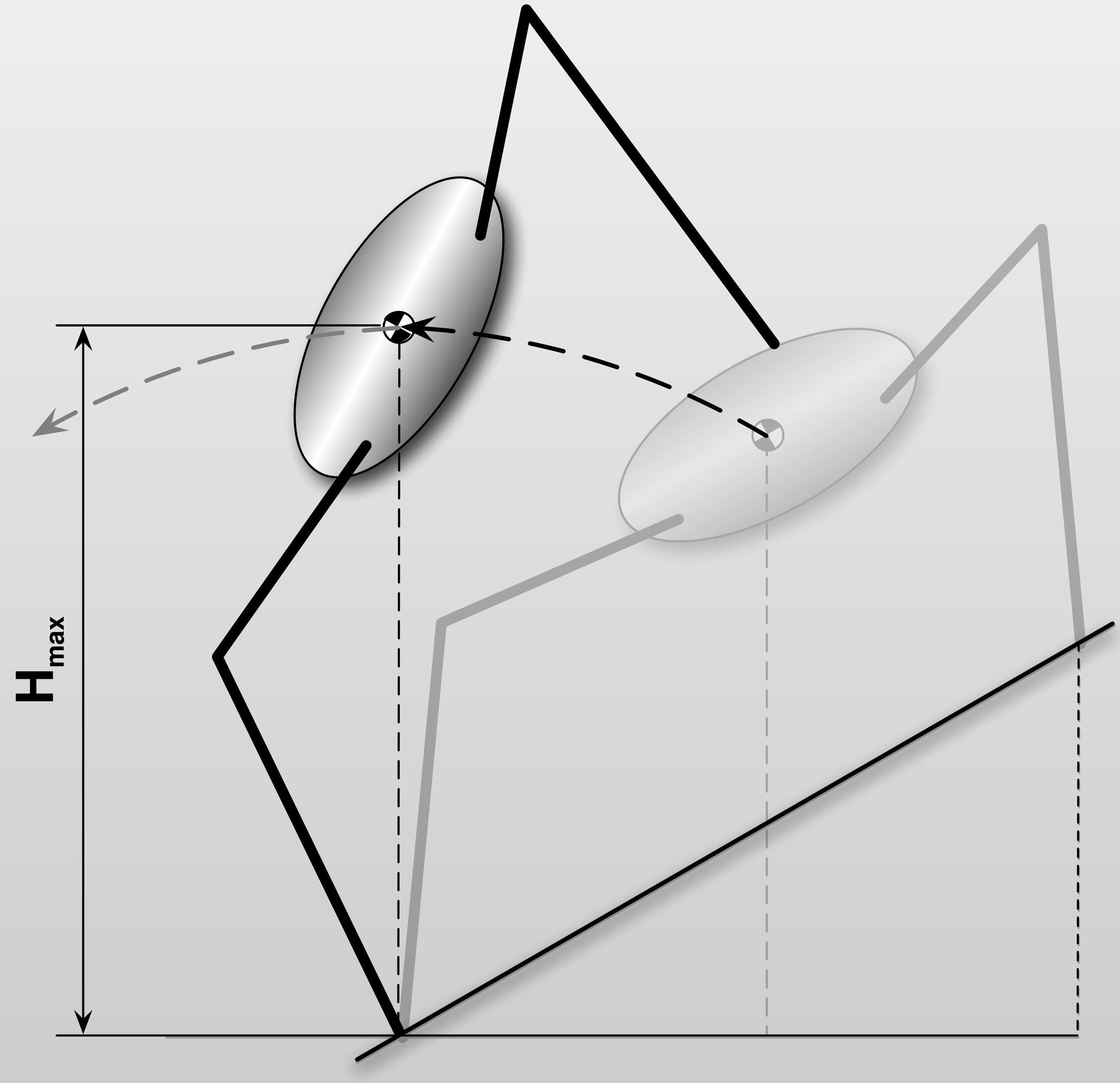
Problems? (other than ignoring dynamics)

- Ignores COM height
- Geometric - can be difficult to calculate on unknown terrain



Energy-based Stability Analysis

Idea: COM will not reach its highest point if the energy input from the disturbance forces is not great enough.

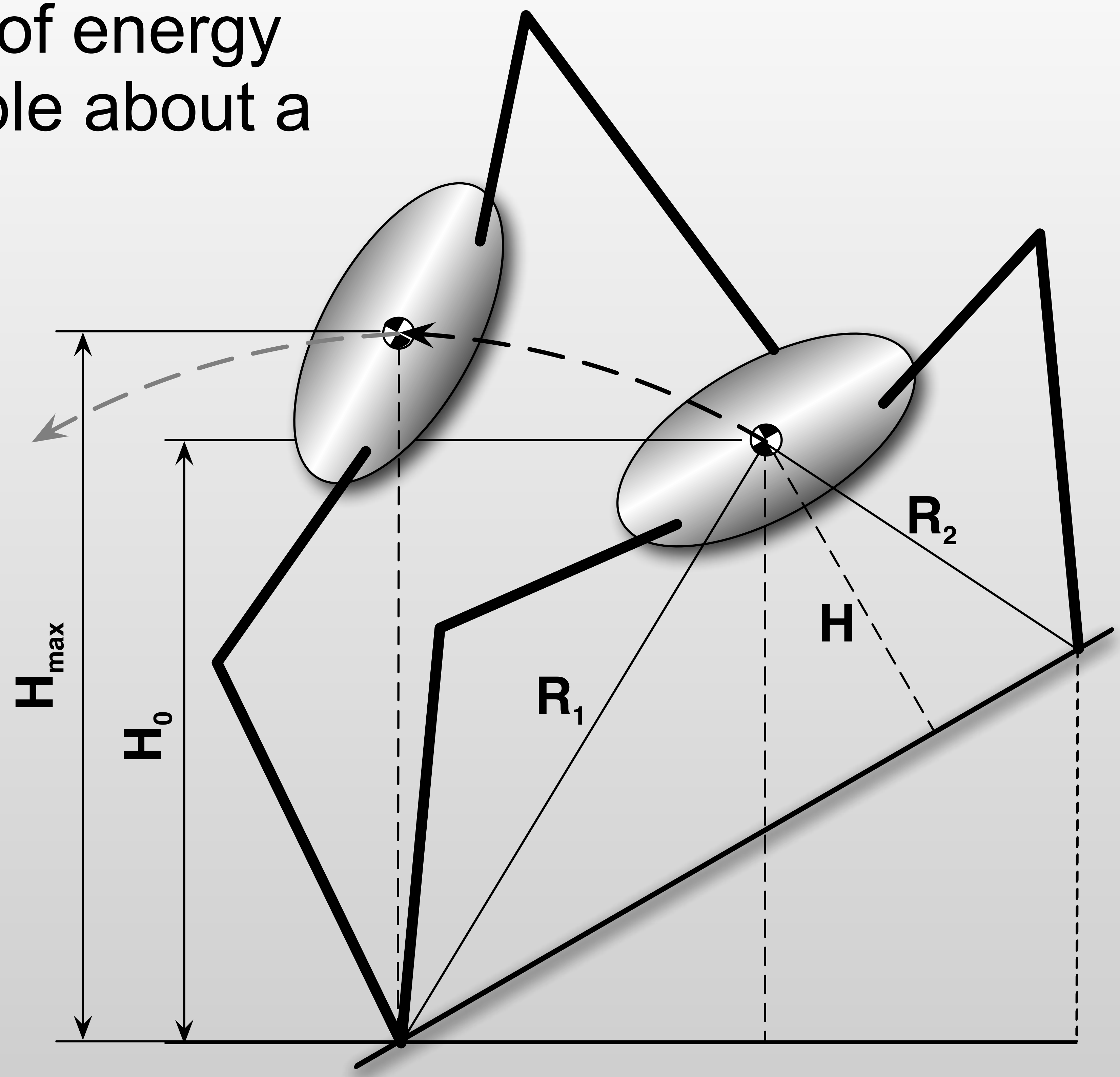


Energy Stability Margin

- Minimum amount of energy necessary to tumble about a support segment

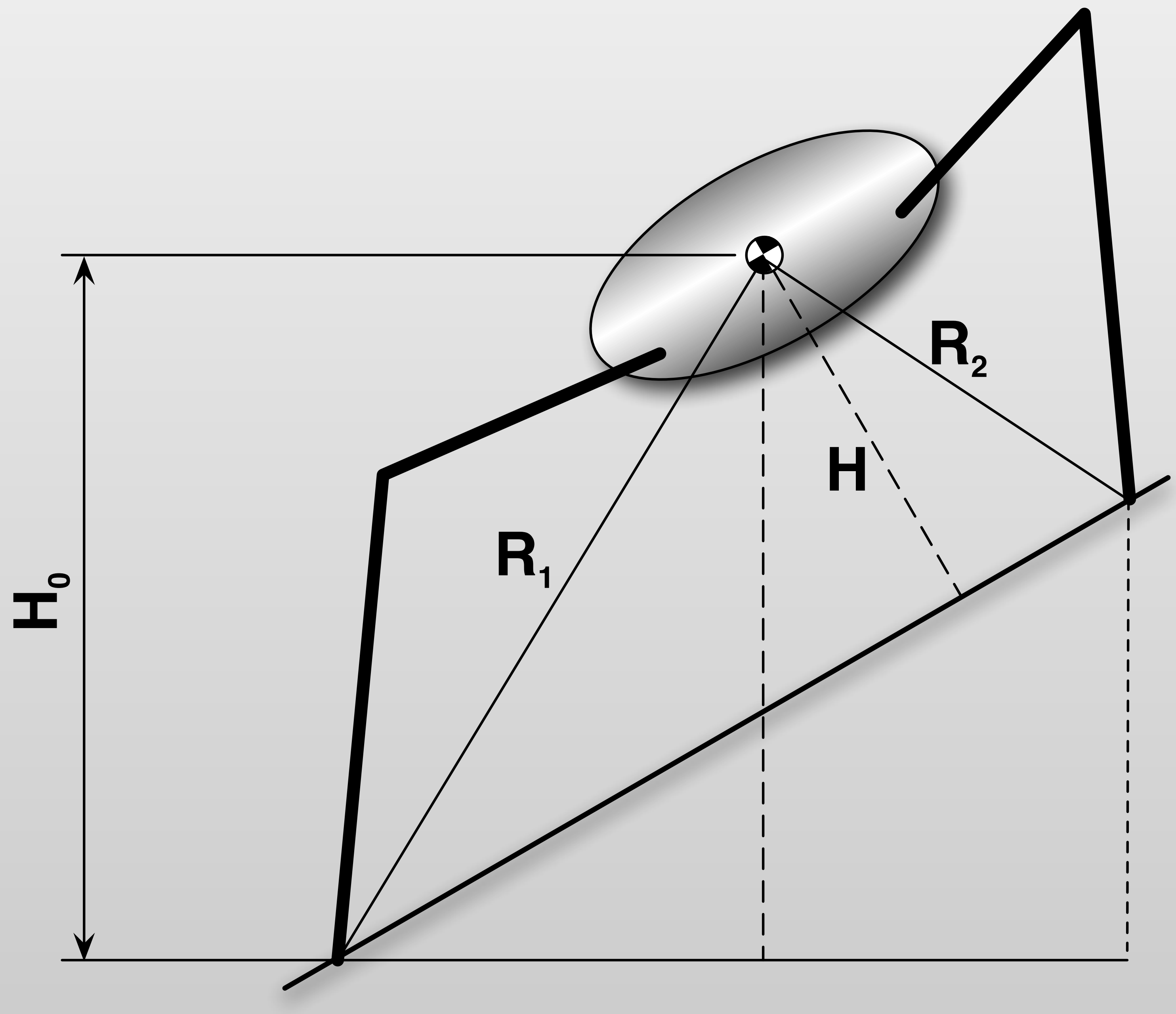
$$S_{ESM} = \min_{i}^{n_s}(mgh_i)$$

$$h_i = H_{max} - H_0$$



Energy Stability Margin

- Minimum amount of energy necessary to tumble about a support segment

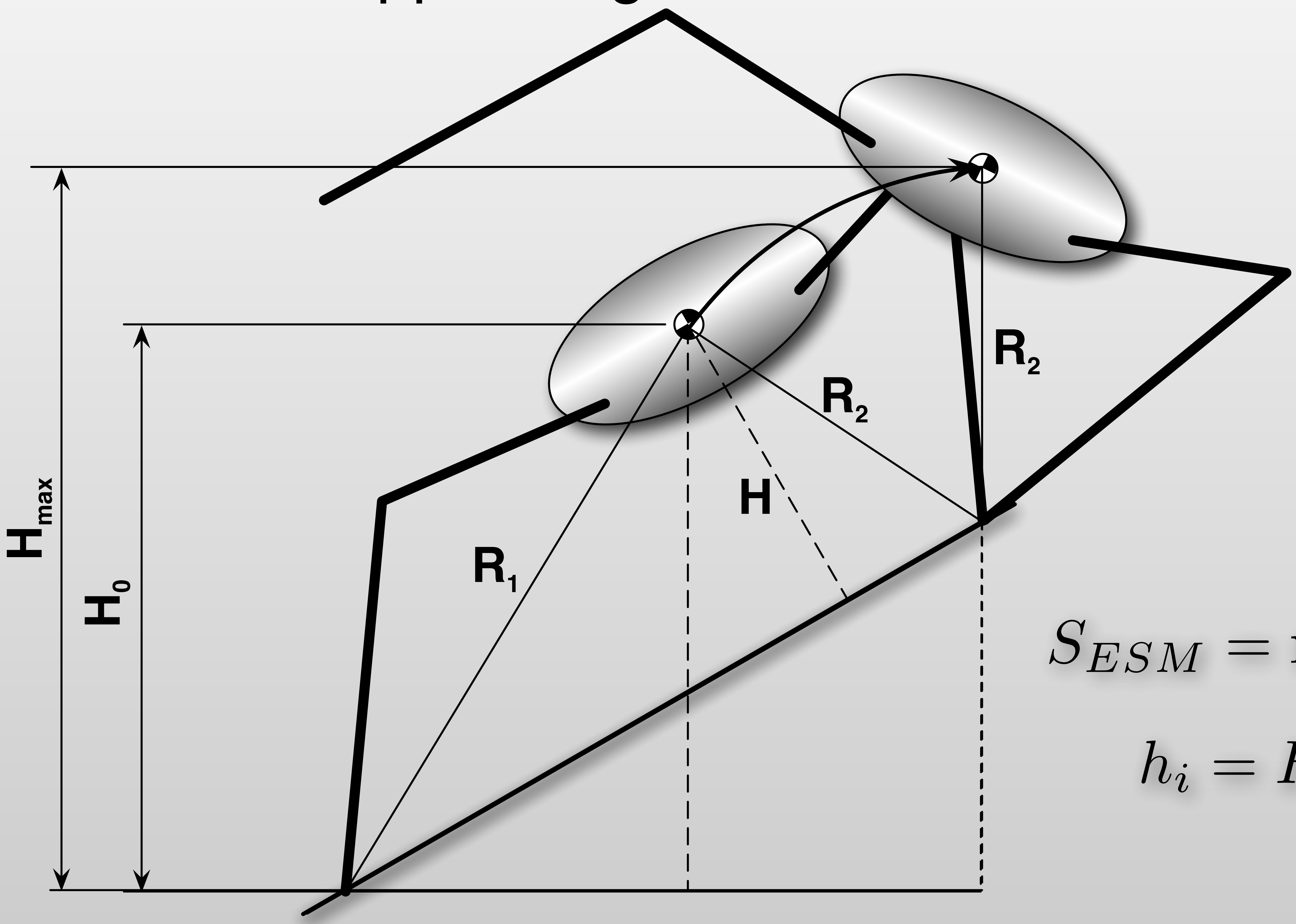


$$S_{ESM} = \min_{i}^{n_s}(mgh_i)$$

$$h_i = H_{max} - H_0$$

Energy Stability Margin

- Minimum amount of energy necessary to tumble about a support segment



$$S_{ESM} = \min_i^{n_s}(mgh_i)$$

$$h_i = H_{max} - H_0$$

Optimal SESM

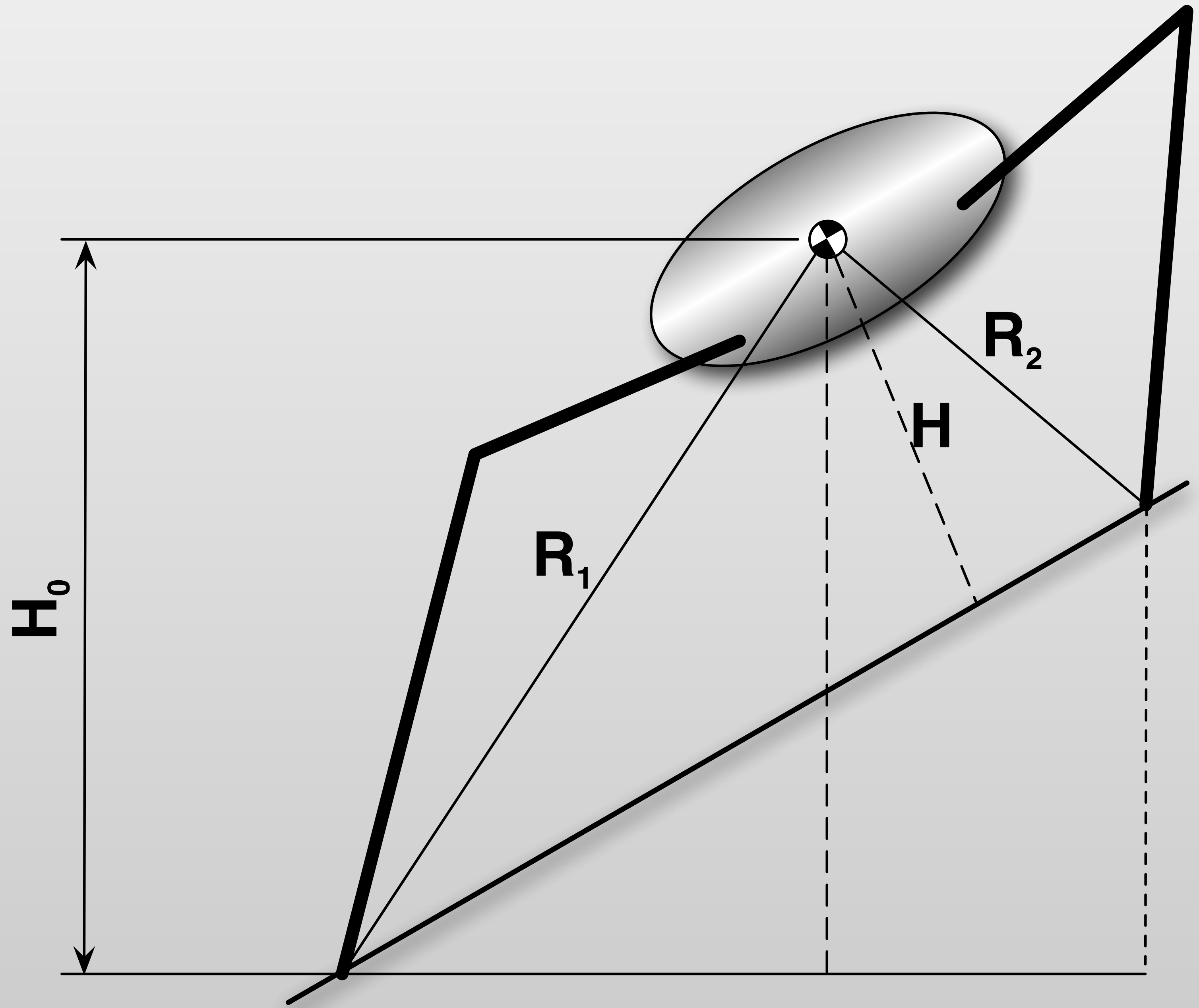
- Position equally likely to tip in any direction = same energy in all directions

$$S_{ESM}(Low) = mg(H_{Lmax} - H_0)$$

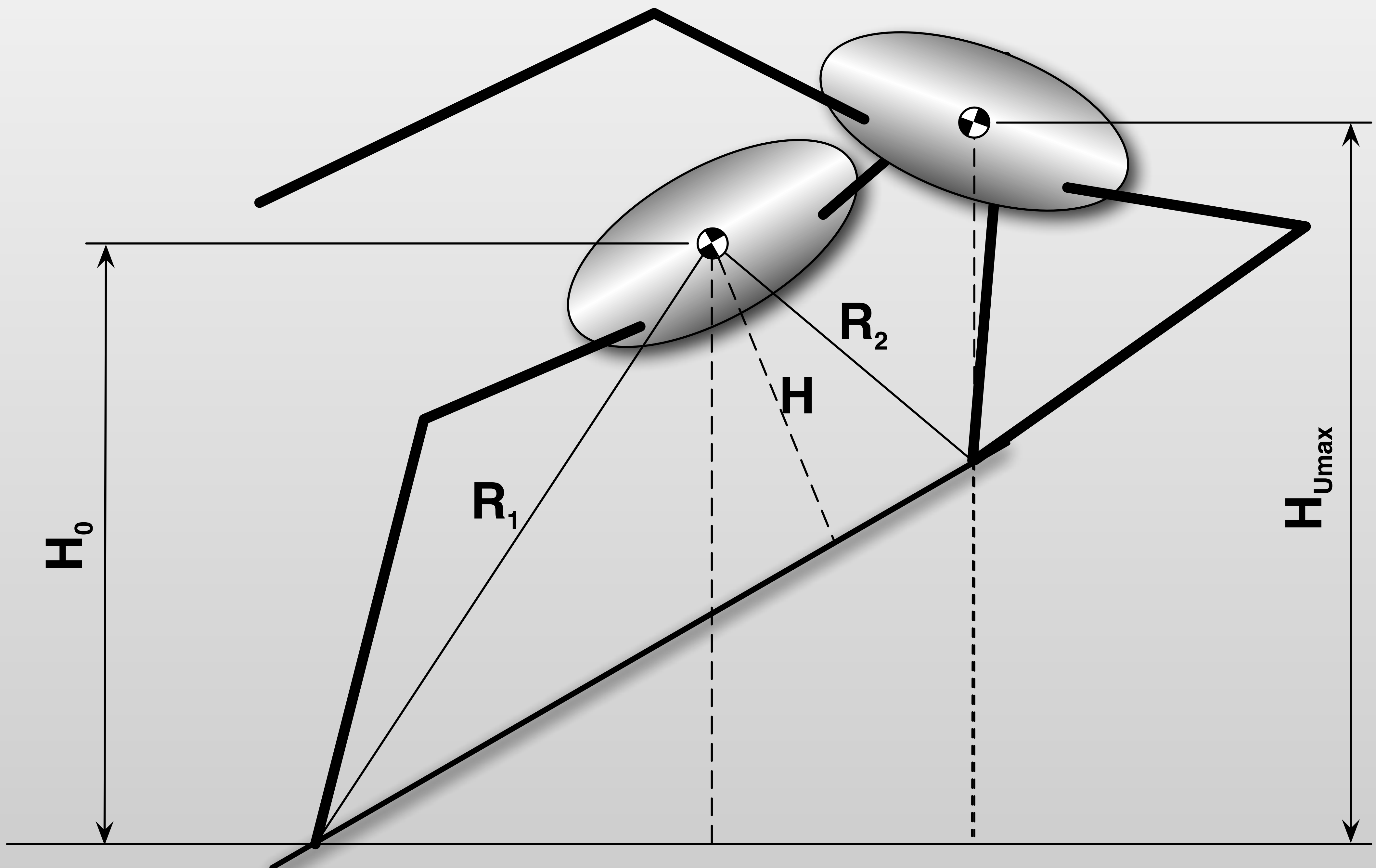
$$S_{ESM}(High) = mg(H_{Umax} - H_0)$$

$$mg(H_{Lmax} - H_0) = mg(H_{Umax} - H_0)$$

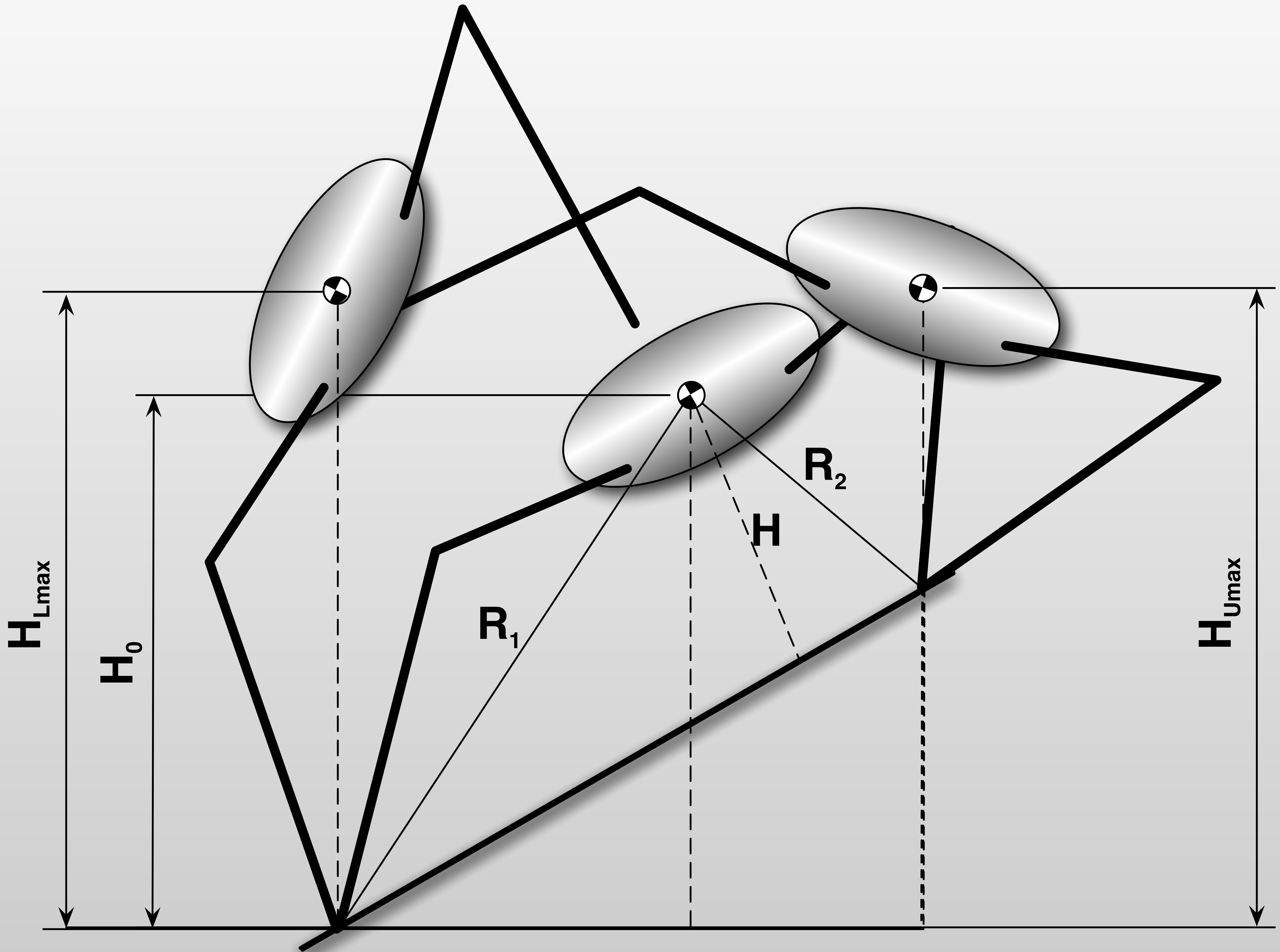
Optimal SESM



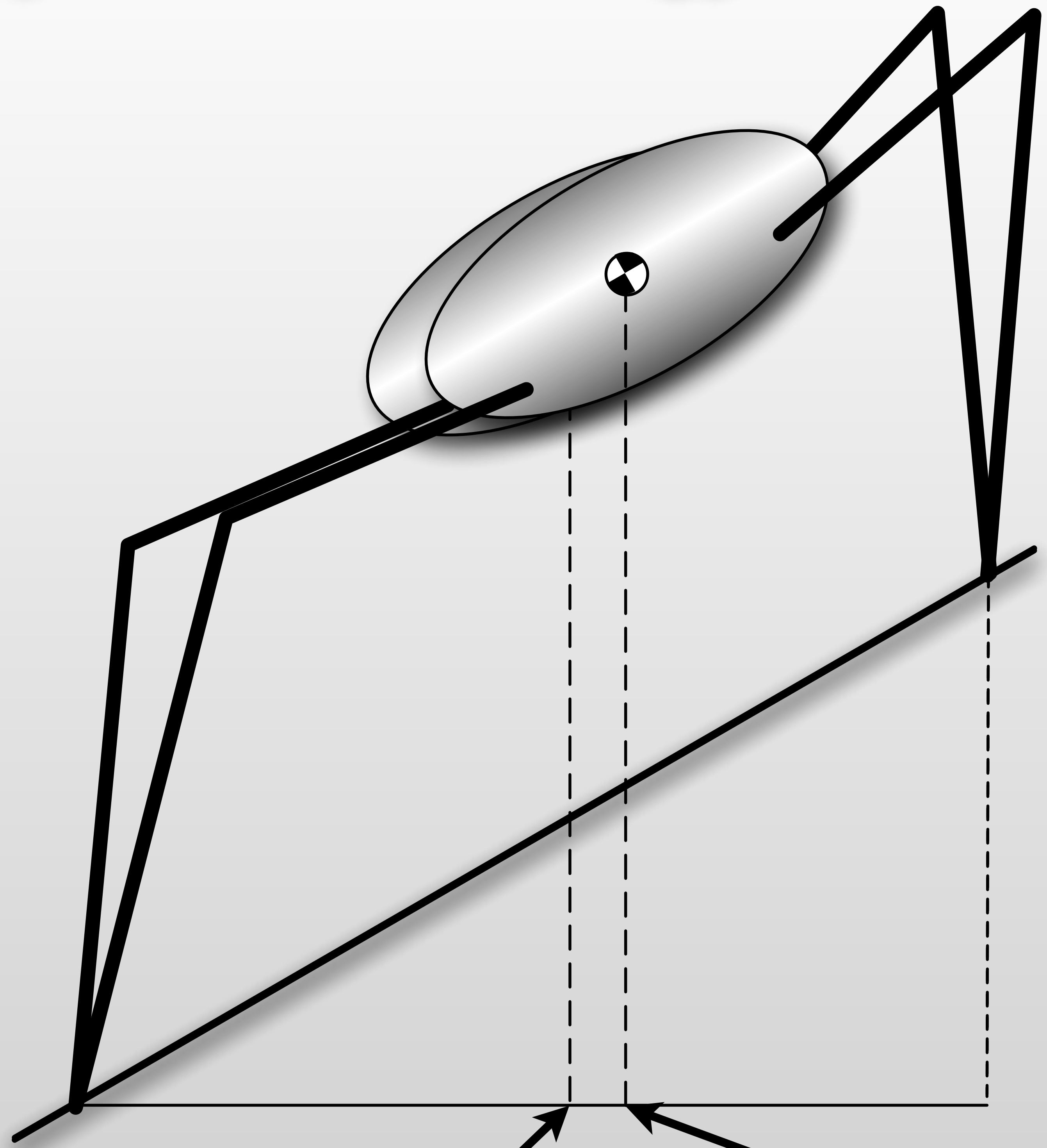
Optimal SESM



Optimal SESM



COM Projection vs. Energy Stability Margin

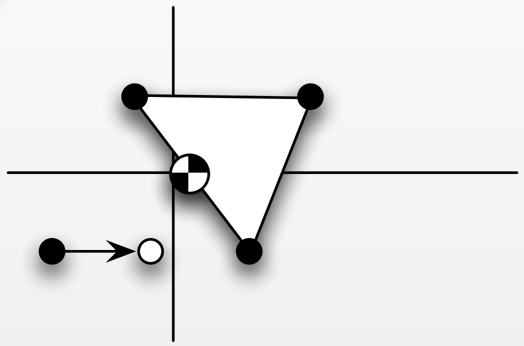


Using Static Stability Margin

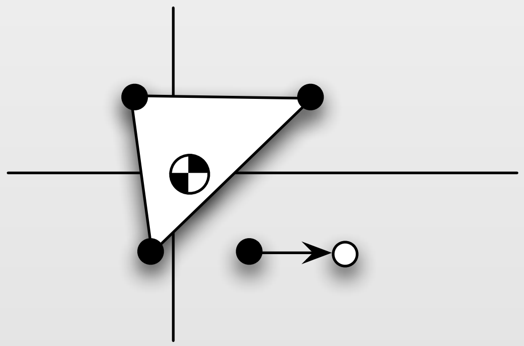
- Gate order: LR-LF-RR-RF

Statically Stable Gate

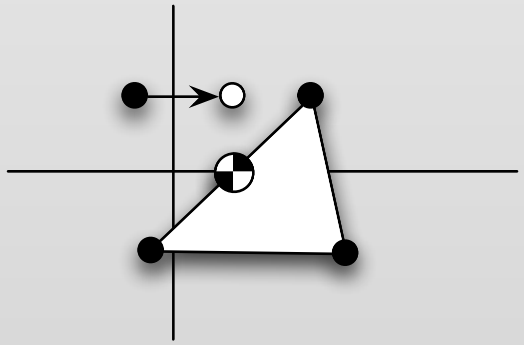
1. Right Rear



2. Right Front



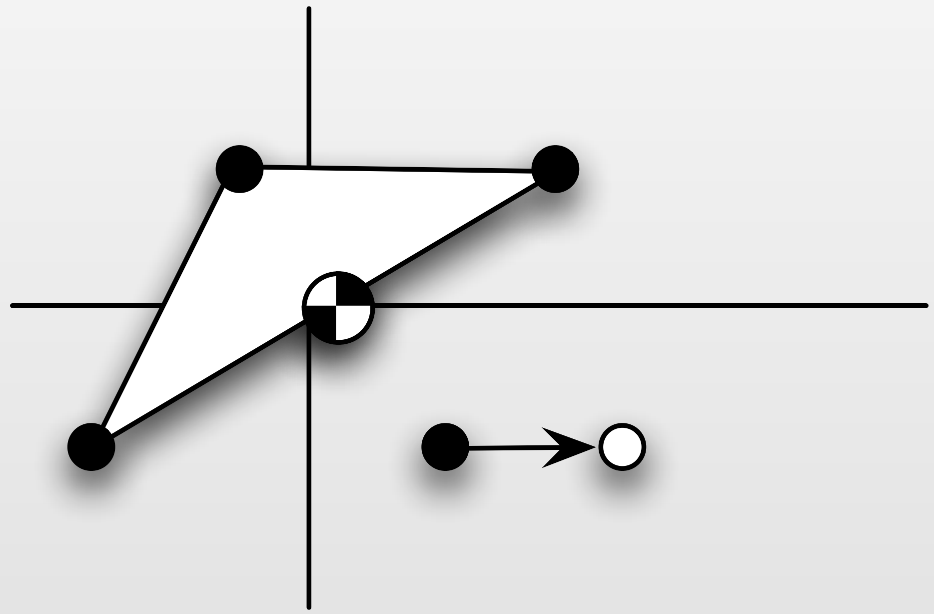
3. Left Rear



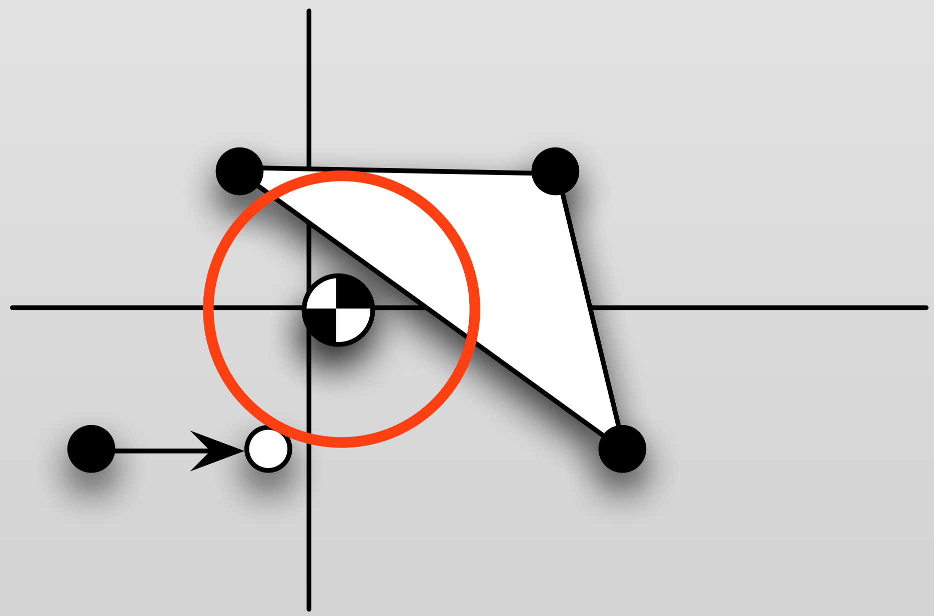
What’s the Problem?

AT-AT Problem

- Wrong gait order, uses LF-LR-RF-RR



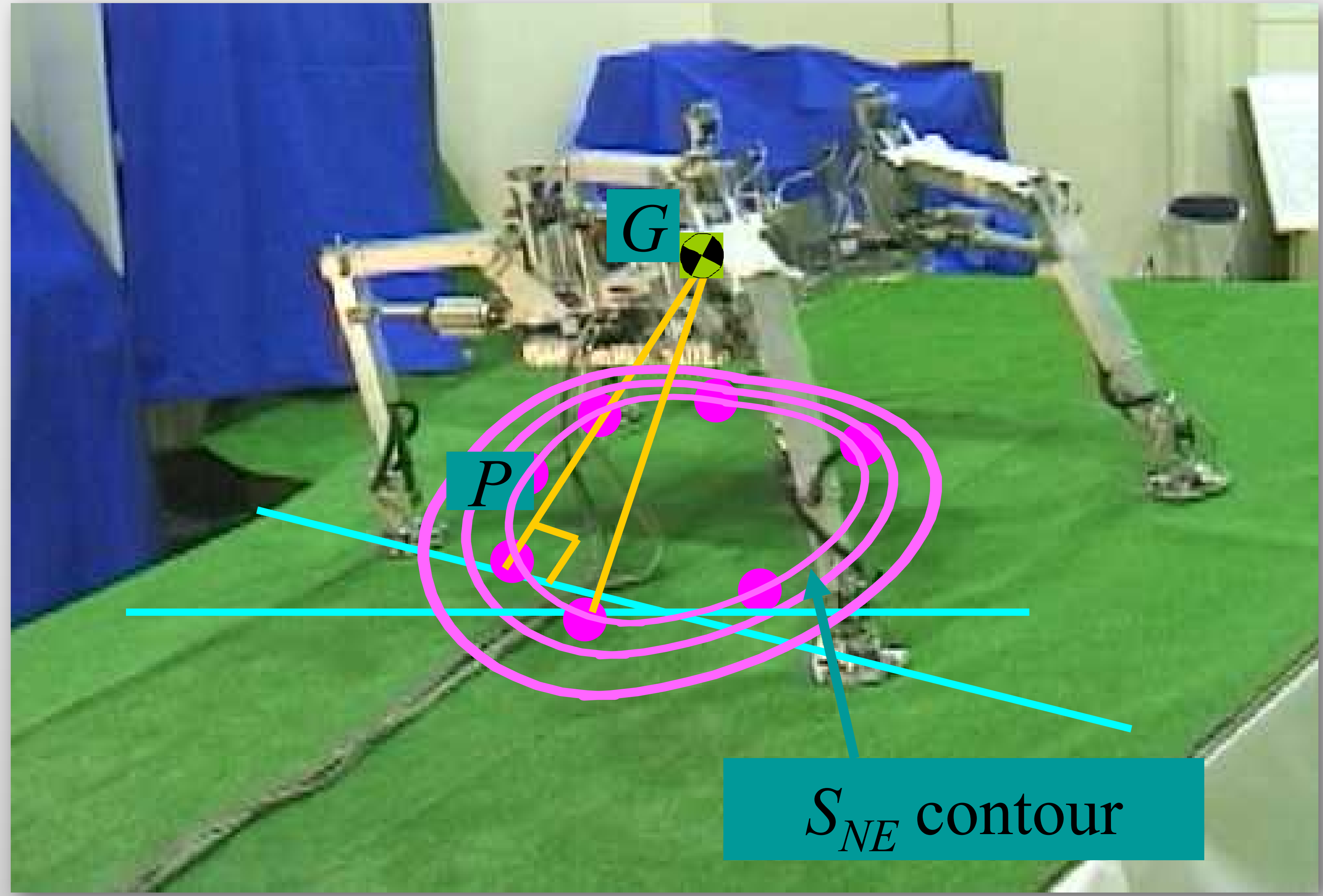
1. Right Front



2. Right Rear

Using Energy Stability Margin

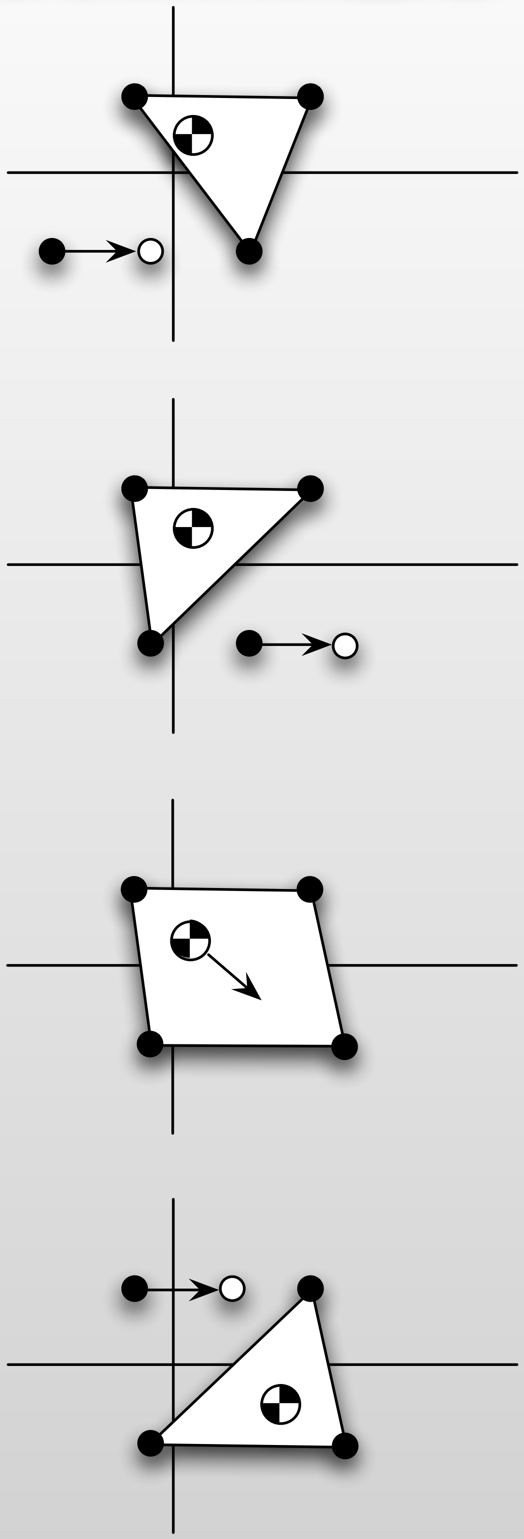
- Can be used to plan foot placements for walking robots
- Generate contour of points with same SESM



Credit: Shigeo Hirose, Tokyo Institute of Technology

Gait to Maximize SESM

1. Right Rear
2. Right Front
3. Shift COM
4. Left Rear
5. …

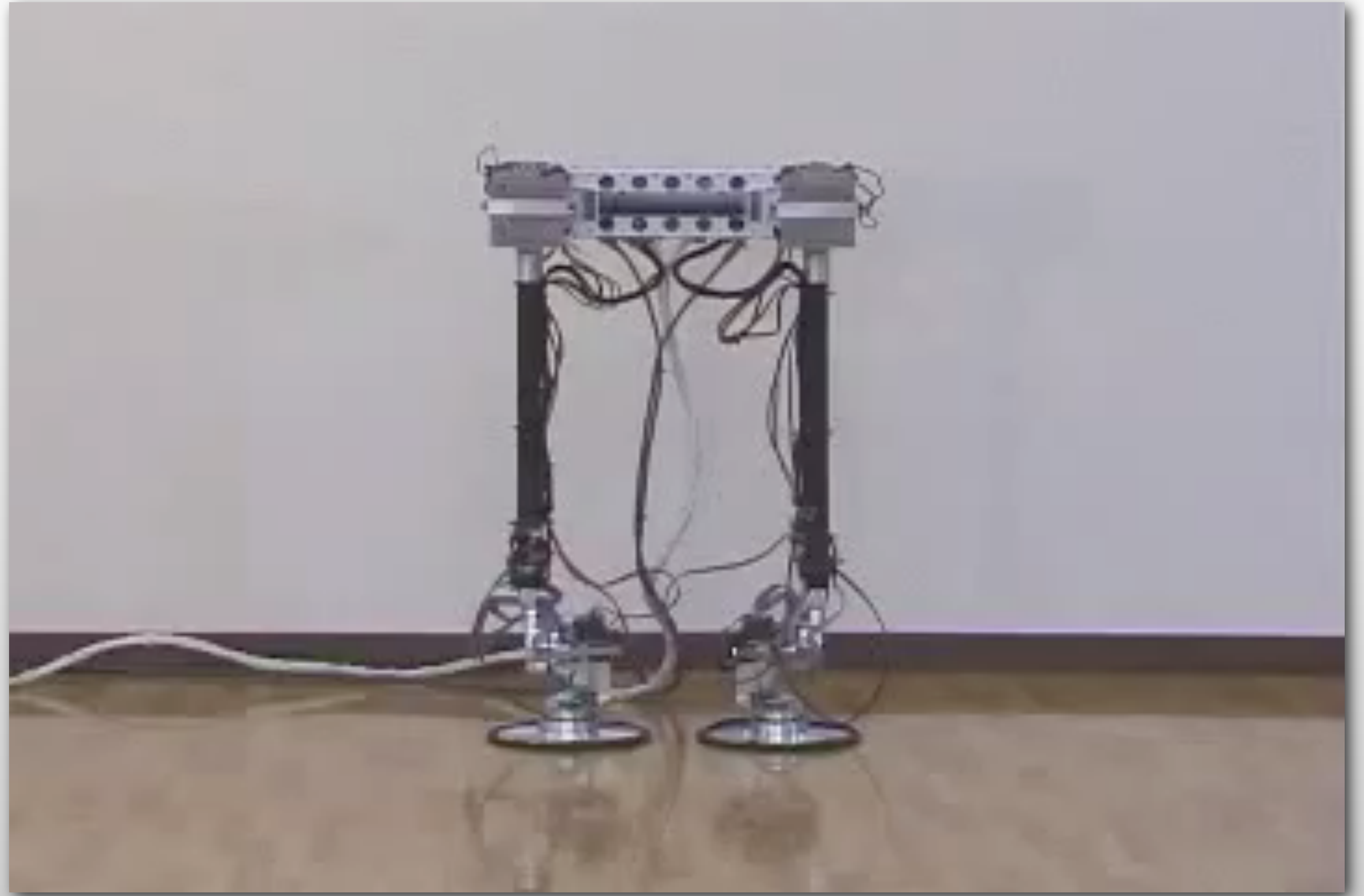


Crawl Gait

Walking Robots on Rough Terrain

Credit: Shigeo Hirose, Tokyo Institute of Technology

How about bipedal walking?

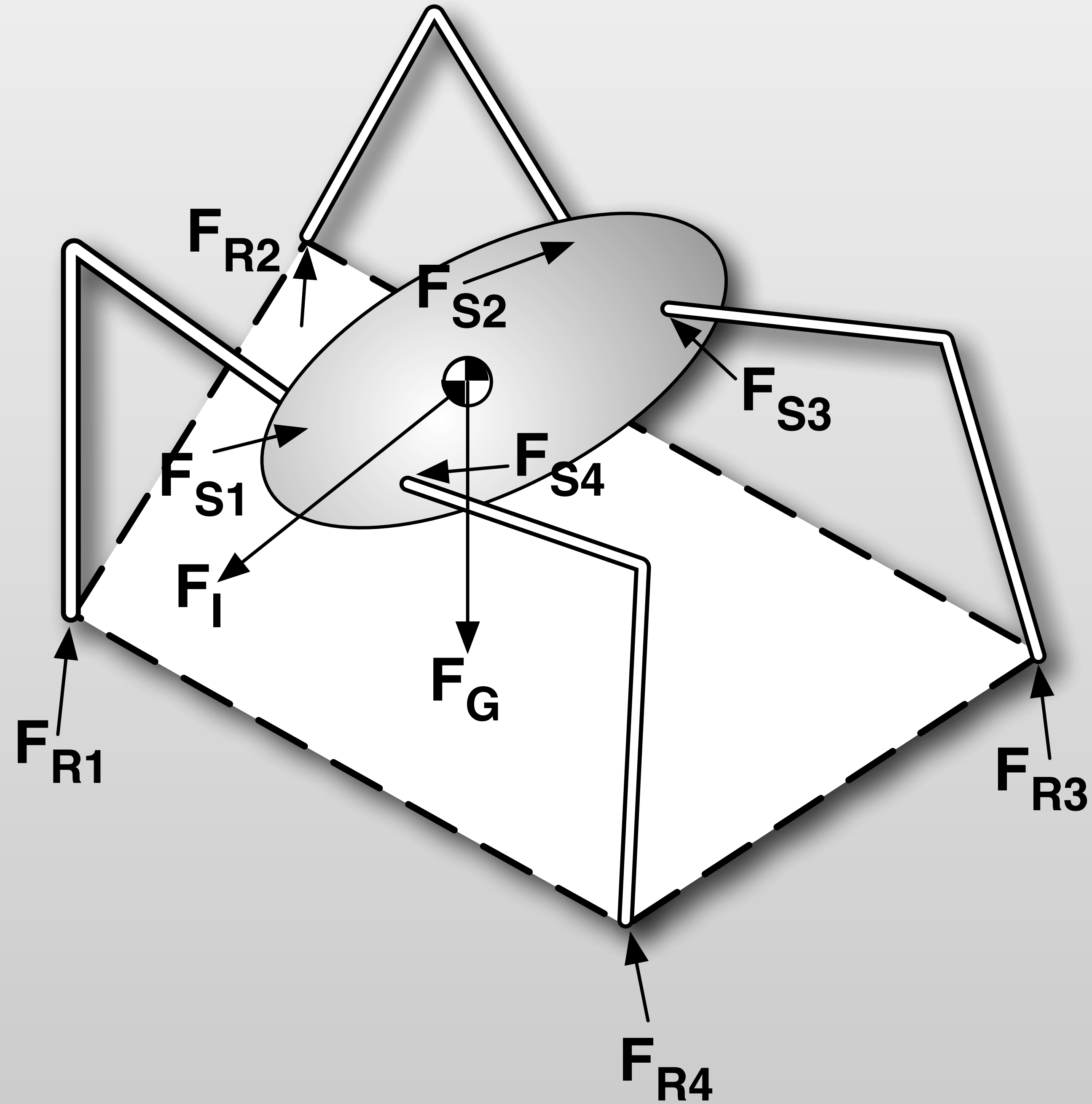


Dynamic Stability

- Considers forces acting on the system
  - Gravity
  - Support forces (terrain interaction)
  - Inertial Forces (acceleration)
  - Manipulation tasks
    - Tool forces required
    - Weight of object grasped
    - ...

Example Forces

- FG - gravity
- FRi - foot reaction forces
- FSi - support forces (coincide with support forces for massless legs)
- FI - inertial force



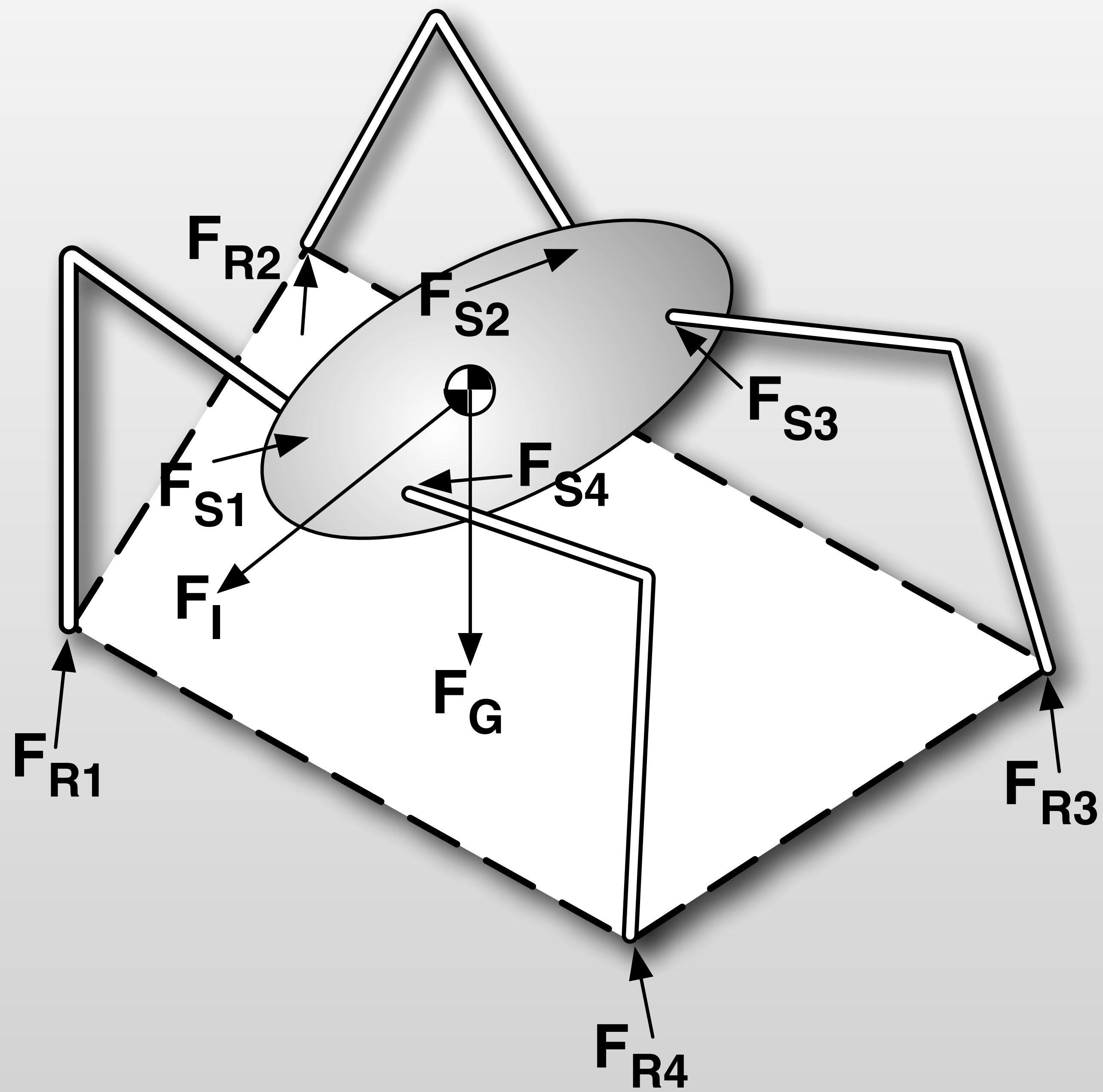
Dynamic Stability Margin

- Dynamic equilibrium:

$$F_I = F_S + F_G$$
$$M_I = M_S + M_G$$

$$F_S = \sum F_{Si}$$

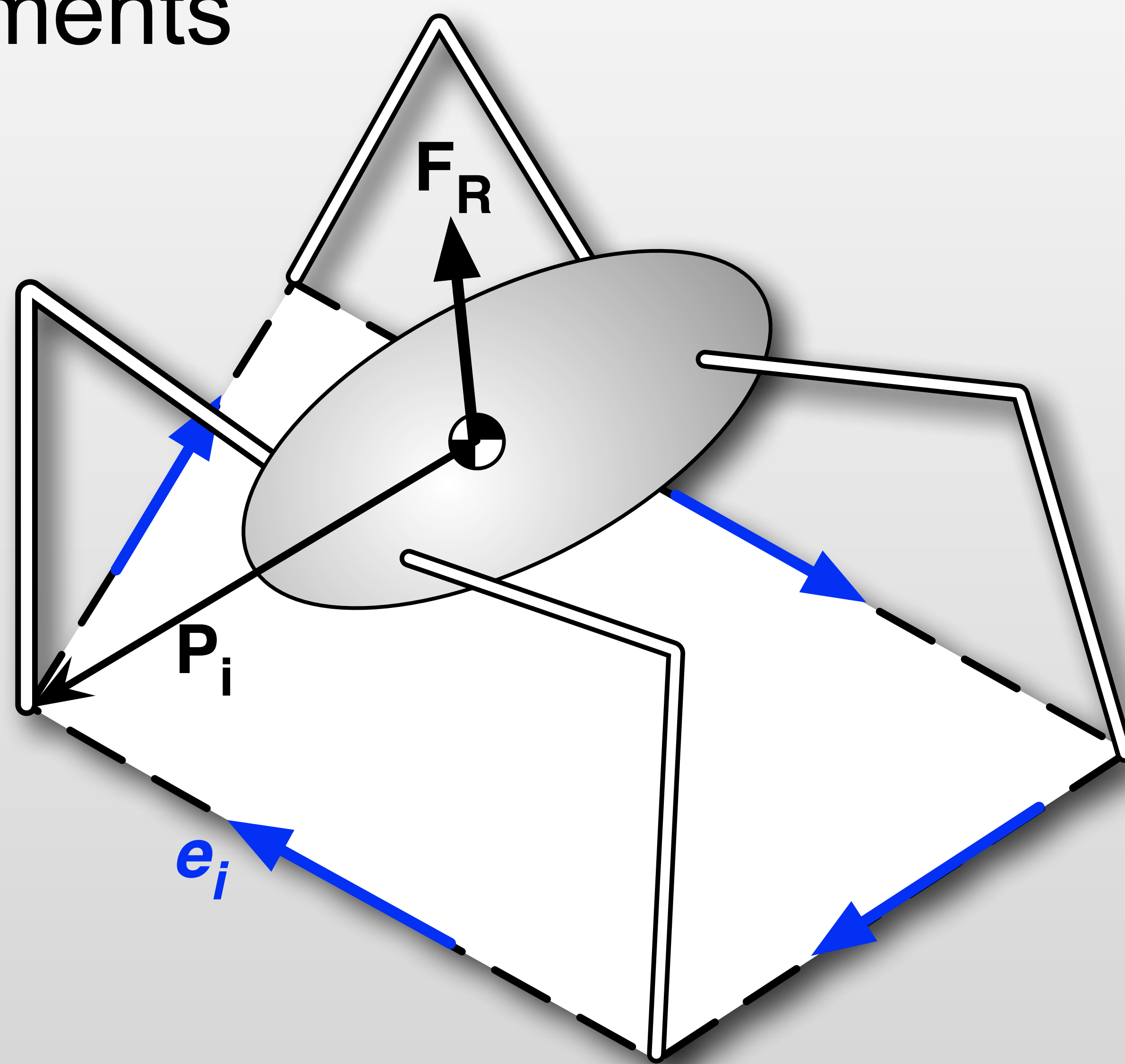
$$\vdots$$



Dynamic Stability Margin

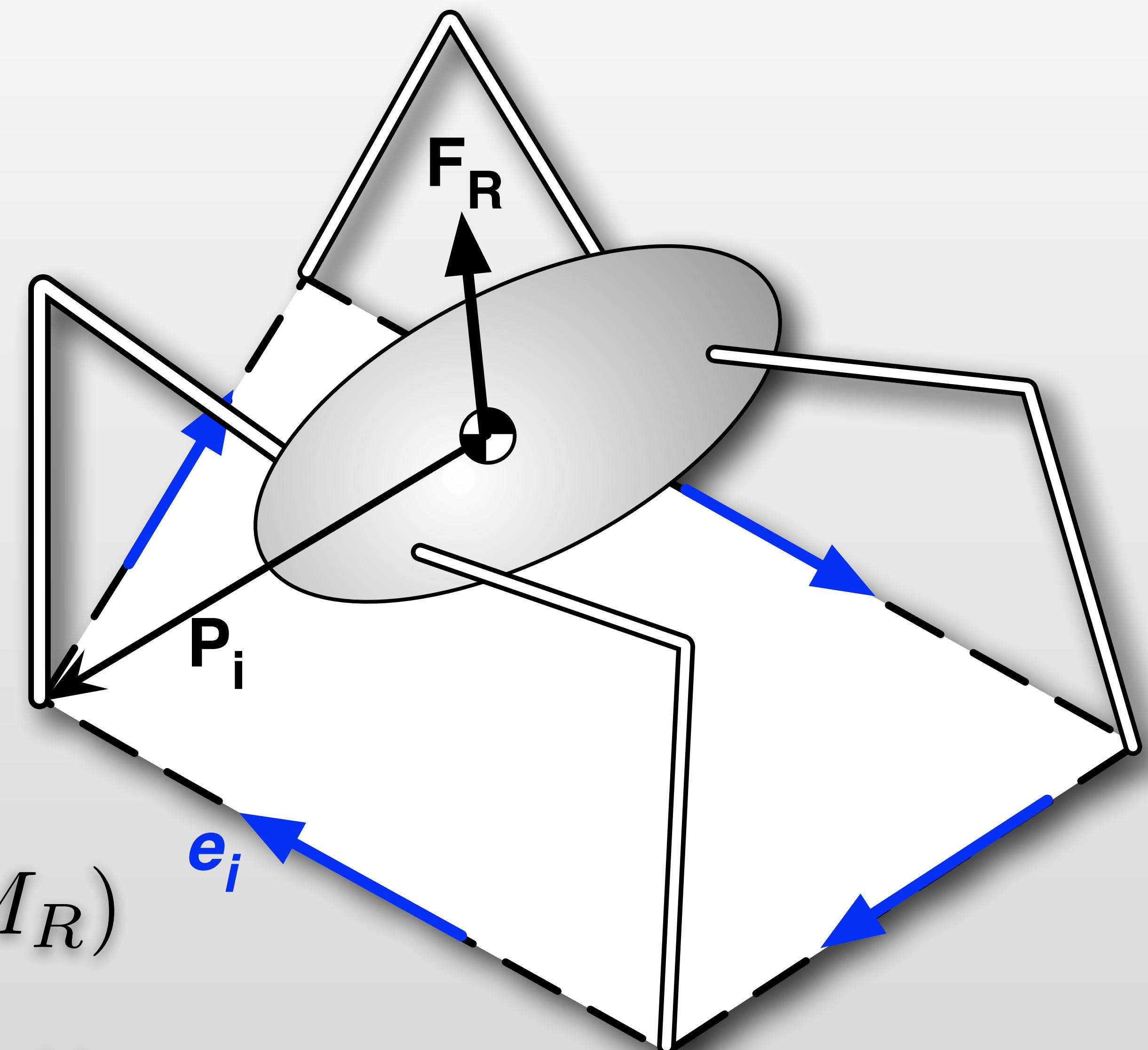
- During a tumble FR and MR must compensate for destabilizing forces and moments
- Moment created by support:

$$M_i = e_i \cdot (F_R \times P_i + M_R)$$



Dynamic Stability Margin

- If all Mi are positive then the system is stable
- Use minimum of Mi as stability margin
- Normalize by weight to allow comparison between systems



$$M_i = e_i \cdot (F_R \times P_i + M_R)$$

$$\min_i (e_i \cdot (F_R \times P_i + M_R))$$

$$S_{DSM} = \min_i \left(\frac{e_i \cdot (F_R \times P_i + M_R)}{mg} \right)$$

How do we find FR and MR?

How do we find FR and MR?

- Measure directly (common in bipedal walking robots)
- Estimate from other parameters (most common)

$$\sum F = ma$$

$$ma = F_S + F_G$$

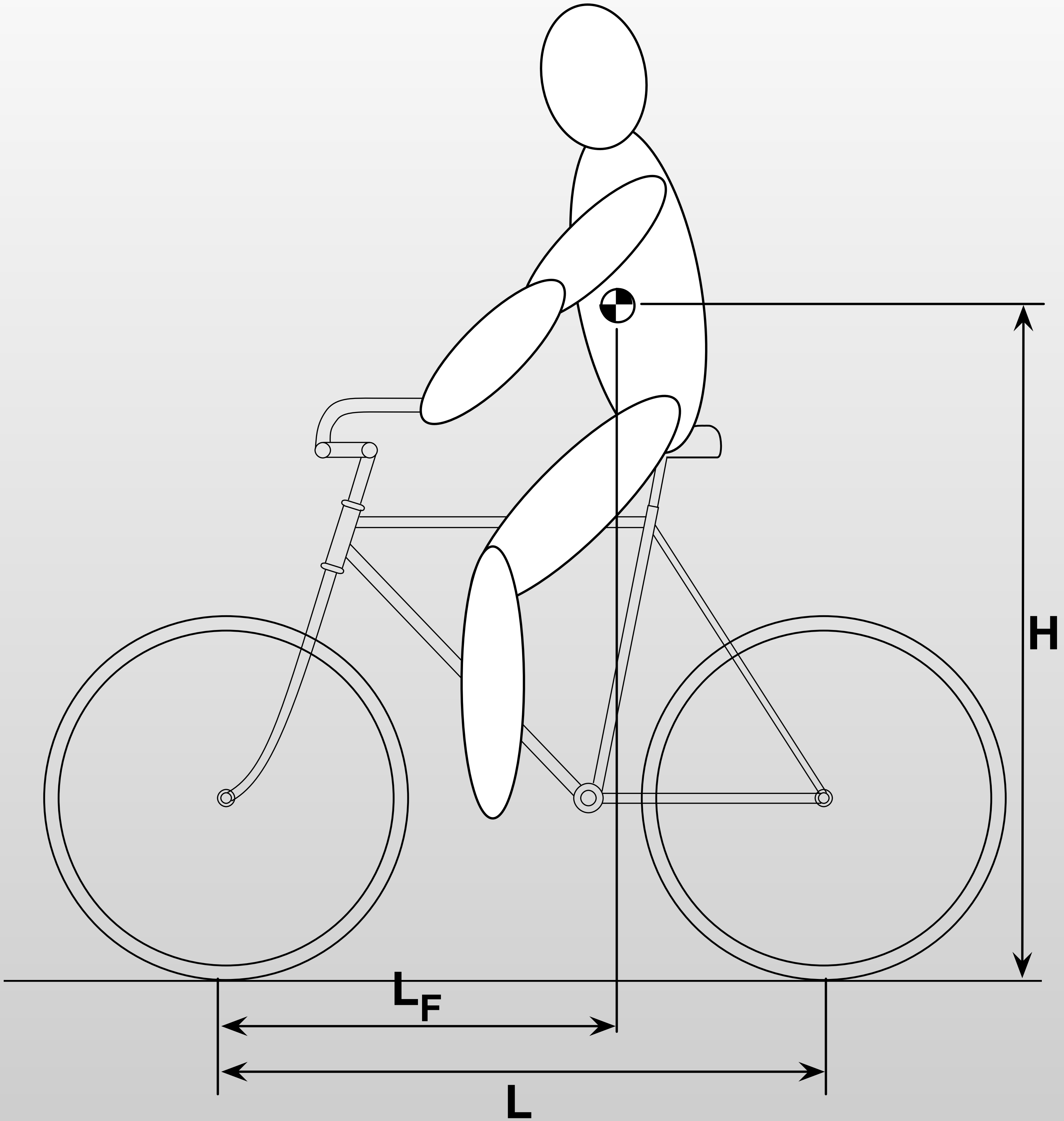
$$F_I = F_S + F_G$$

$$F_S = F_I - F_G$$

Assume Massless legs: $F_S = F_R$

$$\boxed{F_R = F_I - F_G}$$

Bicycle Example



Bike Crash

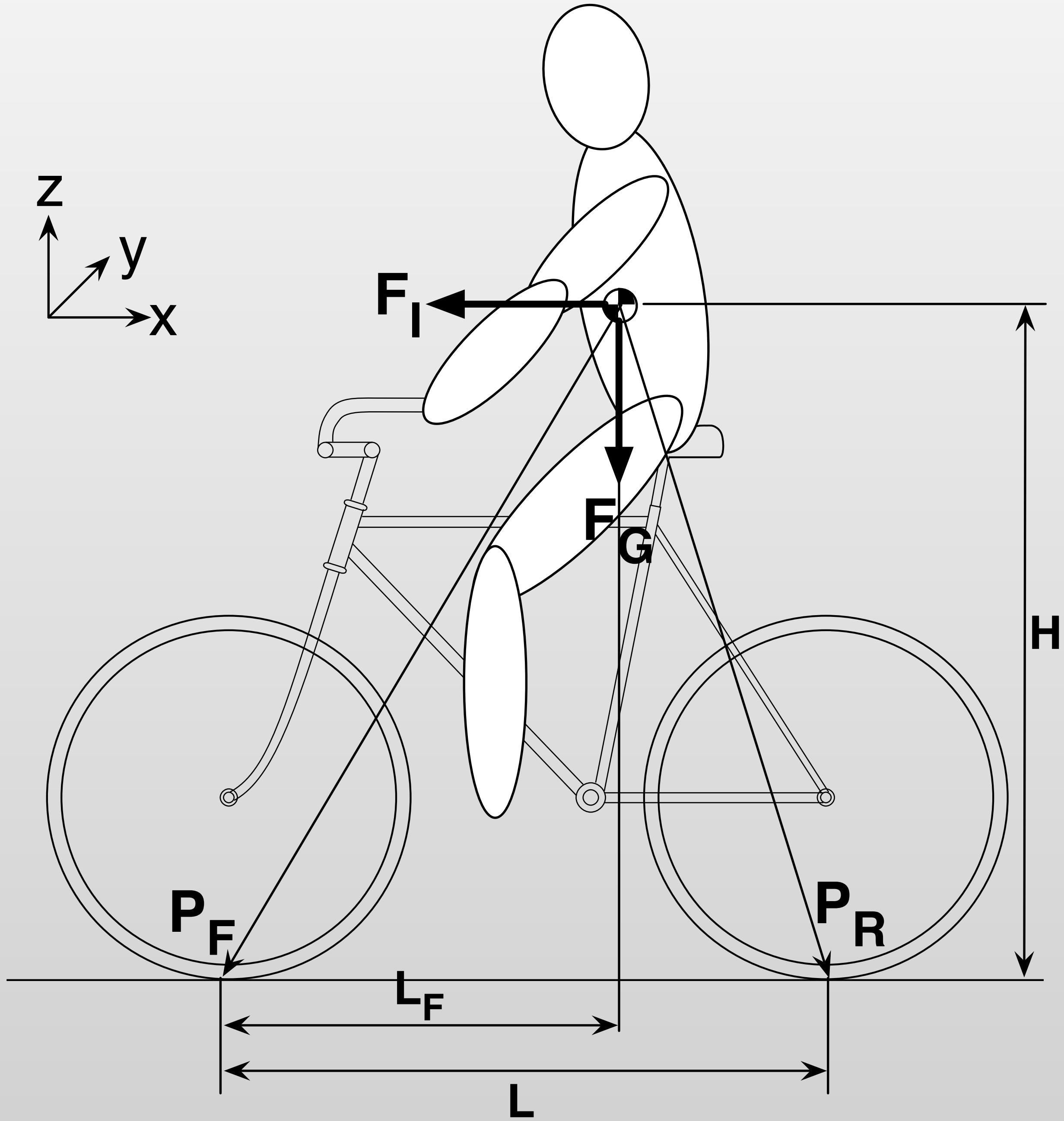
Bicycle Example - Forces

- Stopping immediately
- No skid

$$F_I = (-ma)\bar{i}$$
$$F_G = (-mg)\bar{k}$$

$$P_F = (-L_F)\bar{i} + (-H)\bar{k}$$

$$P_R = (L - L_F)\bar{i} + (-H)\bar{k}$$



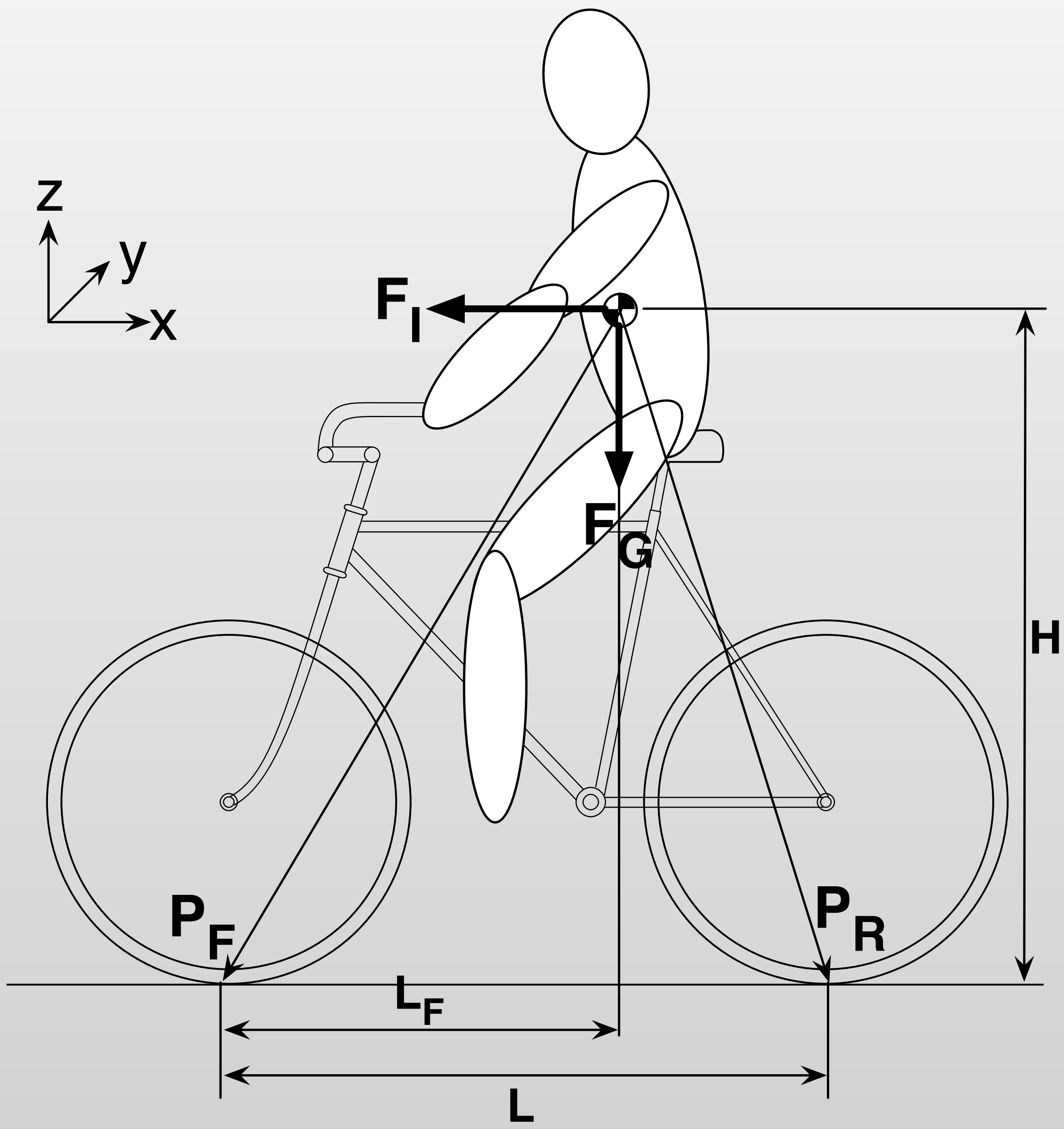
Bicycle Example

- Stopping immediately
- No skid

$$F_R = F_I - F_G$$
$$= (-ma)\bar{i} - (-mg)\bar{k}$$

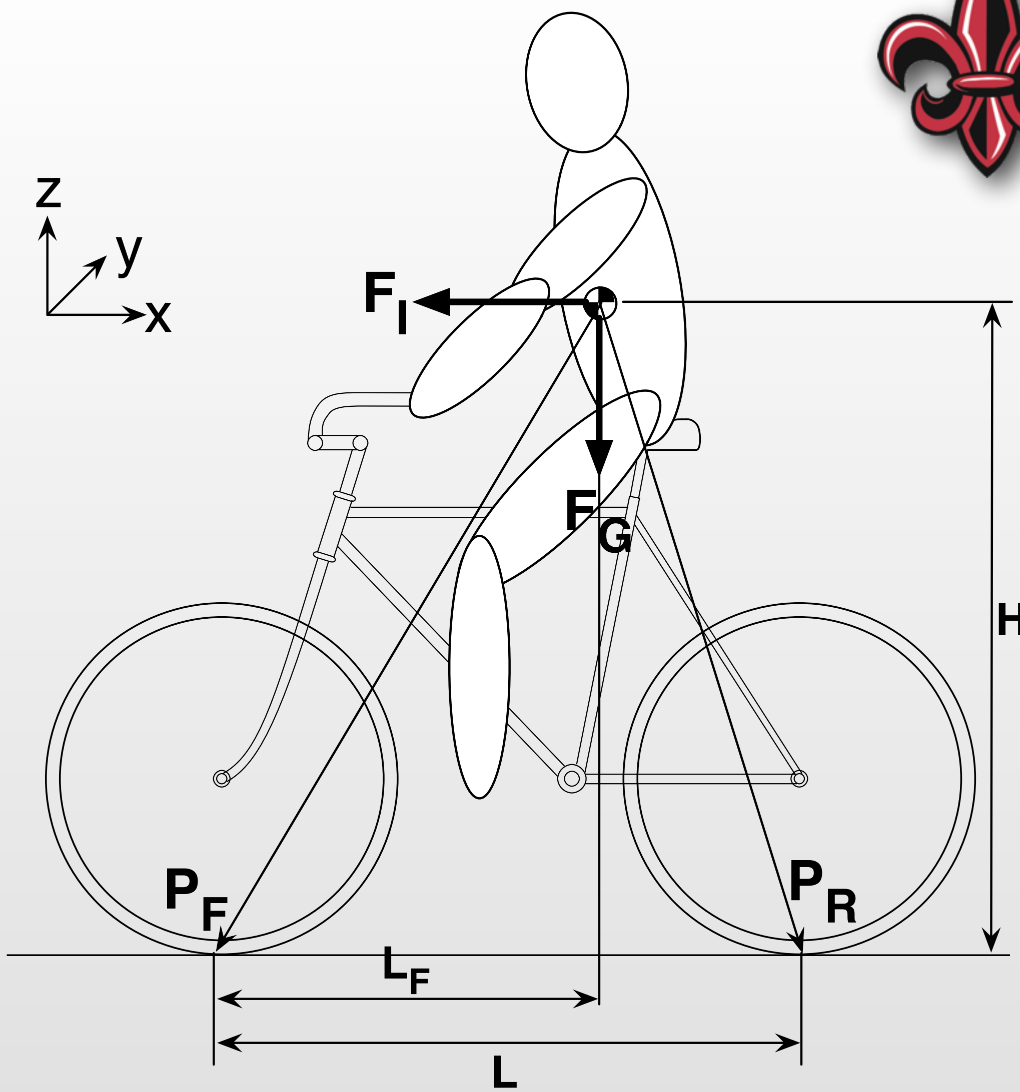
$$e_F = \bar{j}$$
$$e_R = -\bar{j}$$

$$M_F = e_F \cdot (F_R \times P_F)$$
$$M_R = e_R \cdot (F_R \times P_R)$$



Front Axis

- Stopping immediately
- No skid



$$F_R = F_I - F_G$$
$$= (-ma)\bar{i} - (-mg)\bar{k}$$

$$e_F = \bar{j}$$

$$M_F = e_F \cdot (F_R \times P_F)$$

$$M_F = \bar{j} \cdot \left(\left[(-ma)\bar{i} + (mg)\bar{k}\right] \times \left[(-L_F)\bar{i} + (-H)\bar{k}\right]\right)$$
$$M_F = \bar{j} \cdot (-Hma + L_F mg)\bar{j}$$
$$M_F = -Hma + L_F mg$$

Bicycle Dynamic Stability Margin

- Front

$$\frac{M_F}{mg} = -H\frac{a}{g} + L_F \;= S_{DSM}$$

- Rear

$$\frac{M_R}{mg} = H\frac{a}{g} + (L - L_F)$$

Zero Moment Point

- Used heavily in bipedal walking robots (Asimo)
- Point on which reaction force from foot creates no moment
- If this point is within foot support polygon, then stable
- Not valid for non-planar contact