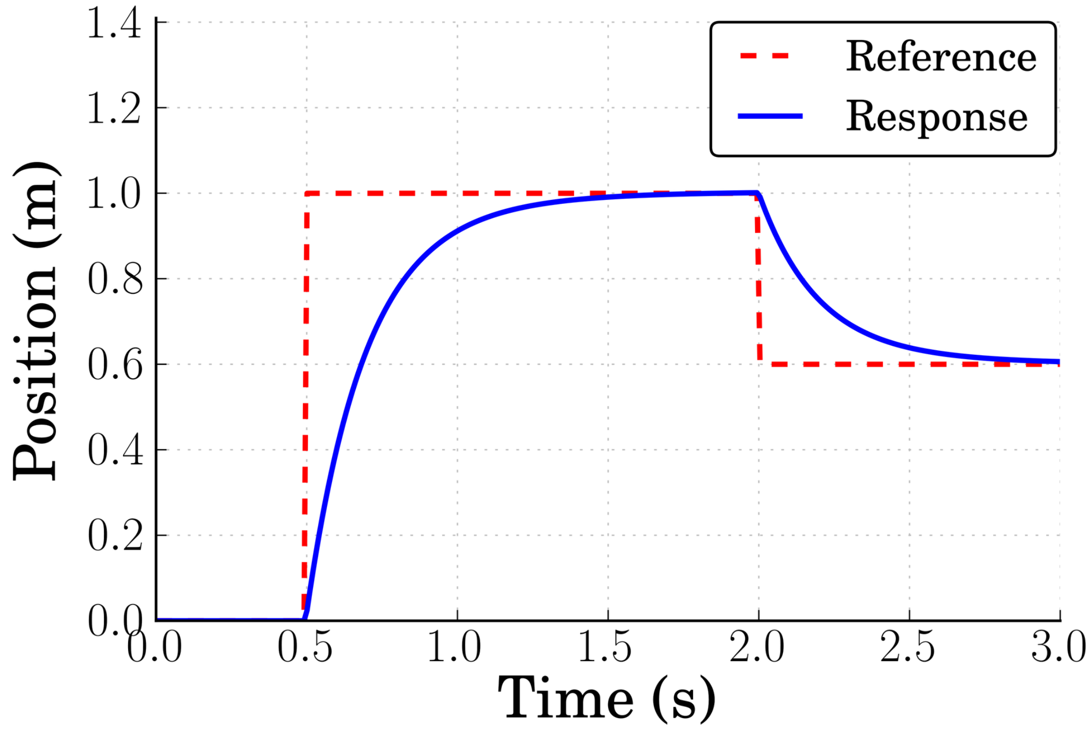
PID Implementation Concerns - Derivative Kick

Note: There are other implementation issues you need to know about. The link below does an excellent job covering them.

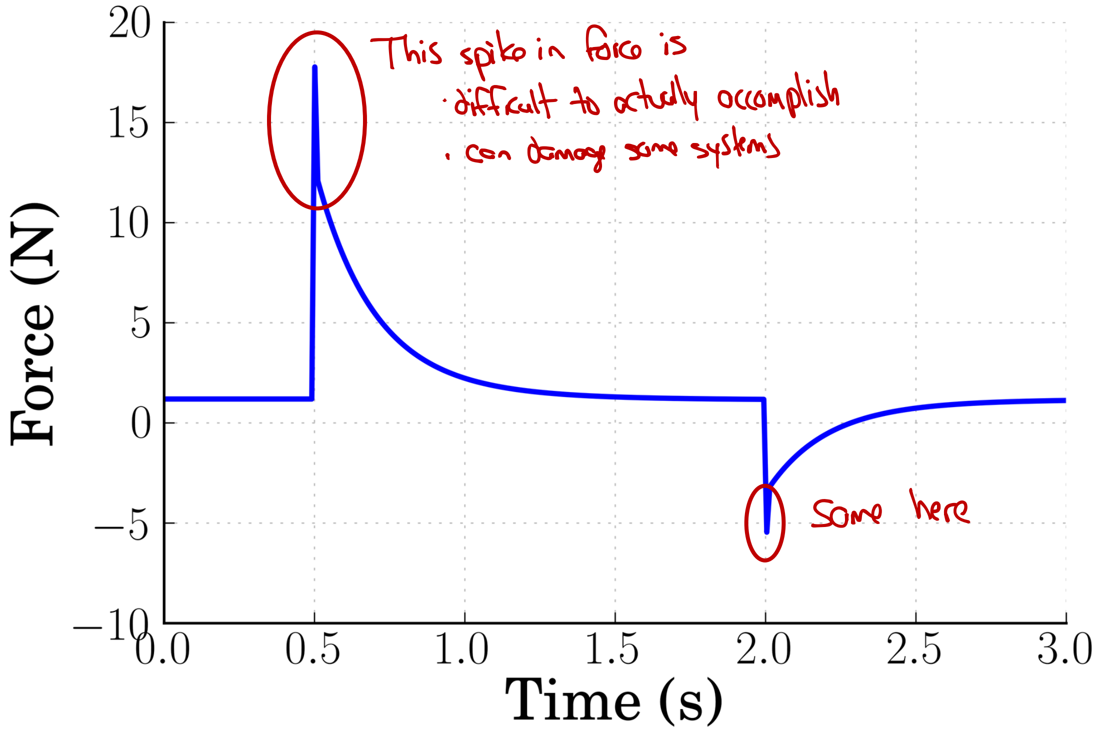
http://brettbeauregard.com/blog/2011/04/improving-the-beginners-pid-introduction

Let's look at an example response:



Q: What force is needed to create this response?

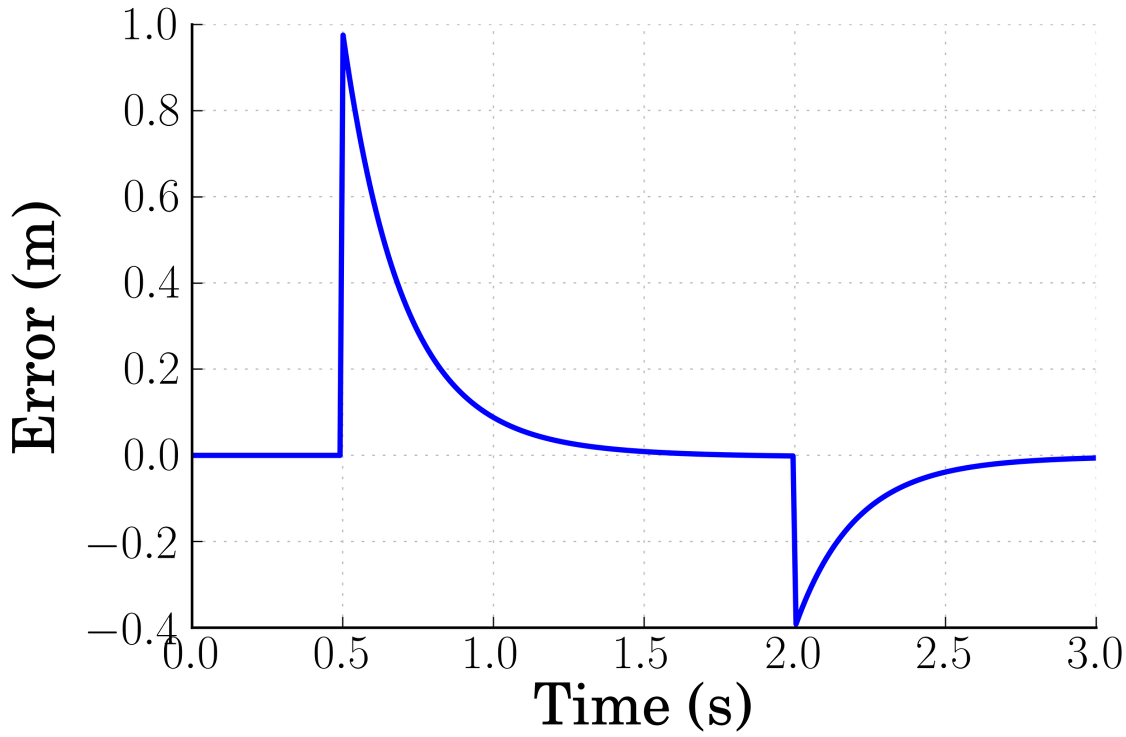
$u(t) = k_p e + k_d \dot{e} + k_i \int e\,dt$



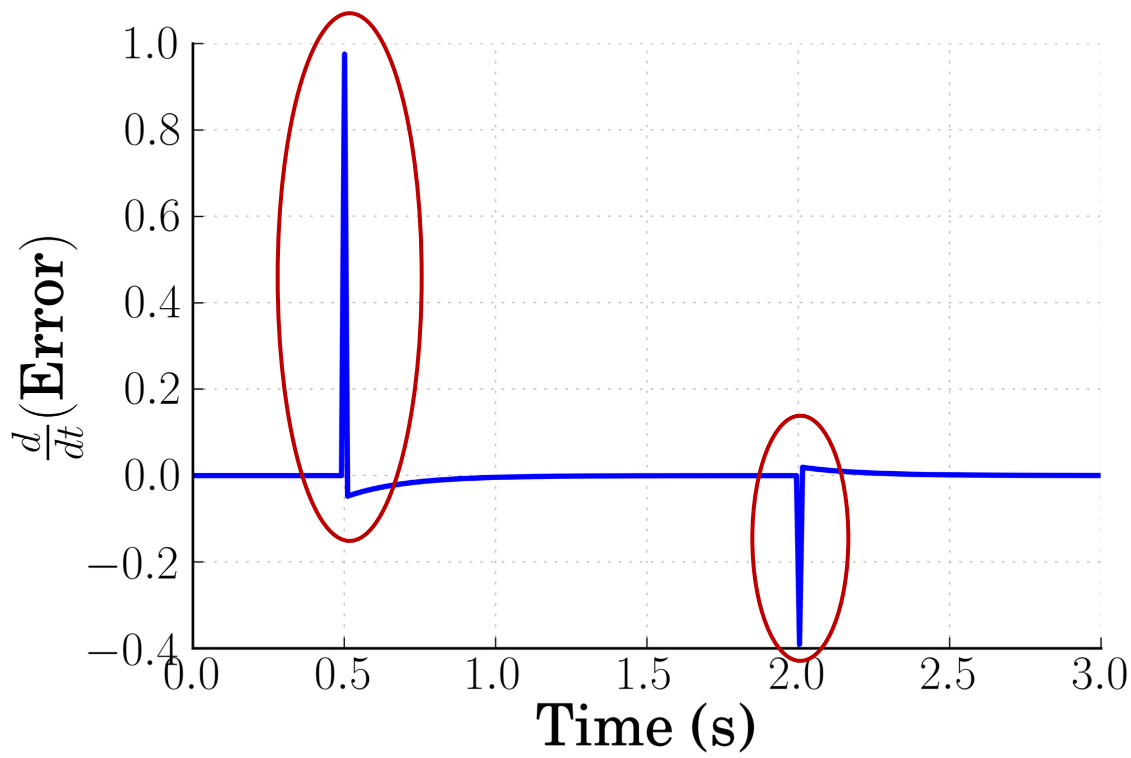
Q: Why does this "derivative kick" happen?

Let's look at the error term to see

PID Implementation Concerns - Derivative Kick (cont.)



Q: What's the problem?



These spikes in $\dot{e}$ show up in the derivative term of the controller.

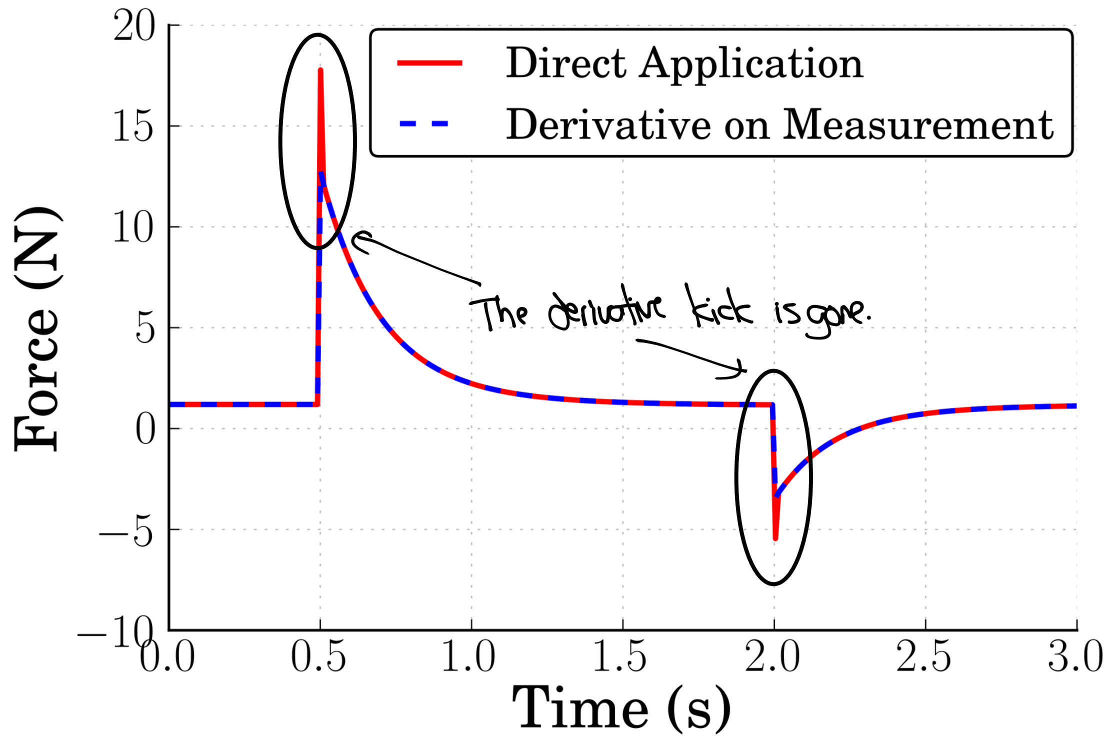
$$u(t) = k_p e + \underline{k_d \dot{e}} + k_i \int e\,dt$$

Q: How do we fix this?

Look at $\dot{e}$:

$\dot{e} = \dot{x}_d - \dot{x}$ ← But $\dot{x}$ will be smooth for most sys.

↑ if x_d contains steps $\dot{x}_d$ will have "spikes"



So, let's try using this term for the derivative gain.

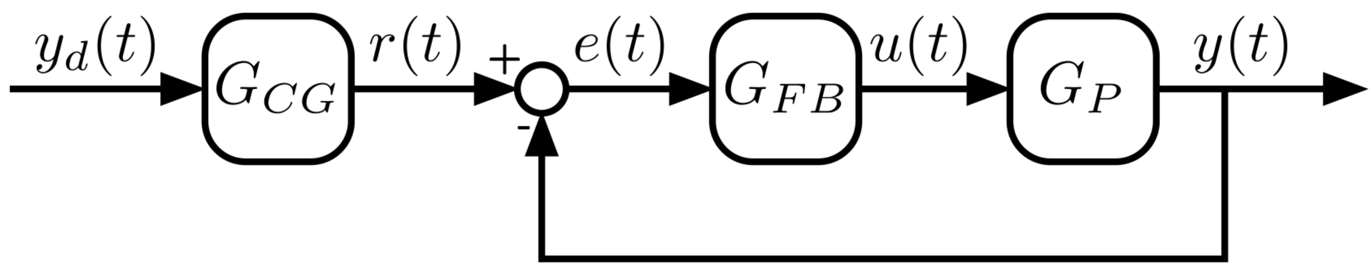
The controller becomes:

$$u(t) = k_p e + k_d(\cancel{\dot{x}_d} - \dot{x}) + k_i \int e\,dt$$

$$u(t) = k_p e + k_d(-\dot{x}) + k_i \int e\,dt$$

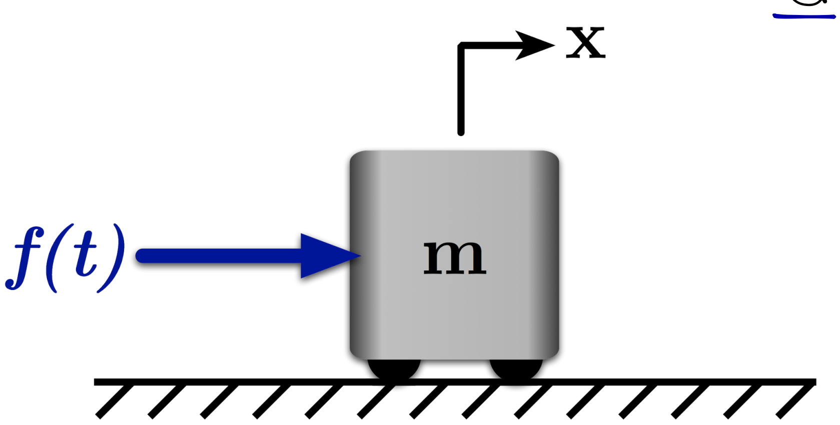
Main point: We can't blindly apply even theoretically-correct controllers, etc.

Command Generation



Q: Given some $y_d(t)$, what should $r(t)$ be?

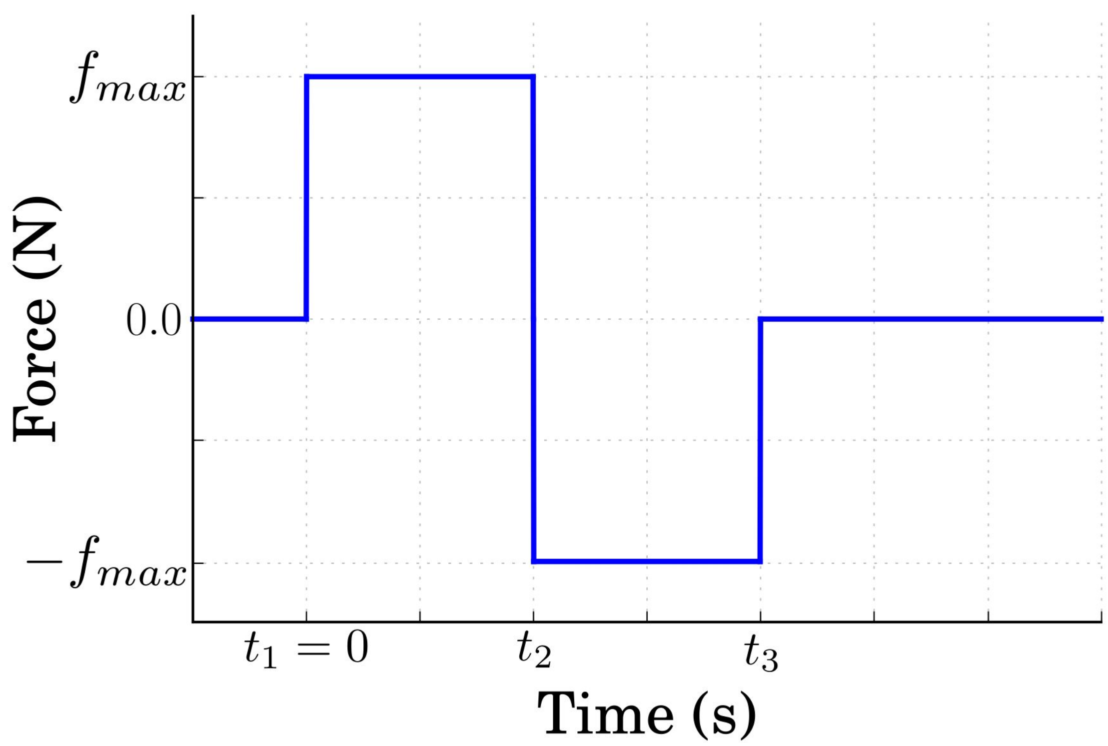
Basic Command Generation



Q: What should $f(t)$ be to move x from x_0 to x_d as fast as possible?

Bang-Bang Input

- actuator is acting at its max amplitude at all times



So, our job is to select t_2 and t_3.

Q: How?

Start with Equation of Motion

$$m\ddot{x} = f(t)$$

$$\ddot{x} = \frac{f(t)}{m} \quad \text{and} \quad -f_{max} \leq f(t) \leq f_{max}$$

$$x(t) = \int_{t_1}^{t_3}\int_{t_1}^{t_3} \frac{f(t)}{m}\, dt^2$$

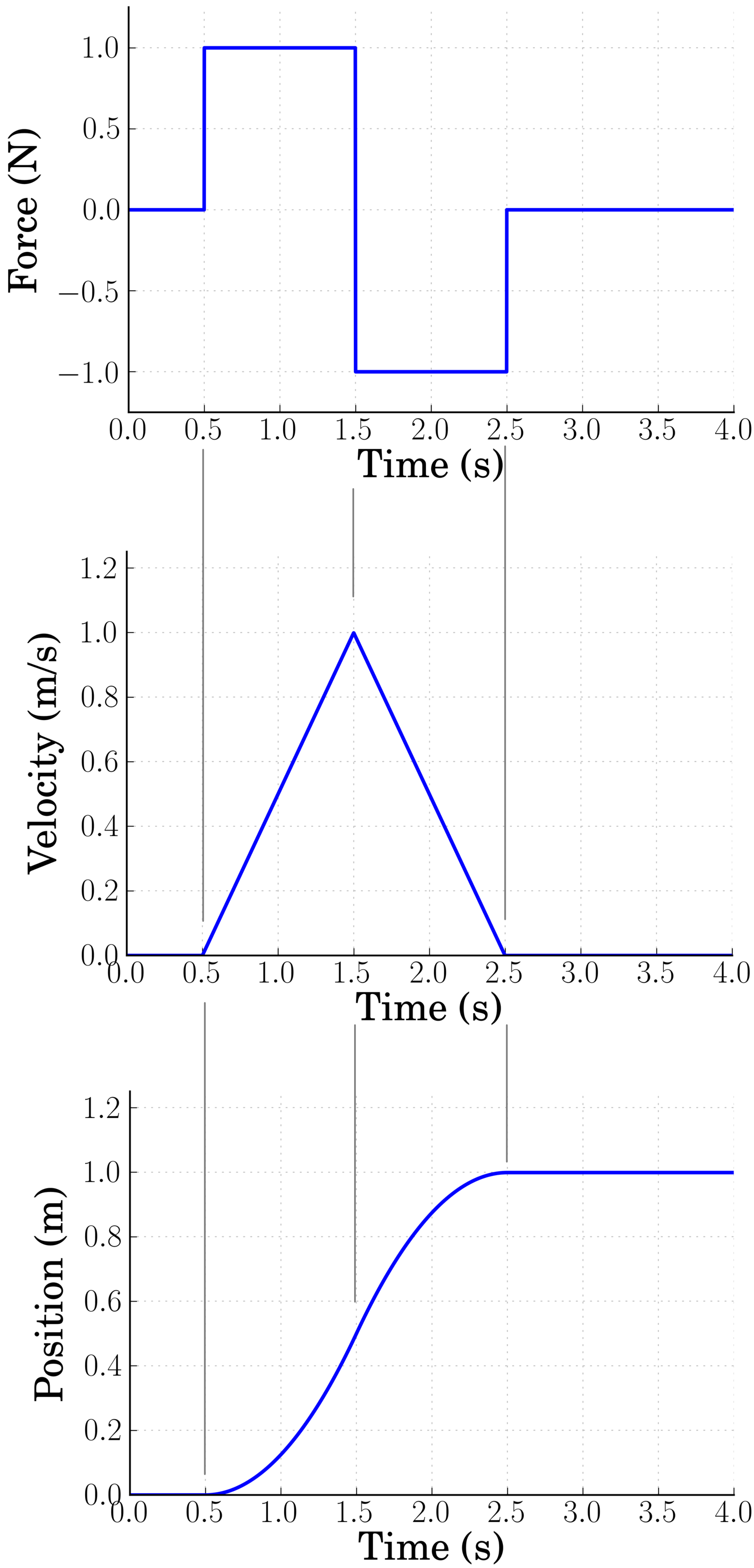
- We can recognize that system will travel some distance over intervals $t_1 \to t_2$ or $t_2 \to t_3$

So, solve integral over $t_1 \to t_2$ and $\frac{x_d}{2}$. (get halfway at t_2 and you'll get to x_d at t_3)

$$x(t) = \int_0^{t_2}\int_0^{t_2} \frac{f_{max}}{m}\, dt^2 \longrightarrow x(t) = \frac{f_{max}}{2m}(t_2)^2 \longleftarrow \text{at } t_2 \rightarrow \frac{x_d}{2} = \frac{f_{max}}{2} t_2^2 \rightarrow \boxed{t_2 = \sqrt{\frac{x_d}{\alpha}} \text{ where } \alpha = \frac{f_{max}}{m}}$$

So, knowing $t_3 = 2t_2 \rightarrow \boxed{t_3 = 2\sqrt{\frac{x_d}{\alpha}}}$

Basic Command Generation (cont.)



A bang-bang force input leads to

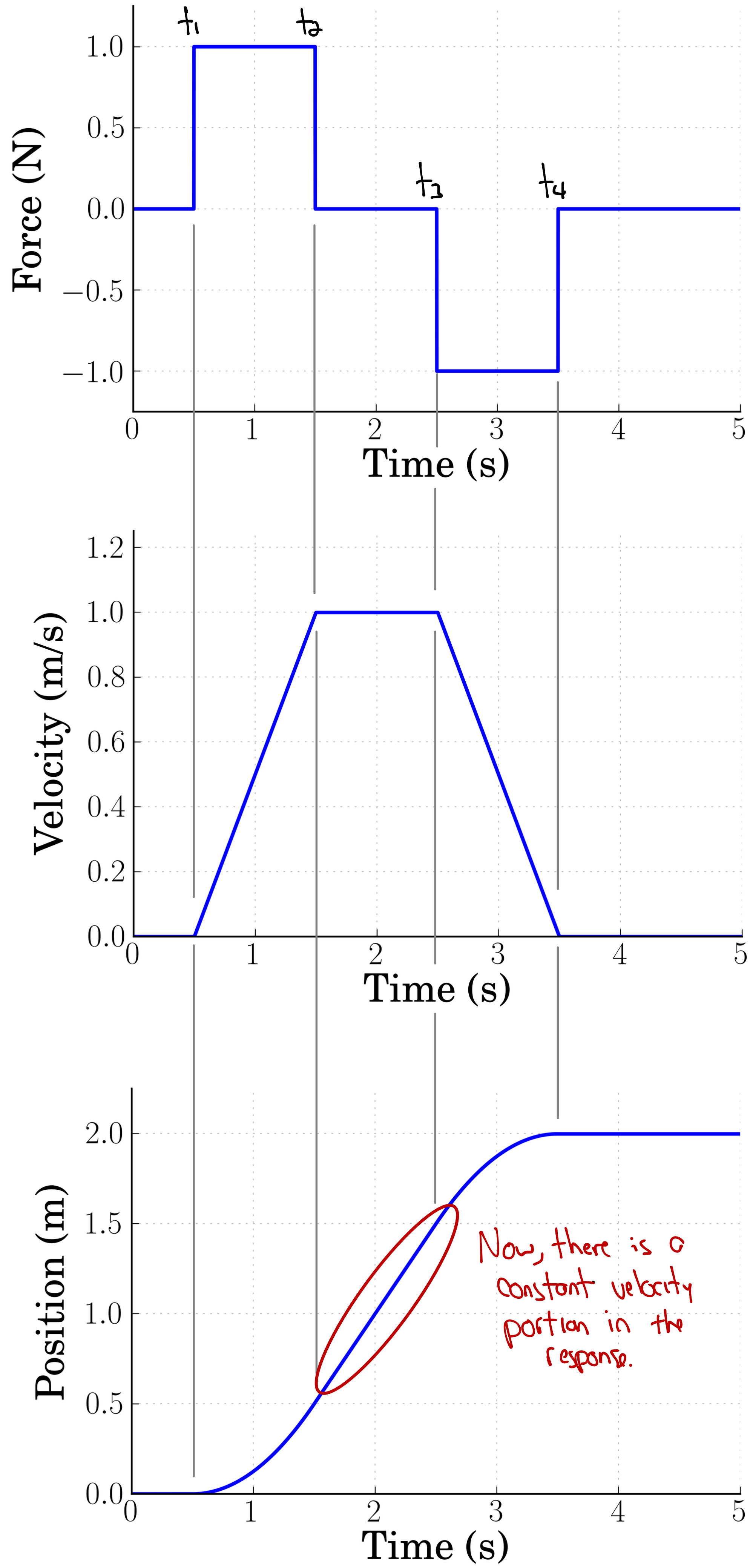
A triangle-like velocity profile

And an S-curve position response.

Basic Command Generation (cont.)

Q: What changes if the is a velocity limit?

Bang-Bang $\longrightarrow$ Bang-Coast-Bang



Now, there are 4 switch times
(can solve for switch times similar to bang-bang)

Let $t_1 = 0$, then

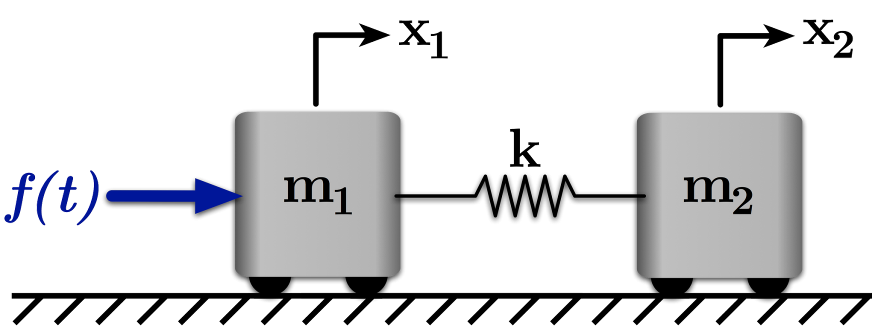
$$t_2 = \frac{V_{max}}{\alpha}$$

$$t_3 = \frac{X_d}{V_{max}}$$

$$t_4 = t_3 + t_2$$

The velocity profile becomes a trapezoidal profile.

Command Generation for Flexible Systems



Q: What should $f(t)$ be to move $x_1 \to x_{1d}$ and $x_2 \to x_{2d}$ as quickly as possible?

Model:

$$m_1 \ddot{x}_1 = f + k(x_2 - x_1)$$
$$m_2 \ddot{x}_2 = -k(x_2 - x_1)$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F \\ 0 \end{bmatrix}$$

Or, in state-space:

$$\dot{\bar{X}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k/m_1 & 0 & k/m_1 & 0 \\ 0 & 0 & 0 & 1 \\ k/m_2 & 0 & -k/m_2 & 0 \end{bmatrix} \bar{X} + \begin{bmatrix} 0 \\ 1/m_1 \\ 0 \\ 0 \end{bmatrix} f(t)$$

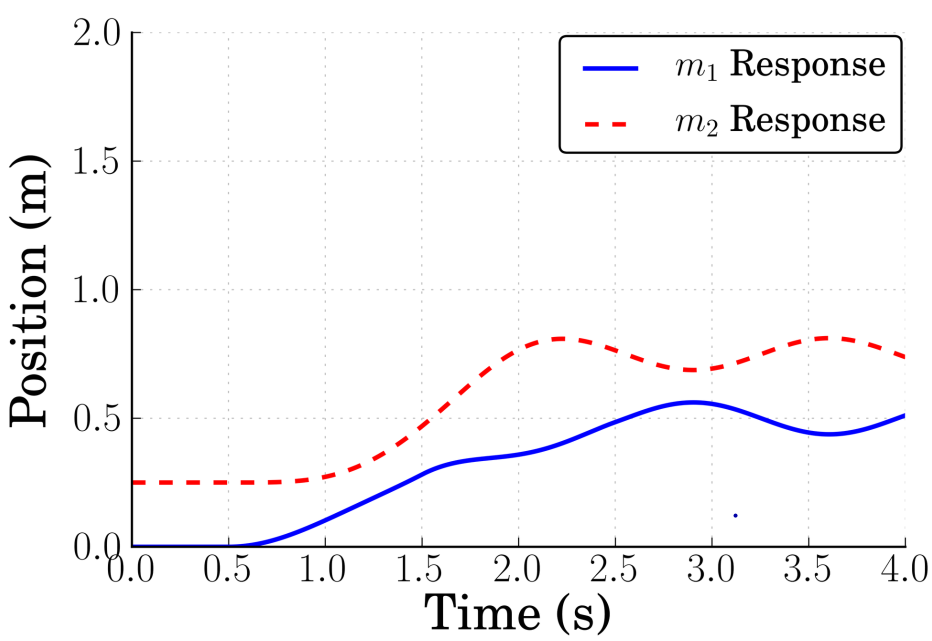
Let:

$m_1 = m_2 = 1\,kg$

$k = 10\ N/m$

$|f_{max}| = 1\,N$

just add spring and mass to previous example



Q: Problems?

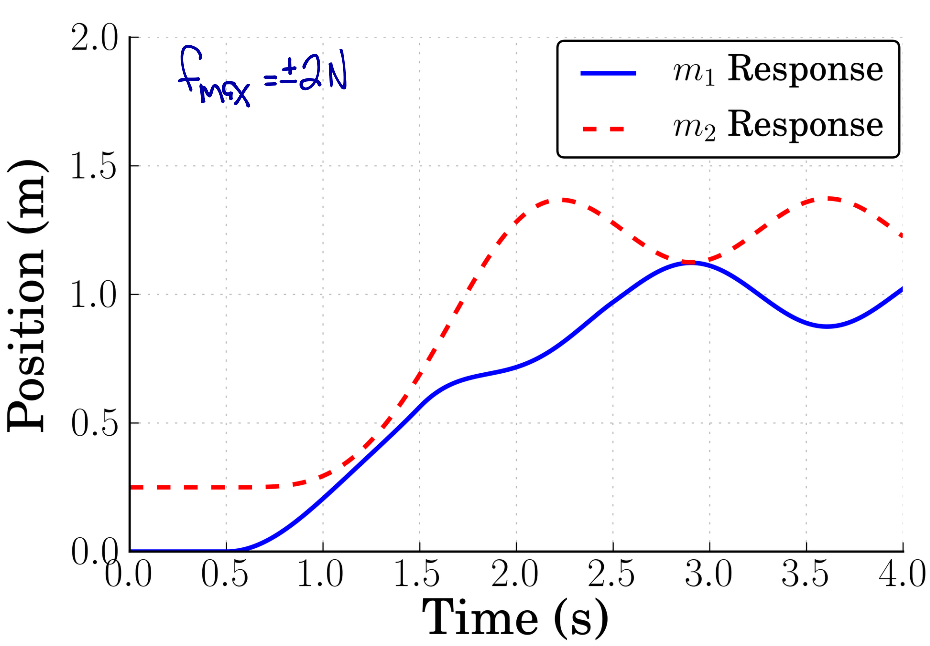
1) vibration

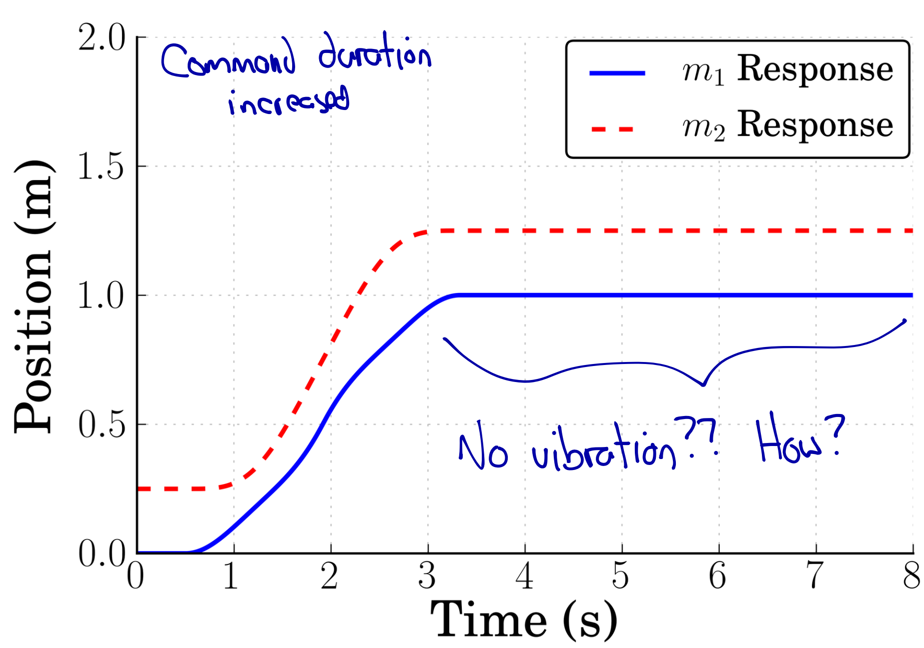
2) Never reaches the desired position (same force, 2x the mass = 1/2 as far)

Q: How can we fix?

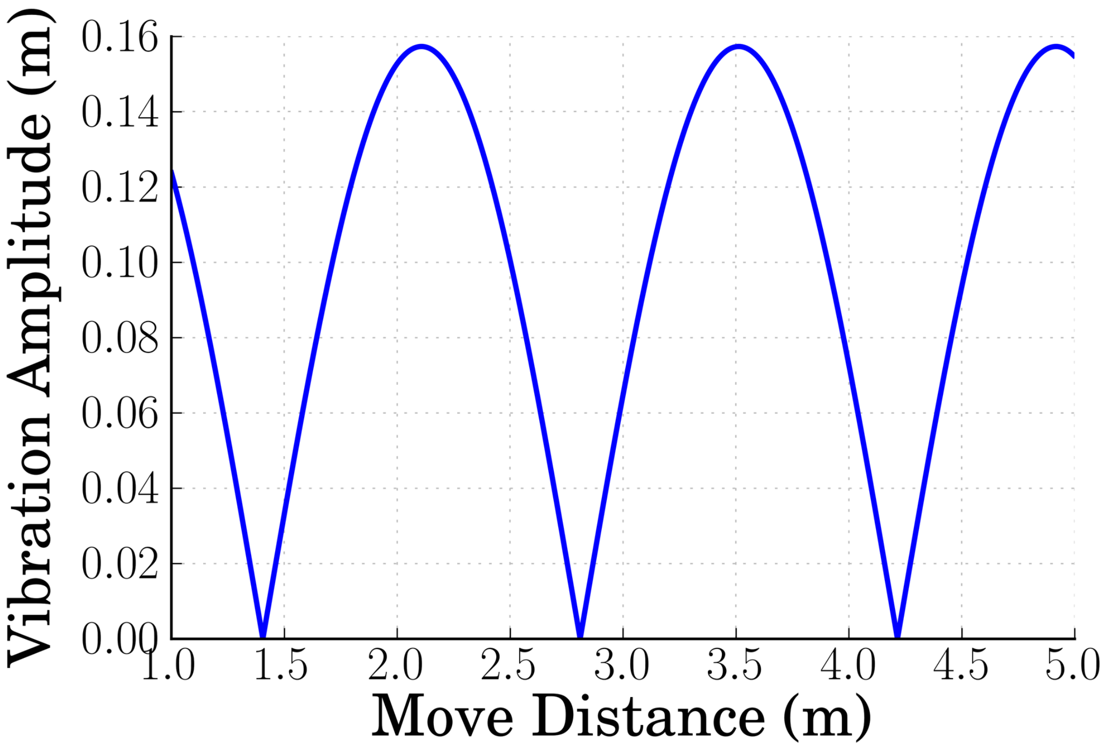
1) ... Coming next lecture...

2) Increase f_{max} or increase command duration





Vibration vs. Move Distance



As the move distance is varied the vibration amplitude changes.

Q: Why?

Varying constructive/destructive interference of accel/decel portions of the command

Q: Can we move any arbitrary distance with low vibration? How?