# MCHE 470: Robotics

Fall 2013 – Mid-term

Tuesday, October 15

Answer Key CLID: Name:

**Directions:** Complete the attached problems making sure to clearly indicate your answer, show your work, and list any assumptions that you have made (with justification for them, if necessary). If you need extra space for any question, you may attach additional sheets of paper.

#### Academic Honesty (just a reminder):

An essential rule in every class of the University is that all work for which a student will receive a grade or credit be entirely his or her own or be properly documented to indicate sources. When a student does not follow this rule, s/he is dishonest and s/he defeats the purpose of the course and undermines the goals of the University.

### Problem 1 - 15 Points

- **a.** What is the primary difference between analog and digital signals?
- **b.** How does the number of bits in an Analog-to-Digital converter (ADC) affect its precision?
- **c.** You are attempting to read an analog signal that can vary continuously between 0 and 5VDC. Using the 10bit ADC on your RedBoard, what analog signal voltage corresponds to an ADC value of 716?
- **d.** What duty-cycle is needed to produce a 3VDC equivalent via the RedBoard PWM output?
- e. Sketch this PWM output. Be sure to clearly label the axes.
- f. What are the main parameters of a serial communication packet?
- a. Analog are continuously variable. Digital signals are on/off.

b. A ADC with a higher number of bits will have a greater precision than one with a lower number of bits. The higher bits allow the analog signal to be divided into a larger number of "buckets", increasing resolution.

c. Use: 
$$\frac{\text{Resolution of ADC}}{\text{System Voltage}} = \frac{\text{ADC value}}{\text{Analog Voltage}}$$
The 10 bit RedBoord ADC has a resolution of  $2^{16} = 1024$   
Its value an range from 6-1023.  
 $\frac{1023}{5V} = \frac{716}{\text{Anolyy}} \longrightarrow \text{Analog V} = \frac{5}{1023}(716) \approx 3.5V$ 

d. The range of voltage output from the RedBoard is 0-5VDC. 3V is 60% of 5V, so the the duty cycle (the time the PWM command is "on") is also 60%.



f. Baud Rate - how fast The data - 5-9 bits usually Parity - rarely used Stop bit - 1 or 2 common

#### Problem 2 - 25 Points

- a. What two functions are needed in every Arduino sketch?
- **b.** What is the purpose of each of these functions?
- **c.** In the code in Figure 2.1:
  - i. What is the purpose of lines 18-20?
  - ii. Line 23 prints a value to the Serial Monitor. What value does it print?
  - **iii.** If you actually tried to compile and run this code, line 14 would raise a compiler error. Why?
  - **iv.** In what line could you insert a variable declaration to create a global variable?
- d. What does the built-in Arduino function pinMode() do? Why is it necessary?
- e. Write an Arduino sketch (pseudo-code is okay, but be sure to include all the elements of a proper Arduino sketch) that will loop through the even numbers between 0 and 20. For each, the code should print the square of the number to the Serial Monitor if it's less than 10 and its cube if the number is greater than or equal to 10.



Line 23 Col 21 Arduino + Unicode (UTF-8) + Unix (LF) + 1 581/55... Figure 2.1: Sample Arduino Sketch

# Problem 2

a. setup() and loop()

b. setup() contains an initialization code that must be run before the "main" program starts. It is run only once.

loop() runs continuously once setup() is finished. It is the "main" part of the sketch.

- c. i. Lines 18-20 declare variables i, j, and k, reserving memory space for them. Lines 18 and 19 also assign initial values to i and j, respectively.
  - ii. Line 23 prints 6 to the Serial Monitor.
  - iii. An error would occur because variable k has not been declared in a scope accessible to the function setup(). It is only declared in function loop().
  - iv. A global variable declaration could be inserted at line 11.

d. pinMode() defines an digital pin as input or output. It is necessary because the digital pins can function as either inputs or outputs.



## Problem 3 - 15 points

For the proposed control system in Figure 3.1:

- **a.** What is the primary purpose of each control block ( $G_{CG}$  and  $G_{FB}$ )? What is  $G_P$ ? **b.** Assume  $G_{FB}$  is a PID controller:
  - i. Briefly describe the goal of the controller.
  - **ii.** Write the form (equation) of this controller. An answer in either the time domain or the Laplace domain is acceptable.
  - iii. How can each of the gains in the PID controller be chosen?
  - iv. Complete Table 3.1 below, which summarizes the effect of increasing each of the gains on various performance measures.
- c. What are some potential problems of the PID controller?
- **d.** What is one possible implementation problem with the D portion of the controller? How can it be mitigated?



Figure 3.1: Control System Block Diagram

Gain	Rise Time	Overshoot	Settling Time	Steady-State Error
$k_p$				
$k_d$				
$k_i$				

Table 3.1: PID Gain Influence

•

a. G\_CG is the command generation block. It creates a reference signal, r(t), from the desired states of the system, yd(t).

G\_FB is a feedback controller. It acts on the error between the current system state and the reference command. Using this error, e(t), an input to the system is generated.

G\_P is the "plant". It is the actual system to be controlled.

b. i. The goal of the feedback controller is to minimize the error between the current states of the system and the reference command (which is often the desired states of the system).

$$\text{ii. } \mu(t) = k_{p}e(t) \cdot k_{i} \int edt + k_{d}\dot{e}(t)$$

$$C_{FB}(s) = k_{p} + k_{d}s + \frac{k_{i}}{s} = \frac{k_{d}s^{2} + k_{p}s + k_{i}}{s} \longrightarrow u(s) = G_{FB}(s) E(s)$$

- iii. The gains can be chosen in a variety of ways. These include:
  - trial-and-error
  - numerical optimization routines
  - Ziegler-Nichols tuning
  - Some combination of the above

iv.	Gain	Rise Time	Overshoot	Settling Time	Steady-State Error
	$k_p$	$\downarrow$	$\uparrow$	$\approx$	$\downarrow$
	$k_d$	$\approx$	$\downarrow$	$\downarrow$	=
	$k_i$	$\downarrow$	$\uparrow$	$\uparrow$	$\rightarrow 0$

- c. General problems of the PID controller include:
  - difficulty sensing the needed states (velocity is often tough to sense)
  - noise in the sensing can result in noisy inputs to the system
  - potential instability
  - incompatibility with human operators

d. The D portion of the PID controller can create "Derivative Kick" if the reference command is discontinuous (like a step command). To mitigate this effect, we can use only the derivative of the output, ignoring the input derivative.

#### Problem 4 - 15 points

The control of a satellite can be modeled as a mass acted upon by a force, as shown in Figure 4.1.

- **bang-bang a.** What force, f(t), would create a timeoptimal rest-to-rest move of this simplified satellite model? Sketch this force command on the axes provided below in Figure 4.2.
- vel -> triangle<sup>b</sup>. What would be the velocity and position pos -> s-curve responses to this force input? Sketch them on the axes in Figure 4.2.



c. How would the command, f(t), and responses change if there were a velocity limit? Sketch them on the axes in Figure 4.3.add coast segment ->



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# Problem 5 - 15 points

Consider the simple mass-spring system in Figure 5.1. In this model,  $x_d$  represents the desired location of mass m and is the input to the system.

- **a.** What problem may arise if we don't correctly choose  $x_d?$
- **b.** What are two ways (there are many more) that the vibration of x can be limited?
- c. Assuming the natural frequency is 1 Hz, design a Zero Vibration (ZV) input shaper that would



- eliminate vibration in this system if used to shape a reference command.
- d. Using the unshaped reference command shown in Figure 5.2:
  - i. Convolve your shaper from Part (c) to create the shaped input. Be sure to properly fill in the amplitudes of the two impulses and the time  $\Delta$ .
  - **ii.** Also sketch the input-shaped response on these axes, assuming the shaper is designed correctly.



Figure 5.2: The Input Shaping Process

- a. Vibration of x(t) (which can lead to many other problems).
- b. To limit vibration of x, you can:
  - Move very slow
  - Add physical damping
  - Lowpass filter the input (which is very similar to just moving slower)
  - Use input shaping

c. The form of the ZV shaper is 
$$ZV \equiv \begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \frac{1}{1+K} & \frac{K}{1+K} \\ 0 & \frac{\tau_d}{2} \end{bmatrix}$$
  
where:  $K = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$   
 $\log_{C}C \leq 0$  and  $\log_{C}C \leq 1$  and  $\log_{C}C \leq$ 

d. See plots above.

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## Problem 6 - 15 points

- **a.** Answer the following true/false statements about the design process, by *clearly* circling T or F.
  - i.  $\mathbf{T}$  (**F**) The design process is linear.
  - ii. T  $(\bar{F})$  The only customer of a design is the consumer or end-user.
  - iii. T (F) Once completed, the design tools (House of Quality, Spec. Sheet, etc.) never need to be revisited.
  - iv. T (F) A design is the best because it receives the highest total in an evaluation matrix.
  - **v.**(**T**) | **F** The function tree should contain only things that the design has to do.
- **b.** Label each part of the Problem Understanding Form (the middle of the House of Quality) in Figure 6.1.
- ${\bf c.}\,$  Create the first iteration of a House of Quality for a new energy drink.



Figure 6.1: The House of Quality

# Problem 6

	0 0	<b>Engineering Characteristics</b>						
	Importance	Caffeine Concentration	Mass of Dissolved Sugar	Container Volume	Acidity	Carbonation Level	Container Aesthetics	Base-water Quality
Customer Requirements								
Low Cost	9		٠	•		٠		•
Boosts Energy	10	•	•			٠		
Tastes good	6		•		•	•	٠	•
Easy to drink	8		•	•	•			
Good mixer	4		•		•	•		
No post-drink crash	2	•		٠	٠	•		
Is "cool"	10			٠		•	•	

# Possibly Useful Equations and Information

$$\begin{split} \bar{f} &= m\bar{a} \\ I_o\bar{\alpha} &= \sum \bar{M_o} \\ \sin(a \pm b) &= \sin(a)\cos(b) \pm \cos(a)\sin(b) \\ \cos(a \pm b) &= \cos(a)\cos(b) \mp \sin(a)\sin(b) \\ x(t) &= ae^{i\omega_n t} + be^{-i\omega_n t} \\ x(t) &= a\cos\omega_n t + b\sin\omega_n t \\ x(t) &= e^{-\zeta\omega_n t} \left[a\cos(\omega_d t) + b\sin(\omega_d t)\right] \\ e^{\pm i\omega t} &= \cos(\omega t) \pm i\sin(\omega t) \\ \int u \ dv &= uv - \int v \ du \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ \delta_{oc}V &\equiv \forall \sum \\ \begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \frac{1}{1+K} & \frac{K}{1+K} \\ 0 & \frac{\tau_d}{2} \end{bmatrix} \\ \begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \frac{1}{1+2K+K^2} & \frac{2K}{1+2K+K^2} & \frac{K^2}{1+2K+K^2} \\ 0 & \frac{\tau_d}{2} & \tau_d \end{bmatrix} \\ K &= e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \\ \omega_n &= \sqrt{\frac{k}{m}} & \frac{c}{m} = 2\zeta\omega_n \qquad f = \frac{\omega_n}{2\pi} \\ \frac{\text{Resolution of ADC}}{\text{System Voltage}} &= \frac{\text{ADC value}}{\text{Analog Voltage}} \end{split}$$

$$i \equiv \sqrt{-1}$$

$$2 + 2 = 2 \times 2 = 2^{2} = 0b0010 = 0x2$$

$$V(\omega, \zeta) \equiv e^{-\zeta \omega t_{n}} \sqrt{[C(\omega, \zeta)]^{2} + [S(\omega, \zeta)]^{2}}$$

$$C(\omega, \zeta) = \sum_{i=1}^{n} A_{i}e^{\zeta \omega t_{i}} \cos(\omega t_{i}\sqrt{1-\zeta^{2}})$$

$$S(\omega, \zeta) = \sum_{i=1}^{n} A_{i}e^{\zeta \omega t_{i}} \sin(\omega t_{i}\sqrt{1-\zeta^{2}})$$

$$x(t) = \int_{0}^{t} f(\tau)h(t-\tau)d\tau$$

$$x(t) = \int_{0}^{t} f(\tau)h(t-\tau)d\tau$$

$$f(t) = \sum_{n=0}^{\infty} a_{n}\cos(n\omega_{0}t) + \sum_{n=1}^{\infty} b_{n}\sin(n\omega_{0}t)$$

$$a_{n} = \frac{\omega_{0}}{\pi} \int_{0}^{\frac{2\pi}{\omega_{0}}} f(t)\cos(n\omega_{0}t)dt$$

$$b_{n} = \frac{\omega_{0}}{\pi} \int_{0}^{\frac{2\pi}{\omega_{0}}} f(t)\sin(n\omega_{0}t)dt$$

$$a_{0} = \frac{\omega_{0}}{2\pi} \int_{0}^{\frac{2\pi}{\omega_{0}}} f(t)dt$$

